

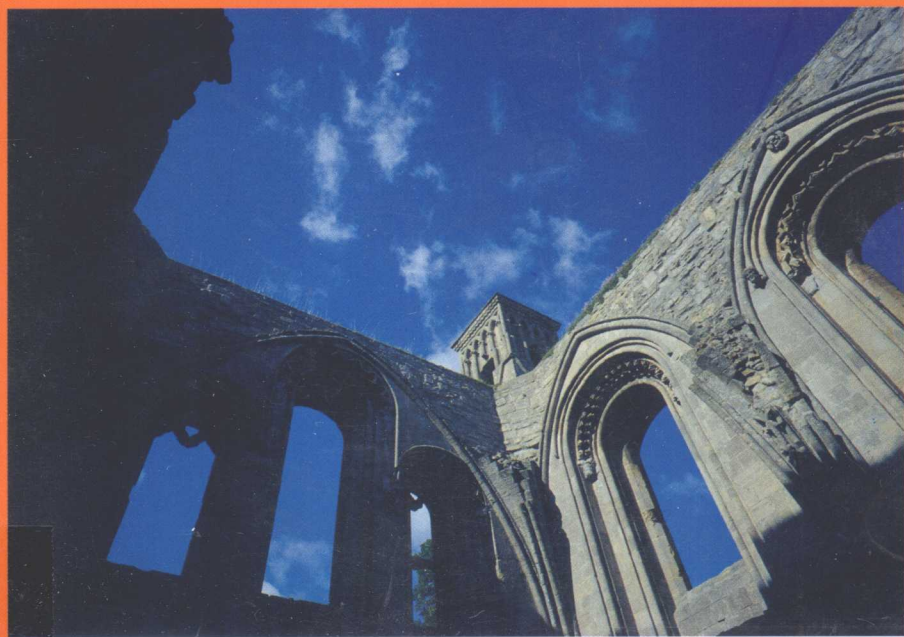
时代教育·国外高校优秀教材精选

(英文版)

# 微积分

——Maple实验教程

## Calculus with Maple Labs



(加) Wieslaw Krawcewicz 著  
(印) Bindhyachal Rai



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引进国外优秀原版教材，在有条件的学校推动开展英语授课或双语教学，自然也引进了先进的教学思想和教学方法，这对提高我国自编教材的水平，加强学生的英语实际应用能力，使我国的高等教育尽快与国际接轨，必将起到积极的推动作用。

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这套教材出版后，我们将根据各高校的双语教学计划，举办原版教材的教师培训，及时地将其推荐给各高校选用。希望高校师生在使用教材后及时反馈意见和建议，使我们更好地为教学改革服务。

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# 序

由于计算机的计算速度和存储量的惊人发展,具有强大功能的数学系统软件(如 Maple 和 Mathematica 等)的开发,使得通常的微分、积分以及代数式变形等符号运算已像数的四则运算一样,可以通过计算机轻易地实现;各种复杂的数值计算、图形绘制在计算机上也能相当简便地完成;过去由于运算太繁琐而无法解决的微积分应用问题现在也变得不那么困难。这样,传统的微积分课程的教学内容和教学方法就面临一个如何改革的问题。

对于这个问题,国内外数学教学界看法不一,做法甚多,但从基本观点看主要有两种。一种观点认为,微积分教学应降低理论要求,强调直观认识,简化运算,多用数学软件,加强实际应用。在这种观点指导下,出版了一批将 Maple 软件或 Mathematica 软件的应用与微积分教学内容相结合的教材,其中不少教材在微积分应用方面引入了许多实际例子,编写得很有特点,值得一阅。另一种观点则认为,微积分的教学内容应加强理论,特别是数学思维的训练,降低运算技巧的要求,加强实际应用。本书就是持这种观点的作者撰写的一本很有个性的教材,它是作者在 Alberta 大学进行单变量微积分课程教学的基础上逐渐形成的。作者认为,正是由于今天的数学软件的功能强大,在微积分教学应用中应该强调数学概念的理解以及数学语言与思维方式的训练,这样才能充分发挥、利用软件的功能,去解决多种复杂问题。因此这本教材理论系统严密,定理的证明比较完善;对一些基本概念和基本知识十分重视,例如实数概念与性质、极限理论及证明方法,甚至对附属的概念和运算以及多项式方程根的存在定理等都有恰当的讲述。

该教材习题相当丰富,分成 3 部分。第一部分是复习题,这些多是一般微积分教材中的常见题。第二部分是补充题,难度大一些,包括应用题、证明题等。这两部分附在每章的后面。第三部分是利用 Maple 软件来做的习题,以附录的形式冠以“实验”的名字放在全书最后,全部共有 34 个实验,汇集了全书全部章节的内容,其中有一些简单的实际应用题。

很有意思的是,本教材在主体内容的叙述之余,作者还因处置宜地引用了一些数学家的格言,相关的数学史料、趣闻和体现数学家有别于其他学者的思维方式的

笑话。这些内容会使读者在理解数学这种严肃和费脑子的材料的同时，被一些生动的故事引得捧腹大笑，这自然对提高教学效率是大有好处的。

清华大学 数学系  
谭泽光

## PREFACE

“CALCULUS with Maple Labs” is a consistent and formal exposition of a single variable calculus course for undergraduate students. This course was initially developed during the years 1998–2000 for honors students in mathematics at the University of Alberta. Yet, later it was worked out to be a general type of a calculus textbook. It was our intention to create a different type of a modern calculus textbook with an emphasis on solving complex and challenging problems, rather than only developing simple technical skills based on repetitions and multiple examples. The presently observed development of advanced computer technology opens great opportunities for use of sophisticated mathematical softwares. Even an average person can use these tools to solve mathematical problems. Such a software today can be compared to calculators that revolutionized arithmetic calculations 40 years ago. It allows a computer to do symbolic and numerical computations of all kinds. In this way, a better and more profound understanding of all the related aspects of a mathematical problem become crucial for its solving. In the same time the individual technical skills become less important and play only a secondary role. With the use of computers we can move the challenges related to calculus on a higher level, where the knowledge of various simple ‘tricks’ becomes less important and is replaced by stronger emphasis on proofs, ideas and problem solving.

For our Calculus textbook we chose one of the best mathematical software presently available on the market — the Waterloo Maple V Release 5 (this is not the most recent version of the Maple software but it is the most affordable one). The Maple system was developed by the Symbolic Computation Group at the University of Waterloo in Ontario, Canada. All students and educators wishing to acquire the Maple software should visit the web site at <http://www.maplesoft.com>. There, they will be able to obtain the information about the most recent software updates, user license conditions, special offers for Student Edition Maple, and possibly to download a demo version of Maple.

Maple is a very powerful tool, combining symbolic, numerical and graphical methods in mathematics, which is widely used by professional mathematician, scientists and engineers. The knowledge and ability of using Maple can also be very helpful in other areas of science. In particular, all those students who consider research as a career option should learn it. In Appendix 2, we included 34 Maple labs written by Dr. Liping Liu. These labs cover all the topics discussed in this textbook. They also provide the reader with a complete explanation of the practical use of the Maple and examples for the use of all its commands. They are



presented in such a way that one can use them as sample worksheets. In fact, all the labs were designed as Maple worksheets (for the Windows operating system) and can be downloaded from the web site: <http://krawcewicz.net/download>.

This textbook is different. In contrast to other similar books on calculus, which emphasize practical applications of calculus at the expense of profoundness and formality of the presentation. We give a reader complete proofs of all the presented theorems. The practice problems, instead of repeating again and again the same idea, were chosen to illustrate various important methods and techniques. They could be later used for solving other, more advanced problems. Our presentation of calculus is based on the classical approach but in modern settings. The way the original ideas were created left a strong mark on the development of the modern mathematics. We believe they are an important part of the mathematical education and culture.

Let us discuss briefly the content of our calculus course. We introduce transcendental functions at the very beginning of the textbook. This approach is motivated by the importance of these functions in all applied areas, including statistics, physics, biology, economics, engineering etc. We also believe that the notion of a sequence should be studied before the limit of a function is introduced. The most of calculus books in North America introduce sequences only at the end of the single variable course as a part of the infinite series chapter. We do not share this point of view. The notions of sequence, convergence, limit, number  $e$ , and even complex sequences, are introduced in Chapter 2. In Chapter 3 we discuss the concepts of function, limit, continuity, and elementary functions. From the practical point of view, elementary functions are functions that can be written by a (single) formula using algebraic, exponential, trigonometric and other expressions. In Chapter 3 we also present the most important properties of continuous functions with proofs. That includes the Intermediate Value Theorem, Weierstrass Theorem, and the fact that all elementary functions are continuous.

In Chapter 4, we introduce the notions of differentiability, derivative, differential, and their fundamental properties. We derive all the differentiation formulas for elementary functions. The classical Rolle's, Lagrange and Cauchy theorems are presented. In addition, we included L'Hôpital's Rule, Leibnitz Formula, Taylor's Formula and Newton's approximation method. We also discuss the concept of differentiability for complex functions and prove the Fundamental Theorem of Algebra.

Chapter 5 is devoted to the notion of indefinite integral and various integration techniques developed in a proper logical order. The first integration method is the partial fraction decomposition and integration of rational functions. Next, we derive several other integration techniques based on various reduction methods to rational integrals. Let us point out that other calculus textbooks favor reduction techniques to special types of trigonometric integrals. With such limitations it is in many cases impossible to solve many kinds of basic integrals. By providing our students with more solid background on the integration methods, we enable them to solve more advanced problems, for which the use of Maple may be unsuccessful.

In Chapter 6 we introduce the notions of definite integral, integrability, improper integral and their properties. Next, in Chapter 7 the definite integral is applied to various geometric problems related to computations of areas, volumes and surface areas. We study geometric objects involving graphs of functions, polar curves and parametric curves. In our presentation we include some cases which are not usually covered by other calculus textbooks. In Chapter 8 we study infinite series and power series, including power series representations of many elementary functions.

One important feature of this textbook is that it contains a systematic exposition of complex numbers merged into the main course. It allows students to acquire better technical skills when dealing with more advanced problems. In particular, some knowledge of complex numbers is very helpful to work with trigonometric expressions, algebraic equations and some geometric problems. By introducing the complex numbers we were able to include the proofs of the Fundamental Theorem of Algebra, Factorization Theorem and even the formulas of Cardan for solving algebraic equations of order 3 and 4.

Each chapter is supplied with an extended selection of additional review and practice problems of various order of difficulties. There are about 1,200 additional problems included in this textbook.

Our “Calculus” also provides the reader with some insight into mathematical folklore and culture. On the margin of the main course, we discuss some interesting yet controversial stories behind mathematical discoveries, history of mathematics, and biographical notes about famous mathematicians. Many anecdotes, jokes and cartoons, which are related to mathematics, were collected for this book to make its reading more enjoyable and not so stressful.

We would like to thank Dr. Dick Peter, the former dean of the Faculty of Science at the University of Alberta, for the financial support and encouragement. We are grateful to Dr. Liping Liu for writing the Maple Labs and preparing the Complete Solution Manual for all the problems included in this textbook. We also thank Zbigniew Jujka and Gabriela Novakova, for some of the cartoons presented in our textbook. We would like to express our thanks to Ivan Baggs, John Bowman, Dragos Hrimiuc, Mikhail Kovalyov, Ted Lewis, Sherman Riemanshneider, Jack Macki, and Mazi Shrivani, for their support, suggestions, comments and peer review. Special thanks go to the Department of Mathematics at the University of Allahabad and the Harish Chandra Research Institute at Allahabad, India, for the support and hospitality during the first author's visit to Allahabad in 2001, where a part of the manuscript was written. We would like to thank Rosy Line for the proof-reading of the manuscript and Karma Krawcewicz for her help in typing the manuscript and preparation of some of the figures.

Finally, we would like to pay special homage to ours wives Grazyna Krawcewicz and Madhuri Rai for their unfailing support, inspiration and the always needed encouragement.

Edmonton, Alberta, Canada  
Allahabad, Utar Pradesh, India

Wieslaw Krawcewicz  
Bindhyachal Rai

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