Mechanics of Functionally Graded Material Structures

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Mechanics of Functionally Graded Material Structures

Dedicated to blessed memories of my sister
Lamara (Dvorah) and brother Moshe.
I.E.

Dedicated to my newborn son Kyriacos.
D.P.

Dedicated to my beloved family.
C.G.

Preface

This book deals with the static and dynamic modelling of structures with variable parameters. A variable parameter is a parameter that can assume different values at different points of the structure. In engineering practice, there are designed variabilities and inherent variabilities.

The former is designed in order to obtain desired responses from the structural system. For example, in the new class of materials, called *functionally graded materials*, the mechanical characteristics are variable through the thickness or any other direction in order the structural element made with them to be able to fulfill the designed response. According to Reimanis (2004), "Functionally graded materials (FGMs) are materials that comprise a spatial gradation in structure and/or composition, tailored for a specific performance or function. FGMs are not technically a separate class of materials but rather represent an engineering approach to modify the structural and/or chemical arrangement of materials or elements. This approach is most beneficial when a component has diverse and seemingly contradictory property requirements, such as the cecessity for high hardness and high toughness in wear-resistant coatings."

Jha, Kant and Singh (2013) write: "Alhtough the concept of FGMs, and our ability to fabricate them, appears to be an advanced engineering invention, the concept is not new. These sorts of materials have been occurring in nature...Bones have functional grading. Even our skin is also graded to provide certain toughness, tactile and elastic qualities as a function of skin depth and location on the body." The naturally available materials include bamboo (Nogata and Takahashi, 1996; Ghavami *et al.*, 2003; Ramachandrarao, 2005; Silva *et al.*, 2006, 2008; Tan *et al.*, 2011) and seashells (Li, X.D., 2007).

Apparently the first monograph on the functionally graded materials was pioneered by Suresh and Mortensen (1998) which got over twelve hundred citations since its appearance, by the time the current book was under composition. One should also mention the definitive books by Miyamoto (1999), Shen Jha, Galgali and Misra (2004), (2009b) and Chung (2010) in addition to several volumes of conference proceedings published around the world.

Several disstertaions were also published in recent years in the book form. The interested reader can consult with works by Ebrahimi (2010), Marur (2011) and Reynolds (2011), for example. It is also important to cite reviews by Markworth, Ramesh and Parks (1995), Aboudi, Pindera and Arnold (1999), Paulino (2002), Birman and Byrd (2007), Hilton, Lee and El Fouly (2008), Jha, Kant and Singh (2013), Birman, Keil and Hosder (2013), and Birman (2014a, 2014b). It should be noted that the first papers on inhomogeneity in thickness direction were written many years prior to the uncovering of the FGMs. The interested readers can consult papers by Stavsky (1965), Zaslavsky (1970), Tvergaard (1976), and Eisenberger (1992), as well as the books by Maugin (1993) and Epstein and Elzanowski (2007) to cite just few. The inhomogeieties in other directions were treated in early papers by Jaiani (1975, 1976, 1977, 1998a, 1998b). Modern setting of FGM plates and shells belongs to Reddy (2000), Paulino (2002a, 2013), Batra, Hilton (2003, 2005), Aboudi, Pindera and Arnold (1996), and their respective coworkers, along with numerous contributors around the world as manifested in about 800 referenced papers.

The inherent variabilites are related to the fact that all the structural systems are affected by uncertainties including the he defin lack of complete information. Such uncertainties result from the imprecision of the manufacturing tools, instruments that measure data, and from our incomplete understanding of the laws governing the behavior of natural phenomena. For example, the crack location and depth in a structural element can be modelled through parameters affected by uncertainties. These kinds of problems represent two different aspects of the engineering practice, and, as a consequence, they have to be faced with two different approaches. In order to deal with the first problem, a deterministic method is used, where all the parameters are known exactly and the structural response is determined with the variation of the design parameters. The second problem requires the use of probabilistic models to explicitly incorporate imprecision into the mathematical models that are to be utilized in the analysis and design. Pertinent reviews of structures with uncertaqinties were given in the reviews by Ibrahim (1987) and Manohar and Ibrahim (1999). It was most natural to develop finite element method in stochastic setting. This task was accomplished by Nakagiri and Hisada (1985), Ghanem and Spanos (1991), Kleiber and Hien (1992), Haldar and Mahadevan (2000) and Elishakoff and Ren (2003). Reviews of stochastic finite element method were written by Elishakoff, Ren and Shinozuka (1996), Elishakoff and Ren (1999), Matthies (2008), Sudret and Der Kiureghian (2009), and Stefanou (2009), among others. The second possibility of inherently inhomogeneous materials deals with deterministically variable modulus of elasticity or mass density or any other structural parameters. Naturally there are many works dealing with such systems. There might be an intersection between these two types of variabilities, since once the new system is created via the functional grading, the material characteristics may turn to be varying along some or all coordinates probabilistically. In this book, we limit ourselves with structures made of FGM, and totally concentrate on structures with *fully determined* rather than uncertain parameters.

The book is organized as follows. The first part is devoted to the three-dimensional analysis of rectangular plates made of functionally graded materials (FGMs). The concept of FGMs has been introduced in the '90 apparently by Koizumi (1993, 1997), with the aim of overcoming the drawbacks of composite materials. Traditional composite materials offer numerous superior properties to metallic materials, such as high specific strength and high specific stiffness. This has resulted in the extensive use of laminated composite materials in aircraft, spacecraft and space structures. For example, a layer of a ceramic material bonded to the surface of a metallic structure acts as a thermal barrier in high temperature applications. However, the sudden change in material properties across the interface between discrete materials can results in large interlaminar stresses leading to delamination. Furthermore, large plastic deformations at the interfaces may trigger the initiation and propagation of cracks in the material. One way to overcome these adverse effects is to use functionally graded materials in which material properties, say, modulus of elasticity, mass density, Poisson's ratio, vary continuously. This is achieved either by gradually changing the volume fraction of the constituent materials, usually in the thickness direction only, with the objective of optimizing the performance of the material for a specific application. Since properties of FGMs vary along a direction, they fall under the category of inhomogeneous materials.

FGMs have found application in various branches of engineering, for example, in aerospace structures, power generation industries, machine parts, etc. In recent years, these new classes of materials have gained considerable attention that motivates the importance of a deep understanding of their static and dynamic behavior. The rectangular plate is the most common structural element for such applications. Thus, rectangular plates are under study. In particular, since FG plates may have substantial thickness a three-dimensional analysis is conducted. Bi-dimensional theories may result inadequate for an appropriate design of such structural components. Special attention is devoted to the effects of the thickness, of the material property variation and boundary conditions on the static and dynamic behavior of such plates.

Whereas in the first part of the present book we deal with grading in thickness direction as proposed by the pioneers of the very concept of FGMs, we devote its second part to the functional grading in either axial direction for beams and columns, or in radial direction for circular plates. We extend methodologies presented for the first time in the monograph by Elishakoff (2005) and some papers cited therein.We concentrate on closed-form solutions shedding light on effects of grading on the involved eigenvalues.

It is anticipated that the future technologies will allow performing functional grading in arbitrary direction in the structure in order to tailor it to desired behavior. It is hoped, therefore, this book to become a useful reference for engineers and scientists involved in developing new materials, in conjunction with desired structural performance.

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