NATIONAL SOCIETY FOR THE STUDY OF EDUCATION

TWENTY-NINTH YEARBOOK

REPORT OF THE

SOCIETY'S COMMITTEE

ON_ARITHMETIC

PUBLIC SCHOOL PUBLISHING COMPANY BLOOMINGTON, ILLINOIS 1930

TWENTY- NINTH YEARBOOK

OF THE

NATIONAL SOCIETY FOR THE STUDY OF EDUCATION

REPORT OF THE SOCIETY'S COMMITTEE ON ARITHMETIC

PART I. SOME ASPECTS OF MODERN THOUGHT ON ARIPHMETIC PART II. RESEARCH IN ARITHMETIC.

Prepared by the Society's Committee

W. A. Brownell, B. R. Buckingham, G. T. Buswell, C. E. Greene,
 R. L. West, and F. B. Knight (Chairman)

Assisted by the Following Active Members of the Society

E. A. Beito, J. C. Brown, L. J. Brueckner, J. R. Clark, W. F. Dearborn, Arthur Edwards, H. L. Harap, Ernest Horn, C. H. Judd, Fred Kelly, L. A. King, R. H. Lane, Josephine MacLatchy, C. R. Mead, R. L. Morton, Elma A. Neal, G. M. Norem, W. J. Osburn, J. R. Overman, Isidoro Panlasigui, Harriet E. Peet, F. G. Pickell, A. C. Repp, G. M. Ruch, C. W. Stone, Florence Stratemeyer, and C. W. Washburne

Edited by
GUY MONTROSE WHIPPLE

THIS YEARBOOK WILL BE DISCUSSED AT THE ATLANTIC CITY MEETING OF THE NATIONAL SOCIETY, FEBRUARY 22 AND 25, 1930, 8:00 P.M.

PUBLIC SCHOOL PUBLISHING COMPANY BLOOMINGTON, ILLINOIS 1930

OFFICERS OF THE SOCIETY

for 1930

Board of Directors

(Term of office expires March 1 of the year indicated)

WILLIAM C. BAGLEY (1931)
Teachers College, Columbia University, New York City

W. W. CHARTERS (1932) Ohio State University, Columbus, Ohio

Frank N. Freeman (1932)*
University of Chicago, Chicago, Illinois

Ernest Horn (1933) State University of Iowa, Iowa City, Iowa

CHARLES H. JUDD (1931) University of Chicago, Chicago, Illinois

LEONARD V. Koos, *Chairman* (1933) University of Chicago, Chicago, Illinois

Guy Montrose Whipple (Ex-officio) Danvers, Massachusetts

Secretary-Treasurer
GUY MONTROSE WHIPPLE (1932)
Danvers, Massachusetts

^{*}Elected to fill the vacancy caused by the resignation of Lewis M. Terman, December 20, 1929.

MEMBERSHIP OF THE SOCIETY'S YEARBOOK COMMITTEE ON ARITHMETIC

- W. A. Brownell, Professor of Educational Psychology, George Peabody College, Nashville, Tennessee.
- B. R. Buckingham, Lecturer, Graduate School of Education, Harvard University, Cambridge, Massachusetts.
- G. T. Buswell, Professor of Educational Psychology, School of Education, University of Chicago, Chicago, Illinois.
- C. E. Greene, Director of Research, Denver Public Schools, Denver, Colorado.
- F. B. Knight (Chairman), Professor of Educational Psychology, College of Education, State University of Iowa, Iowa City, Iowa.
- R. L. West, Director of Teacher Training, State Department of Public Instruction, Trenton, New Jersey.

ACTIVE MEMBERS OF THE SOCIETY ASSISTING THE COMMITTEE IN PART II

- E. A. Beito (Recently), Mechanics Arts High School, St. Paul, Minnesota.
- L. J. Brueckner, University of Minnesota, Minneapolis, Minnesota.

W. F. Dearborn, Harvard University, Cambridge, Massachusetts.

ARTHUR EDWARDS, Superintendent of Schools, Geneva, Iowa.

FRED KELLY, Gettysburg College, Gettysburg, Pennsylvania.

JOSEPHINE MACLATCHY, Ohio State University, Columbus, Ohio.

C. D. MEAD, University of California, Berkeley, California.

G. M. Norem, State Normal School, Minot, North Dakota.

ISIDORO PANLASIGUI, University of the Philippines, Manila, Philippine Islands.

HARRIET E. PEET, Harvard University, Cambridge, Massachusetts.

A. C. Repp, State Teachers College, Flagstaff, Arizona.

G. M. Ruch, University of California, Berkeley, California.

C. W. STONE, State College of Washington, Pullman, Washington.

FLORENCE STRATEMEYER, Teachers College, Columbia University, New York, New York.

C. W. WASHBURNE, Superintendent of Schools, Winnetka, Illinois.

ACTIVE MEMBERS OF THE SOCIETY SERVING ON THE SPECIAL REVIEWING COMMITTEE

- J. C. Brown, Superintendent of Schools, Pelham, New York.
- L. J. Brueckner, University of Minnesota, Minneapolis, Minnesota.
- J. R. CLARK, Teachers College, Columbia University, New York, N. Y.
- H. L. HARAP, Cleveland School of Education, Cleveland, Ohio.

ERNEST HORN, State University of Iowa, Iowa City, Iowa.

C. H. Judd, University of Chicago, Chicago, Illinois.

L. A. King, University of Pennsylvania, Philadelphia, Pennsylvania.

R. H. LANE, Deputy Superintendent of Schools, Los Angeles, California.

WORTH McClure, State Teachers College, Buffalo, New York.

R. L. Morton, Ohio University, Athens, Ohio.

ELMA A. NEAL, Director of Elementary Education, San Antonio, Texas.

W. J. OSBURN, State Department of Public Instruction, Columbus, Ohio.

J. R. OVERMAN, State College, Bowling Green, Ohio.

F. G. PICKELL, Superintendent of Schools, Montclair, New Jersey.

G. M. Ruch, University of California, Berkeley, California.

EDITOR'S PREFACE

Mindful of the cordial reception accorded the Yearbook on Reading in 1925, the Board of Directors has for several years cherished the idea of issuing eventually a series of yearbooks dealing with the other basic subjects of the elementary school. At the Dallas meeting of the Directors, in February, 1927, it was voted to put two hundred fifty dollars at the disposal of Directors Horn and Judd for the purpose of arranging conferences between them and Professors G. T. Buswell and F. B. Knight to discuss the possibilities of a yearbook on arithmetic, to be published not later than 1930. At the Boston meeting of the Directors in February, 1928, Professor Knight was appointed Chairman of a Yearbook Committee on Arithmetic and was granted a preliminary appropriation of twelve hundred dollars to cover the first year's work on this undertaking. At the Rochester meeting of the Directors in November, 1928, the personnel of the Arithmetic Committee formally presented by Professor Knight was unanimously endorsed, as follows: B. R. Buckingham (Harvard University), G. T. Buswell (Chicago University), J. R. Clark (New York University), C. E. Greene (Denver Public Schools), R. L. West (State Department of Instruction, New Jersey), and F. B. Knight (State University of Iowa), chairman. Following the subsequent resignation of Professor Clark, the Directors approved at their Cleveland meeting in February, 1929, the addition to the Committee of W. A. Brownell (George Peabody College for Teachers). At their Chicago meeting in May, 1929, the Directors placed at the disposal of the Committee for the completion of its work the additional sum of twelve hundred dollars.

In all, the Society has expended through its Committee approximately two thousand dollars in preparing and bringing together in manuscript form the material of this volume. It is a fortunate thing for the members of the Society that its publications have acquired prestige and influence such that men of the professional caliber represented in these pages are willing to devote uncommon amounts of time and energy to these yearbooks with no reward other than the approval of their colleagues. As one of the contributors remarked: "If I were asked to set a price upon what I have done as my share of this book on arithmetic, I would be entirely reasonable if I set the figure at five thousand dollars."

The essential features of the treatment of arithmetic in this Yearbook are so fully set forth in the chapters introducing Part I and Part II and in the summaries of the reviewers embodied in the Appendix that they need not be repeated here. It may be well to mention here, however, that the Committee has eliminated from the Yearbook a considerable amount of material of undoubted value consisting of comparisons between well-known textbooks in arithmetic with respect to the amount of space devoted to various aspects of instruction and drill. This elimination arose from the apprehension of the Committee that such contributions might be construed as propaganda. however carefully the identity of the texts was concealed. Again, the Committee decided for several reasons to refrain from recommending specific details to be observed in the construction of textbooks. workbooks, and drill exercises or in the conduct of classroom teaching. It may be felt that these omissions have reduced the practical usefulness of the Yearbook to the classroom teacher, but they have surely been more than compensated by the wealth of stimulating suggestions that the Yearbook contains for arithmetic supervisors, textbook writers. and professional students of arithmetic in general.

Another feature of the Yearbook worthy of mention is the incorporation along with the report of the Arithmetic Committee of another report made by a Reviewing Committee whose personnel differs from that of the Committee that produced the volume. The plan under which these reviewers operated is explained in the Appendix and in the editor's prefatory note to it.

G. M. WHIPPLE.

TABLE OF CONTENTS

Едітон	R'S PREFACE	PAGE IX
	PART I. SOME ASPECTS OF MODERN THOUGHT ON ARITHMETIC	
I.	Introduction	1
II.	THE SOCIAL VALUE OF ARITHMETIC. B. R. BUCKINGHAM	9
III.	THE ARITHMETIC CURRICULUM	65
IV.	Some Considerations of Method	145
V.	TESTING, DIAGNOSIS, AND REMEDIAL WORK IN ARITHMETIC CHARLES E. GREENE AND G. T. BUSWELL	269
VI.	THE TRAINING OF TEACHERS OF ARITHMETIC B. R. BUCKINGHAM	319
	PART II. RESEARCH IN ARITHMETIC	
I.	THE PURPOSE AND PLAN OF PART TWO	411
II.	THE TECHNIQUES OF RESEARCH EMPLOYED IN ARITHMETIC. W. A. BROWNELL	415
III.	A CRITICAL SURVEY OF PREVIOUS RESEARCH IN ARITHMETIC G. T. BUSWELL	445
IV.	THE NUMBER ABILITIES OF CHILDREN WHEN THEY ENTER GRADE ONE B. R. BUCKINGHAM, Harvard University, Cambridge, Massachusetts, and Josephine MacLatchy, Ohio State University, Columbus, Ohio	473
V.	A CRITICAL EVALUATION OF METHODS OF ANALYZING PRACTICE IN FRACTIONS LEO J. BRUECKNER, University of Minnesota, Minneapolis, Minnesota, and Fred Kelly, Gettysburg College, Gettysburg, Pennsylvania	525
VI.	MIXED VERSUS ISOLATED DRILL ORGANIZATION	535
VII.	THE LEARNING OF THE ONE HUNDRED MULTIPLICATION COMBINATIONS G. M. NOREM, State Normal School, Minot, North Dakota, and F. B. KNIGHT, State University of Iowa, Iowa City, Iowa	551

vii

CHAP.	PAGE	
VIII. A MEASUREMENT OF TRANSFER IN THE LEARNING OF NUMBER COMBINATIONS	569	
E. A. Beito, Mechanic Arts High School, St. Paul, Minnesota, and Leo J. Brueckner, University of Minnesota, Minnesota		
IX. AN EXPERIMENTAL STUDY IN IMPROVING ABILITY TO REASON IN ARITHMETIC. C. W. STONE, State College of Washington, Pullman, Washington	589	
X. A TEST IN ARITHMETIC FOR MEASURING GENERAL ABILITY OF PUPILS IN THE FIRST SIX GRADES HARRIET E. PEET and WALTER F. DEARBORN, Harvard University, Cambridge, Massachusetts	601	
XI. THE EFFECT OF AWARENESS OF SUCCESS OR FAILURE ISIDORO PANLASIGUI, University of the Philippines, Manila, Philippine Islands, and F. B. KNIGHT, State University of Iowa, Iowa City, Iowa	611	
XII. A STUDY OF ERRORS IN PERCENTAGE	621	
XIII. THE GRADE PLACEMENT OF ARITHMETIC TOPICS CARLETON W. WASHBURNE, Superintendent of Schools, Winnetka, Illinois	641	
XIV. A REVIEW OF EXPERIMENTS ON SUBTRACTION	671	
APPENDIX: A CRITIQUE OF THE YEARBOOK	681	
Constitution of the National Society for the Study of Education.		
Minutes of the Cleveland Meeting of the Society		
Synopsis of the Proceedings of the Board of Directors During 1929		
Audit of the Treasurer of the Society for 1928		
List of Honorary and Active Members of the Society		
Information Concerning the Society		
List of the Publications of the Society	.747	

CHAPTER I

INTRODUCTION

I. PURPOSES OF THE YEARBOOK

In this Twenty-Ninth Yearbook of the Society the attempt has been made to discuss certain crucial problems of arithmetic in such a way as to bring out the theoretical aspects in so far as our present knowledge permits, and also to bring out numerous practical applications which, we trust, will make the discussions useful to supervisors and teachers. Our attempt has been facilitated by certain circumstances among which the following seem outstanding: first, the Arithmetic Committee, in ten full days of conference, secured the manifest advantage of frank exchange of opinion; second, the widespread interest in arithmetic made available for our use a large body of written discussion; third, a reviewing committee received most of the material in galley form in time to prepare a concluding critical chapter, so that this Yearbook contains not only discussions by the committee, but also a deliberate appraisal of these discussions by an independent and competent group of students of arithmetic.

II. SOME LIMITATIONS

There are certain omissions in this volume which will be recognized immediately by the reader. He will note, for instance, the absence or the scantiness of discussions directed to several important aspects of arithmetic. Thus, problems and problem-solving receive but a partial discussion, because lack of rigorous data permits no thorough and penetrating discussion at present, and in general, the main reason for our neglect of certain issues which stand in obvious need of treatment is the simple fact that there is at present an insufficient amount of consistent scientific findings to furnish a satisfactory basis for such treatments. It seemed better to remain silent when discussion could go little beyond the limits of personal opinion.

A second reason for certain omissions arises, in some instances, from a decision not to proceed beyond general principles to definite and specific suggestions and detailed directions for classroom procedure. Several considerations led to this decision. In some cases sufficiently

close agreement on details could not be reached to present them without, at the same time, presenting a minority opinion. Too frequent expression of minority opinion becomes confusing to the reader. On the whole, the Yearbook is the expression of unanimous agreement of the Committee. While, of course, each member of the Committee has mental reservations about occasional paragraphs in the several chapters (on the score of pertinency as well as probable truth), the Yearbook as a whole represents the judgment of the entire Committee in the interpretation of available data. An exception to this is, of course, the concluding chapter, which is the appraisal of an independent reviewing committee.

Another factor influencing our decision to omit specifics was the desire of members of the Committee to avoid the promotion of their own particular interests in arithmetic, which can be found in print elsewhere. The Committee believes that the Yearbook does not propagandize any particular course of study, textbook, or type of instruction. In leaning backward in its effort to accomplish this purpose, the Committee may have omitted specific suggestions which would prove useful to the supervisor or to the teacher. The student may, however, by using the general points of view presented herein, be enabled to evaluate specific methods found in materials published elsewhere.

III. THE EDUCATIONAL PHILOSOPHY UNDERLYING THE YEARBOOK

Some readers may feel that this Yearbook is too conservative, that it lacks a bold and daring spirit of progressiveness. There has been a conscious attempt to avoid the urging of any point of view not supported by considerable scientific fact. It has seemed preferable to proceed slowly and on sure ground, to be content with sane and moderate progress, rather than to expound a theory of instruction which, though supported by fine hopes and splendid aspirations, has as yet no basis in objective data. The justification of the contention of those who feel that the spirit of the Yearbook is not progressive will hinge on the definition given to 'progressive.' The changes we have supported are changes which can be made and held, changes based on a sober psychology of learning and of human nature, rather than extravagant changes based on a psychology which in its enthusiasm stubbornly refuses to view all the factors involved. The Committee feels that the treatment of arithmetic in this Yearbook is

as progressive as modern thinking, courses of study, textbook construction, and scientific experimentation give it a right to be, and much more progressive than much of present classroom practice.

The spirit in which this Yearbook was written, while not extreme. assumes the desirability of a liberal school in contrast to the lockstep, teacher-driven school of the '90's. It recognizes the child as the center of interest. The final criterion of all values is considered to be the effect any technique of teaching or any content of instruction has upon the child. The Committee has held in mind, however, that it is the whole child, not a part of him, which is the reality to be kept constantly in mind. A child's present life is but a part of himself, and an educational philosophy based upon the assumption that the present interests, needs, strengths, weaknesses, whims of the child comprise the sole or dominating aspect of the child will in practice render but limited service. The child's future is a part of him. In a sense, that the child will soon become an adult is a reality of childhood which must not be forgotten. Further, it is of importance to realize that the child, whose most precious attribute is 'soon to become an adult,' is destined to live in an actual society, a total environment, which will be, roughly, the United States between the years 1940 and 1980. The actualities of this total environment are not likely to be the ideal ones of which we now dream with fondness. In our more realistic moments we see that the child will be an adult living his life in terms of needs, duties, responsibilities, successes, failures, satisfactions, and monotonies very much like our own.

It is not enough to cast education in terms of children's present interests and desires alone. A child's life at the moment is not a thing in itself, nor is it at all self-sufficient and self-contained. His life is a continuum; his future is more real than his present. He is essentially an organism which is becoming something other than a child. Hence the education of the child must be cast in terms of his becoming an adult as well as in terms of his present status. The child must, of course, learn (and be taught) in ways which utilize the principles of child psychology, but the aims of his education must be influenced by two considerations: (a) his real nature, a potent part of which is his rapid leaving of his present status and his constant becoming an adult, and (b) the demands which life will place upon him to-morrow. These demands are not those of our wishing, but

those which will exist in the United States in the next generation, many of which can be predicted with reasonable certainty.

Certain educational philosophers seem to slight this reality of the future. Whether or not these philosophers have this intention, their pronouncements are interpreted to mean that childhood (having its own rights as it surely does) should serve the purposes of childhood only or mostly. The fact that a given unit of material is preparation for adult living is to these persons no sufficient defense for its inclusion in a child's curriculum, even if it be adapted to childhood in accord with good psychology. The aims of the school and the aims of childhood can be cut loose from adult aims and can be considered as quite separate from the aims of the future. Aims, as aims, are thus located in the present rather than in the future. Let the child eat, drink, and be merry, for to-morrow he will-become only an adult. A further present tendency which seems to the Committee dubious philosophy is the attitude of helplessness in predicting the demands which life will place upon the adult of the immediate future, say, 1940 This helplessness is often alleged to be a necessary consequence of the rapid changes now occurring in society. Present society is changing in its technicalities, but the fundamentals of competent and useful living are not changing so rapidly that reasonable predictions of demands a generation or two in the future are either impossible or undesirable.

The revolt against the conventional school is a blessing, but it carries with it certain abuses. The Arithmetic Committee feels that a gradual evolution is more desirable than either a revolution or an impatient, extravagant attempt at reform. A temperateness in change may not be quite as thrilling to the philosopher as sweeping radical changes, but it is better sense.

The philosophy of this Yearbook, then, finds aims in the future as well as in the present. It suggests the desirability of preparation for adult living and holds it to be evident that a prediction of the demands of the future is feasible to a reasonable and useful degree of certainty. We should teach, then, those skills, informations, judgments, attitudes, habits, ideals, and ambitions which the child will find adequate and satisfying to the most important part of his whole self; that is, to his future adulthood as well as to his present childhood. How to teach the child can be separated, in discussion, from what to teach—and how to teach is fundamentally more a matter of

psychology based on research and investigation than a matter of philosophy.

IV. THE PSYCHOLOGY OF LEARNING ASSUMED IN THE YEARBOOK

Theoretically, the main psychological basis is a behavioristic one, viewing skills and habits as fabrics of connections. This is in contrast, on the one hand, to the older structural psychology which has still to make direct contributions to classroom procedure, and on the other hand, to the more recent *Gestalt* psychology, which, though promising, is not yet ready to function as a basis of elementary education.

The psychological point of view pervading this Yearbook emphasizes the fact that teaching based upon felt needs and interest only is inadequate. Not that felt needs and interest are lacking in vitality and importance, but that neglect of other matters (even if the neglect is only by way of inference) weakens effective teaching and learning to an intolerable degree. Use of all the dynamics of learning, rather than a use of some and a neglect of others, is the position taken. In the older school there was an overconfidence in drill-too often so stupidly administered that it could not possibly effect learning-and a corresponding neglect of interest and of the significance of the work to the worker. In short, there was a failure to capitalize the energyreleasing power of felt needs, together with a very naïve view of the psychological nature of the curriculum. One might suspect that today we are being led by our laudable attention to interest, significance, and the creation or discovery of felt needs to a neglect of other dynamics of human learning. We are in danger of slighting the contributions of properly organized practice; we may allow felt needs to degenerate into whims (often requiring less effort); we may continue (through neglect) to teach subject matter with but a superficial view of its psychological nature. A pedagogy based on a use of all the dynamics of learning in proper proportion is highly preferable to overemphasis upon a single aspect of learning in one generation followed by a corresponding neglect of it in the next generation.

In some quarters it seems the fashion to think that anything that at all resembles conventional practices is necessarily wrong and vicious. There is almost an emotional antipathy to anything that in any way reminds us of the kind of schools we attended as children.

The assumption that children must be unhappy and will develop frightful personalities if exposed to a pedagogy which uses the principles upon which the older schools were based is a sort of inverted oldoaken-bucket delusion. Many aspects of the older school are, after all, distinctly good psychology; they are based on correct theorems relating to human nature. Two instances will illustrate the caution against a lopsided psychology of learning based solely on interest and the felt needs of the child:

(a) A very modern teacher, sure that anything which is conventional is wrong, decided to allow the curriculum of her first-grade class to be guided by the wisdom of her pupils. "Now to-day," she said, "we can do anything we like to do." She suggested several possible projects which were anything but customary school work. Silence followed this attempt to discover felt needs. Finally a child summoned up courage and said: "I want to learn to read just as older people do." Should this teacher keep on trying to find something else to do or should she follow her philosophy and assist the youngster in his learning to read with a use of the best techniques of learning at her command?

(b) A third-grade teacher told her class on a certain morning that they could do anything they wished to do—at the same time waving some raffia in her hand and having previously laid in conspicuous places equipment for activities of various kinds. A hand promptly went up, followed by this suggestion: "Let's do some more drill in long division, so that it

will not be so hard." What should this teacher do?

The notion that regular school work is or must be essentially intolerable to children is not true. Even skill subjects can be taught, and often are taught, in ways which are even more than tolerable to children. A curriculum laid down before the child enters school can be lived through with zest and enthusiasm. The 'going to school' may, in itself, be an abiding and sufficient project. When an adequate psychology underlies the daily work, many of the criticisms directed against the 'orthodox' school lose their force. Many of the attacks on the 'conventional' school so describe it that the attack is against a straw man.

Much may be said for the following point of view: What to teach should be decided by as wise adults as are available for the task, who will base their decisions as far as possible upon the available body of objective scientific data. How to teach should be based upon a virile psychology of learning which uses all the dynamics of learning—not on the one hand, running to seed, as in the case of the older school, on dead tasks for the tasks' sake; and not, on the other hand, de-

claring a Roman holiday, and becoming concerned only with the factor of interest. The actualities are such that until we are much more skillful than we are, the interest value of many moments in school is not particularly high. But, in spite of this, a total psychology of learning would, on occasion, defend such moments as moments to keep on doing the same thing while improving the interest in it, rather than to avoid doing it altogether. There must be no retreat from the position that felt needs be utilized, that the tasks be made significant. that the sustaining effects of interest be earnestly sought and capitalized. Further, we must continue to seek both increased knowledge of the psychology of the subject matter taught (a field quite neglected by the left wing) and increased skill in the use of such aspects of learning as are suggested by the phrases: drill, continued effort, ability to withstand distraction, persistence though momentarily bored, effort sustained not by rewards at hand but by confidence in values forthcoming in the future, and the intent to master the matter in hand as a permanent possession rather than as a temporary accomplishment.

> For the Committee, F. B. Knight.

GUIDE TO CHAPTER II

		AGE
	Introductory—In a Library	9
I.	Number Concepts	10
	I. Concepts Functionally Considered	13
	2. Concepts in Relation to Units	16
	3. How Number Ideas Are Known	20
II.	Notation	24
III.	The Number Facts	27
	I. How Many Number Facts Should Be Taught?	27
	2. How to Teach the Facts So That They Will Be of the	
	Greatest Social Value	31
IV.	Skill in Processes	36
	I. Accuracy and Speed	36
V.	The Content of the Curriculum	37
	I. Other Criteria Beside Frequency of Use	38
	2. All the Usage Cannot Be Secured	39
	3. Topics of Low Frequency	42
	4. When Should Regular Number Work Begin?	48
VI.	Forms of Thought	52
	I. Generalization	52
	2. The Relation between Numbers	53
	3. Functional Thinking	54
	4. Ideas of Type	54
	5. Ideas of Variation	55
VII.	Problem-Solving	56
VIII.	Attitudes	60
	I. The Importance of Attitude	60
	2. Rationalizing the Processes	61
*	3. The Procedure versus the Answer	61