ST(P) Technology Today series

Mathematics for Technicians

A Series for Technicians

Level II
Building Construction Mathematics

A. Greer
& G.W. Taylor

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AUTHORS' NOTE ON THIS VOLUME

This book contains all the work needed to cover the objectives for TEC Level II Building Mathematics.

Each topic and exercise has been prefixed by one or more of the letters A, B and C. All students at Level II start by studying a General Course (items marked A). Afterwards the student takes either Practical Mathematics (items marked B) or Analytical Mathematics (items marked C).

A. Greer
G.W. Taylor

Gloucester, 1980

CONTENTS

1. TRIGONOMETRY

A GENERAL

On reaching the end of this chapter you should be able to:

1. *Define the secant, cosecant and cotangent ratios in terms of the sides of a right-angled triangle.*
2. *Determine, using tables, the secant, cosecant or cotangent of angles from 0° to 90° inclusive.*
3. *Derive the relationship* $\tan A = \dfrac{\sin A}{\cos A}$ *and* $\cot A = \dfrac{\cos A}{\sin A}$ *for a right-angled triangle.*
4. *Determine values for the six trigonometrical ratios for angles between 0° and 360°.*
5. *State the reciprocal relationships between the trigonometrical ratios for all angles.*
6. *Plot the graph of* $y = \sin A$, $y = \cos A$ *and* $y = \tan A$ *for angles between 0° and 360°.*
7. *Use the formulae for (i) sine rule (ii) cosine rule (iii) area of a triangle.*
8. *State the sine rule for a labelled triangle in the form* $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.
9. *Recognise conditions under which the sine rule can be used.*
10. *Apply the sine rule to the solution of practical problems.*
11. *State the cosine rule for a labelled triangle in the form* $a^2 = b^2+c^2-2bc \cos A$.
12. *Recognise conditions under which the cosine rule can be used.*
13. *Apply the cosine rule to the solution of practical problems.*
14. *Calculate the area of a triangle using the formulae* $\frac{1}{2}ab \sin C$ *and* $\sqrt{s(s-a)(s-b)(s-c)}$.

B PRACTICAL

On reaching the end of this chapter you should be able to:

1. *Relate lengths and areas on an inclined plane to corresponding lengths and areas on plan.*
2. *Solve problems, arising from practical situations, involving trigonometry.*
3. *Plot the graphs of* $\sin A$, $\sin 2A$, $\cos A$, $\cos 2A$, $\sin \frac{1}{2}A$ *and* $\cos \frac{1}{2}A$ *for values of A between 0° and 360°.*
4. *Sketch graphs of* $\sin 3A$, $\sin^2 A$, $\cos^2 A$, $2\sin A$ *and* $2\cos A$ *for values of A between 0° and 360°.*
5. *Determine graphically the single wave resulting from a combination of waves within the limitations of 3 and 4.*
6. *Identify amplitude and frequency.*
7. *Derive the following relationships for angles up to 90°:* $\sin^2 A+\cos^2 A = 1$; $\sec^2 A = 1+\tan^2 A$.

C ANALYTICAL

On reaching the end of this chapter you should be able to:

1. *Derive the following relationships for angles up to 90°:*
$$\sin^2 A+\cos^2 A = 1$$
$$\tan^2 A+1 = \sec^2 A$$
$$\cot^2 A+1 = \operatorname{cosec}^2 A$$
2. *Solve equations of the type:*
$$a\sin^2 A + b\sin A + c = 0$$
for values of A between 0° and 360° inclusive; for cases (i) $a = 0$, *(ii)* $b = 0$, *(iii)* a, b, c, *all non zero.*
3. *Solve equations using the identities to reduce equations to the form shown in 2.*
4. *Determine the single wave resulting from a combination of two waves of the same frequency using phasors and a graphical method.*
5. *Solve problems on triangles and quadrilaterals involving the use of the sine rule, cosine rule and formulae for the area of a triangle.*

A TRIGONOMETRICAL RATIOS

Consider any angle θ which is bounded by the lines OA and OB as shown in Fig. 1.1. Take any point P on the boundary line OB. From P draw the line PM perpendicular to the other boundary line OA to meet OA at the point M. Then:

the ratio $\frac{MP}{OP}$ is called the *sine* of the angle AOB

the ratio $\frac{OM}{OP}$ is called the *cosine* of the angle AOB

and the ratio $\frac{MP}{OM}$ is called the *tangent* of the angle AOB

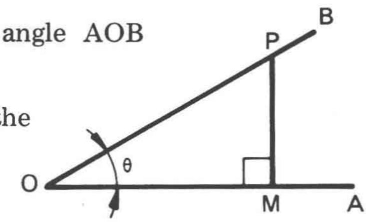

Fig. 1.1

A THE SINE OF AN ANGLE

In any right-angled triangle (Fig. 1.2)

$$\text{the sine of an angle} = \frac{\text{side opposite the angle}}{\text{hypotenuse}}$$

$\therefore$ $$\sin A = \frac{BC}{AC}$$

or $$\sin C = \frac{AB}{AC}$$

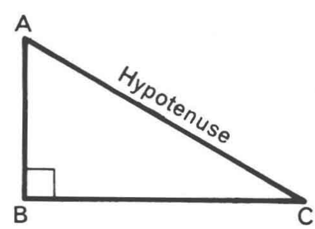

Fig. 1.2

The abbreviation 'sin' is usually used for 'sine'.

EXAMPLE 1

Find the length of the side AB in Fig. 1.3.

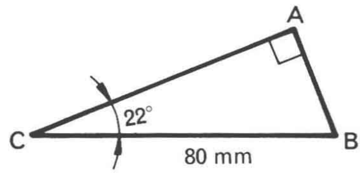

Fig. 1.3

AB is the side opposite to $\angle ACB$.
BC is the hypotenuse since it is opposite to the right angle.

Thus $$\frac{AB}{BC} = \sin 22^\circ$$

$$\therefore \quad AB = BC \times \sin 22^\circ = 80 \times 0.3746 = 29.97 \text{ mm}$$

EXAMPLE 2

Find the length of the side AB in Fig. 1.4.

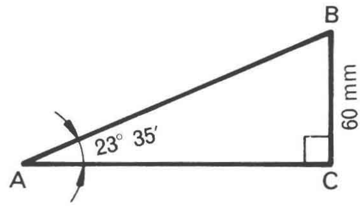

Fig. 1.4

Now $$\frac{BC}{AB} = \sin 23^\circ 35'$$

or $$BC = AB \times \sin 23^\circ 35'$$

or $$AB = \frac{BC}{\sin 23^\circ 35'} = \frac{60}{0.4000} = 150 \text{ mm}$$

EXAMPLE 3

Find the angles A and B in the triangle ABC which is shown in Fig. 1.5.

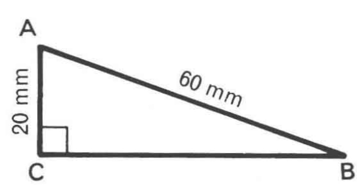

Fig. 1.5

Now $$\sin B = \frac{AC}{AB} = \frac{20}{60} = 0.3333$$

and from the sine tables

$$B = 19^\circ 28'$$

$$A = 90^\circ - 19^\circ 28' = 70^\circ 32'$$

A THE COSINE OF AN ANGLE

In any right-angled triangle (Fig. 1.6)

$$\text{the cosine of an angle} = \frac{\text{side adjacent to the angle}}{\text{hypotenuse}}$$

$\therefore$ $$\cos A = \frac{AB}{AC}$$

or $$\cos C = \frac{BC}{AC}$$

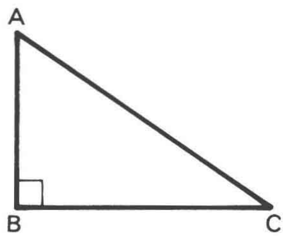

Fig. 1.6

The abbreviation 'cos' is usually used for 'cosine'.

EXAMPLE 4

Find the length of the side BC in Fig. 1.7.

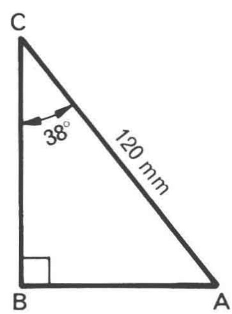

Fig. 1.7

Now BC is the side adjacent to $\angle BCA$ and AC is the hypotenuse.

Thus $$\frac{BC}{AC} = \cos 38°$$

$\therefore$ $$BC = AC \times \cos 38° = 120 \times 0.7880 = 94.56 \text{ mm}$$

EXAMPLE 5

Find the length of the side AC in Fig. 1.8.

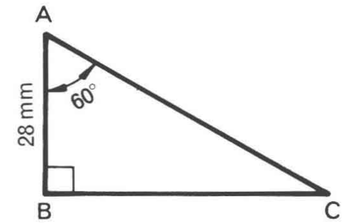

Fig. 1.8

Now $\frac{AB}{AC} = \cos 60°$

or $AB = AC \times \cos 60°$

$\therefore$ $AC = \frac{AB}{\cos 60°} = \frac{28}{0.5000} = 56 \text{ mm}$

EXAMPLE 6

Find the angle θ shown in Fig. 1.9.

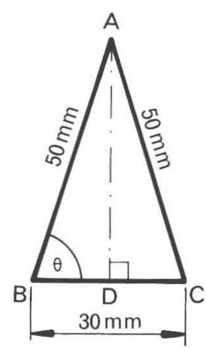

Fig. 1.9

Since triangle ABC is isosceles the perpendicular AD bisects the base BC and hence BD = 15 mm. Hence from $\triangle ABD$

we have $\cos\theta = \frac{BD}{AB} = \frac{15}{50} = 0.3$

$\therefore$ $\theta = 72° 32'$

A THE TANGENT OF AN ANGLE

In any right-angled triangle (Fig. 1.10)

$$\text{the tangent of an angle} = \frac{\text{side opposite to the angle}}{\text{side adjacent to the angle}}$$

$\therefore$ $$\tan A = \frac{BC}{AB}$$

or $$\tan C = \frac{AB}{BC}$$

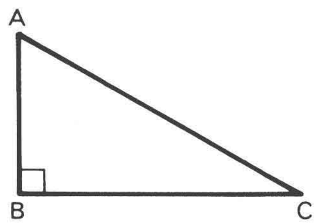

Fig. 1.10

The abbreviation 'tan' is usually used for 'tangent'.

EXAMPLE 7

Find the length of the side AB in Fig. 1.11.

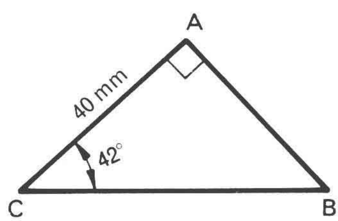

Fig. 1.11

Now $\tan 42^\circ = \dfrac{AB}{40}$

$\therefore$ $$AB = 40 \times \tan 42^\circ = 40 \times 0.9004 = 36.02\,\text{mm}$$

EXAMPLE 8

Find the length of the side BC in Fig. 1.12.

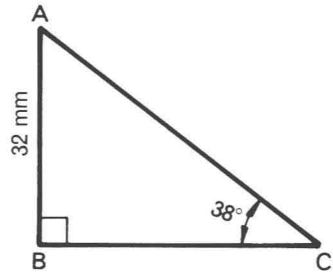

Fig. 1.12

There are two ways of doing this problem.

a) Now $\frac{AB}{BC} = \tan 38°$

$\therefore \qquad BC = \frac{AB}{\tan 38°} \quad \frac{32}{0.7813} = 40.96 \text{ mm}$

b) Since $C = 38°$, $A = 90° - 38° = 52°$

Now $\qquad \frac{BC}{AB} = \tan 52°$

$BC = AB \times \tan 52° = 32 \times 1.280 = 40.96 \text{ mm}$

Both methods produce the same answer but method (**b**) is better. Wherever possible the ratios should be arranged so that the quantity to be found is the numerator of the ratio.

A Exercise 1

1) Find the lengths of the sides marked x in Fig. 1.13, the triangles being right angled.

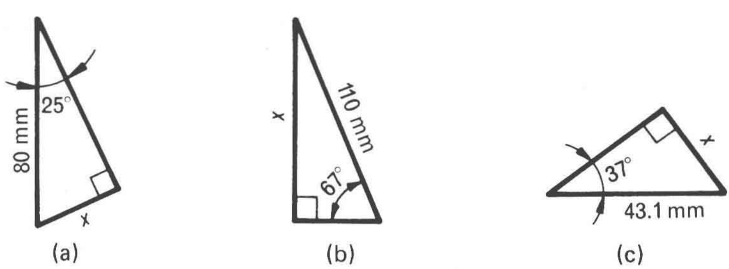

Fig. 1.13

2) Find the angles marked θ in Fig. 1.14, the triangles being right angled.

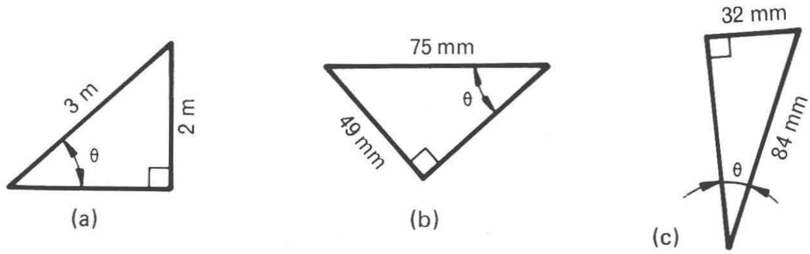

Fig. 1.14

3) An equilateral triangle has an altitude of 187 mm. Find the length of the equal sides.

4) Find the altitude of an isosceles triangle whose vertex angle is 38° and whose equal sides are 7.9 m long.

5) The equal sides of an isosceles triangle are each 270 mm long, and the altitude is 190 mm. Find the angles of the triangle.

6) An isosceles trapezium is shown in Fig. 1.15. Find the length of the equal sides.

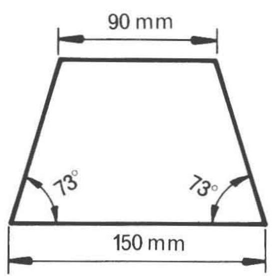

Fig. 1.15

7) Find the lengths of the sides marked x in Fig. 1.16 the triangles being right angled.

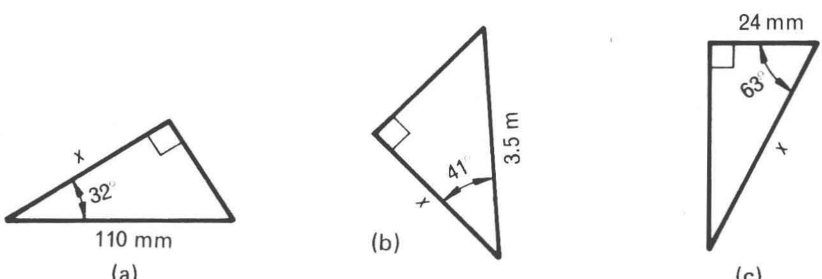

Fig. 1.16

8) Find the angled marked θ in Fig. 1.17, the triangles being right angled.

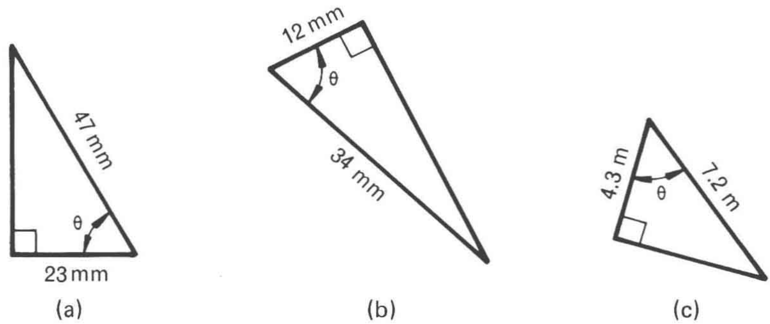

Fig. 1.17

9) An isosceles triangle has a base of 34 mm and the equal sides are each 42 mm long. Find the angles and the altitude of the triangle.

10) In Fig. 1.18 calculate $\angle BAC$ and the length BC.

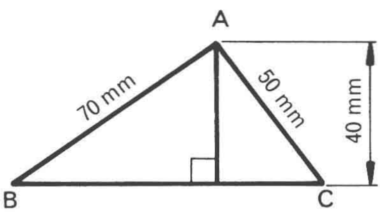

Fig. 1.18

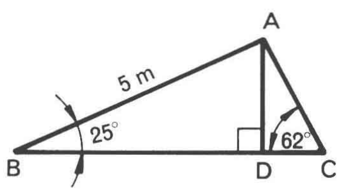

Fig. 1.19

11) In Fig. 1.19 calculate:

(a) BD (b) AD (c) AC (d) BC

12) Find the lengths of the sides marked y in Fig. 1.20, the triangles being right angled.

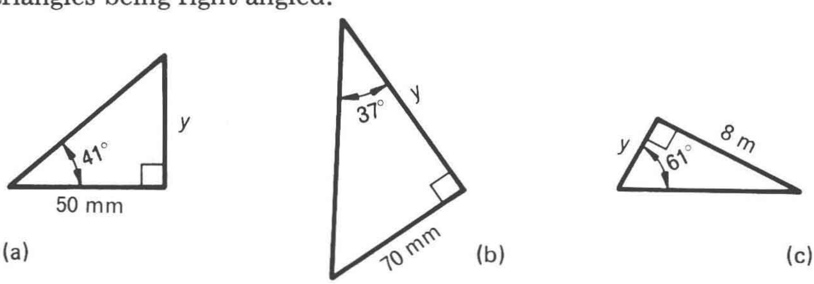

Fig. 1.20

13) Find the angles marked α in Fig. 1.21, the triangles being right angled.

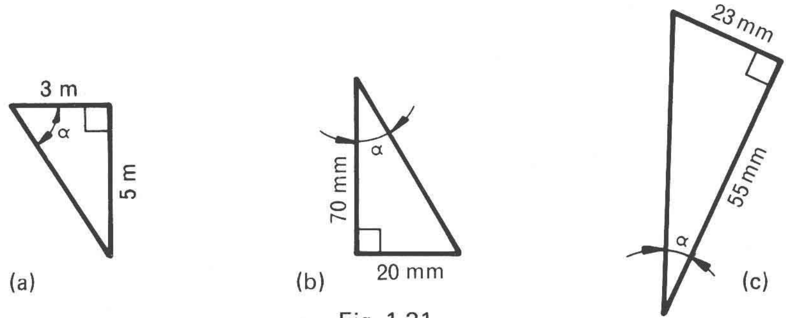

Fig. 1.21

14) An isosceles triangle has a base 100 mm long and the two equal angles are each $57°$. Calculate the height of the triangle.

15) Calculate the distance l in Fig. 1.22.

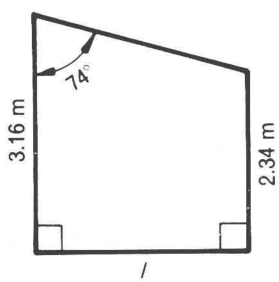

Fig. 1.22

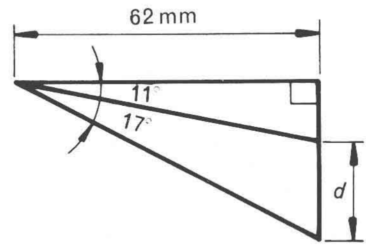

Fig. 1.23

16) Calculate the length d in Fig. 1.23.

A THE STANDARD NOTATION FOR A TRIANGLE

In △ABC (Fig. 1.24) the angles are denoted by the capital letters as shown in the diagram. The side a lies opposite the angle A, the side b opposite the angle B and the side c opposite the angle C. This is the standard notation for a triangle.

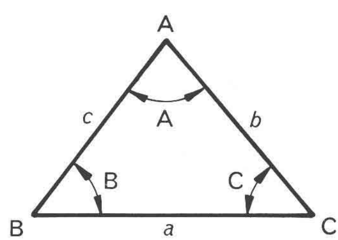

Fig. 1.24