THE QUANTUNE WERS

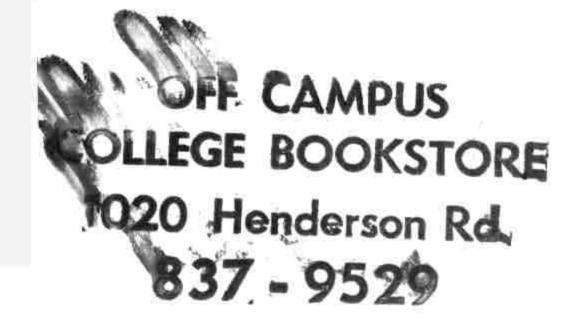
Tony Hey and Patrick Walters

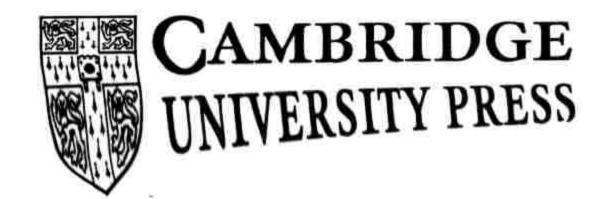
# The quantum universe

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### PREFACE

The goal of this book is to present the essential ideas of quantum physics as simply as possible and demonstrate how quantum physics affects us all in our everyday life. We therefore do not focus our discussion on philosophical debates about the meaning of quantum mechanics. Rather, we describe how the seemingly bizarre and 'academic' notions of quantum mechanics have been successfully applied to an enormously diverse range of fields. Some idea of the scope of these applications can be gained from the contents list of this book.

There is, however, one formidable problem to be faced before such a 'popular' approach to quantum mechanics can become a reality. The problem is this. The natural language in which to express the ideas of physics is mathematics. For this reason, understanding physics, and quantum mechanics in particular, presents those without the necessary mathematical background with a huge obstacle to be overcome. Nonetheless, the impact of quantum mechanics on our lives is so enormous that we firmly believe that almost any attempt to bridge the gulf between the literate and the numerate is worthwhile. To quote from G. K. Chesterton: 'If a thing is worth doing, it's worth doing badly!' It is therefore at the risk of accumulating considerable scorn from some of our academic colleagues, that we attempt in this book to explain the whole of quantum physics without recourse to mathematics or equations!

Our approach is based on the belief that once some idea of the fundamental peculiarity of quantum motion — crudely speaking, that 'particles' can behave like 'waves' — has been gained, then much that was formerly incomprehensible about quantum mechanics becomes 'obvious'. In this way, the simple example of waves on a string leads directly to 'energy quantization', and thence to much of atomic and nuclear physics. Similarly, the strange phenomenon of quantum tunnelling, responsible for nuclear fission, hydrogen burning in the stars, and much else besides, is directly related to similar behaviour observed for water waves. The surprises come only from the basic wave—particle duality of quantum mechanics.

To make clear our goal that this book should be as close to a 'coffee-table quantum mechanics' book as we can make it, we have toyed with many possible titles. At various times we have referred to the book as *Quantum Mechanics for Bank Managers* (financial expediency), *Quantum Mechanics for Housewives* (frankly sexist), *Quantum Mechanics for Builders* (evidently practical), and so on. Perhaps most relevant would be a title such as *Quantum Mechanics for Politicians*. It is a truly appalling comment on our education system that those who guide our economic destiny often have little or no conception of the impact that quantum mechanics and 'basic physics' research has had, and continues to have, on our modern way of life. One example will suffice to make the point. In conversation, the research director of the 'high-tech' division of one of the UK's leading companies dismissed quantum mechanics as being irrelevant to the physics observed at ordinary 'room' temperatures. He believed that some sort of classical explanation would always be adequate. In fact, from the standpoint of

classical physics even the floor that he was standing on, let alone his industry, would not exist! This type of statement, however, seems to typify the feeling prevalent among industrialists and government today that investment in 'basic science' is less beneficial than investment in 'applied' research. One of the physicists mentioned in this book, Henrick Casimir, is exceptional in that he not only made contributions to our understanding of quantum mechanics but also became director of Philips Research Laboratories in Holland, and therefore had experience of both academic and industrial research. Casimir's refutation of this viewpoint is memorable:

I have heard statements that the role of academic research in innovation is slight. It is about the most blatant piece of nonsense that it has been my fortune to stumble upon.

Certainly, one may speculate idly whether transistors might have been discovered by people who had not been trained in and had not contributed to wave mechanics or the theory of electrons in solids. It so happened that inventors of transistors were versed in and contributed to the theory of solids.

One might ask whether basic circuits in computers might have been found by people who wanted to build computers. As it happens, they were discovered in the thirties by physicists dealing with the counting of nuclear particles because they were interested in nuclear physics. One might ask whether there would be nuclear power because people wanted new power sources or whether the urge to have new power would have led to the discovery of the nucleus. Perhaps — only it didn't happen that way, and there were the Curies and Rutherford and Fermi and a few others.

One might ask whether an electronic industry could exist without the previous discovery of electrons by people like Thomson and H. A. Lorentz. Again it didn't happen that way.

One might ask even whether induction coils in motor cars might have been made by enterprises which wanted to make motor transport and whether they would have stumbled on the laws of induction. But the laws of induction had been found by Faraday many decades before that.

Or whether, in an urge to provide better communication, one might have found electromagnetic waves. They weren't found that way. They were found by Hertz who emphasized the beauty of physics and who based his work on the theoretical considerations of Maxwell. I think there is hardly any example of twentieth century innovation which is not indebted in this way to basic scientific thought.

This point of view was also shared by men like Faraday and J. J. Thomson. Faraday, when asked by the famous prime minister Gladstone as to the practical use of the discovery of electricity replied: 'One day, Sir, you may tax it'. Similarly, Thomson, who discovered the electron, remarked that while research into applied science leads to improvement and development of older methods, research in pure science can result in entirely new and more powerful methods. He concluded that 'research in applied science leads to reform, research in pure science leads to revolutions, and revolutions, whether political or industrial, are exceedingly profitable things if you are on the winning side'.

Our book can therefore be seen as a defence of basic science – but that was not our real reason for writing it. We wrote it because we ourselves find the quantum universe a source of endless fascination, and we would like to bring a sense of this excitement to as wide an audience as possible. We hope that our book will stimulate young people to find out more, and discover the true power of quantum mechanics obtained by adjoining mathematics to the qualitative descriptions given in this book. We also hope that it will appeal to older readers who wish to know something about the way in which the quantum world appears to work. We believe it desirable that more

people should understand what physics can and cannot do, and appreciate how physics has made possible the new 'high technologies' that are changing our lives.

This book has grown out of lectures given in Southampton and Swansea Universities and we wish to acknowledge all the valuable suggestions made by many of our loyal students. We also thank many of our friends and colleagues for their interest and help. In particular, Tony Hey wishes to thank Ian Aitchison and Malcolm Coe for helpful comments, Tessa Coe and Charlie Askew for help with computer graphics, and, most of all, Garry McEwen, for his tireless assistance throughout this project without which this book would be much more obscure. Patrick Walters also thanks Colin Grey Morgan, Steve Hibbs, Ann Jenkins and Howard Miles for their help in making this book what it is. Needless to say, the remaining flaws are all ours. Last, but obviously not least, we wish to thank Jessie and Marie for their tolerance and forebearance throughout what was intended as a very short project but ended up as a very much longer one!

We also wish to record our thanks to all the staff of Cambridge University Press who have contributed in producing a book so close to our original conception. We are particularly indebted to Simon Capelin and Robin Rees for not only having faith in the project but also seeing us through the difficult times with good humour and patience. Irene Pizzie and Jeanette Hurworth also made invaluable contributions to our preparation of the text and photographs for publication. To them all, many thanks.

One final 'thank you' is in order. The reader may wonder why we have chosen to introduce each chapter with a quotation from Richard Feynman. The informal flavour of the quotations gives some indication of Feynman's non-pompous, plain-speaking style of communication. Moreover, Feynman is without equal both as a research physicist and as an educator. He is a passionate believer in the importance of understanding as opposed to ritual learning of formulae and definitions. For anyone not familiar with his career and personality, his recent collection of anecdotes Surely You're Joking, Mr. Feynman! makes absorbing reading. Feynman's style of teaching is truly inimitable, combining as it does, great understanding of the physics with great originality of presentation. In this book, we try to follow Feynman's example, and hope that some of the freshness of his style of presentation comes through in our text. In a very real sense, therefore, this book is dedicated to Richard Feynman, for all that he has given to physics. We are not at all sure that he will like the results of our labours: one thing we are sure of is that he would have done it differently!

# **PROLOGUE**

Poets say science takes away from the beauty of the stars — mere globs of gas atoms. Nothing is 'mere'. I too can see the stars on a desert night, and feel them. But do I see less or more? The vastness of the heavens stretches my imagination — stuck on this carousel, my little eye can catch one-million-year-old light . . . Or see them [the stars] with the greater eye of Palomar, rushing all apart from some common starting point when they were perhaps all together. What is the pattern, or the meaning, or the why? It does not do harm to the mystery to know a little about it. For far more marvellous is the truth than any artists of the past imagined! Why do the poets of the present not speak of it?

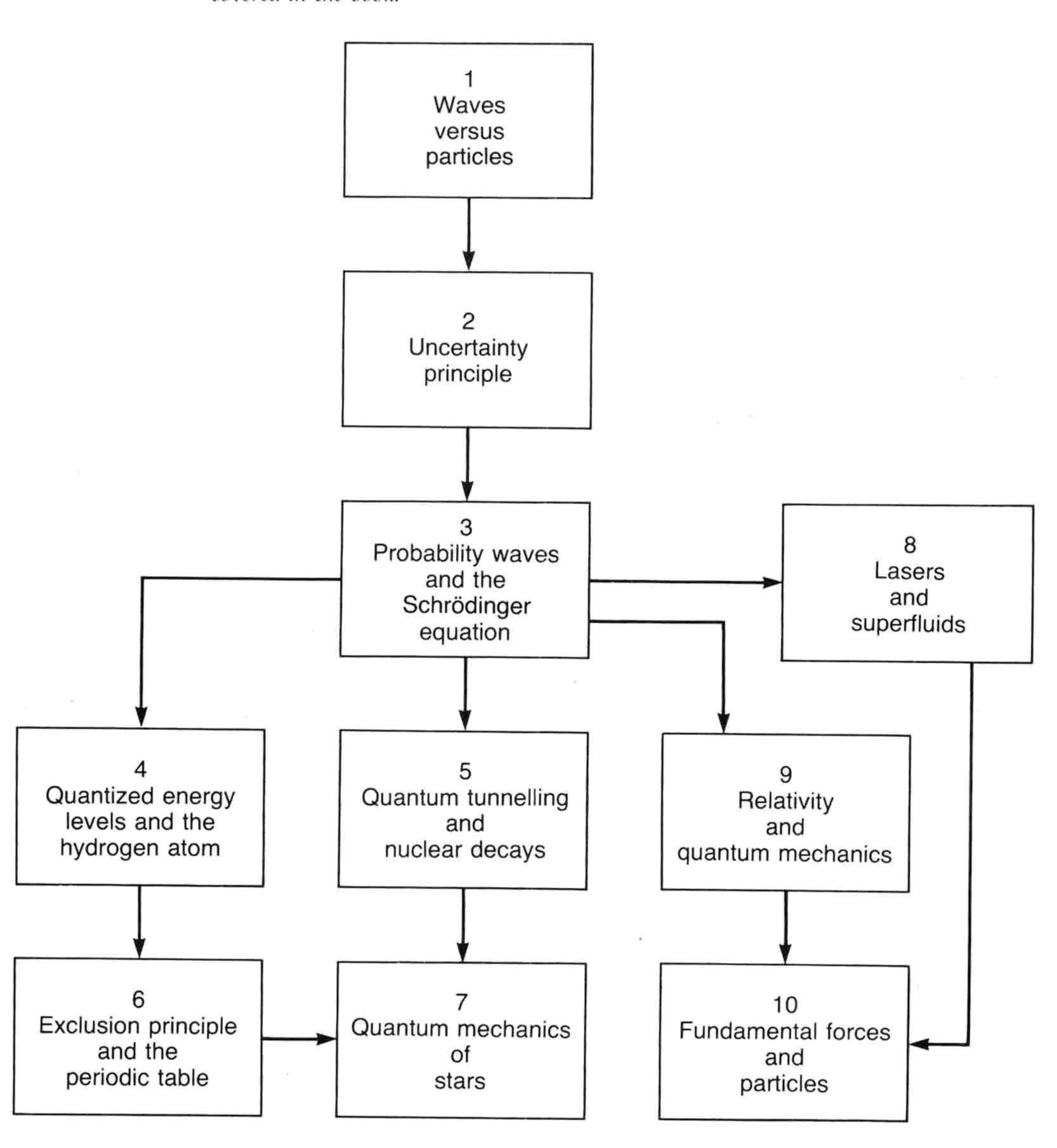
Richard Feynman



Sylvia Posner

## ROUTE MAP

The diagram below shows the major interconnections of the various chapters and topics covered in the book.



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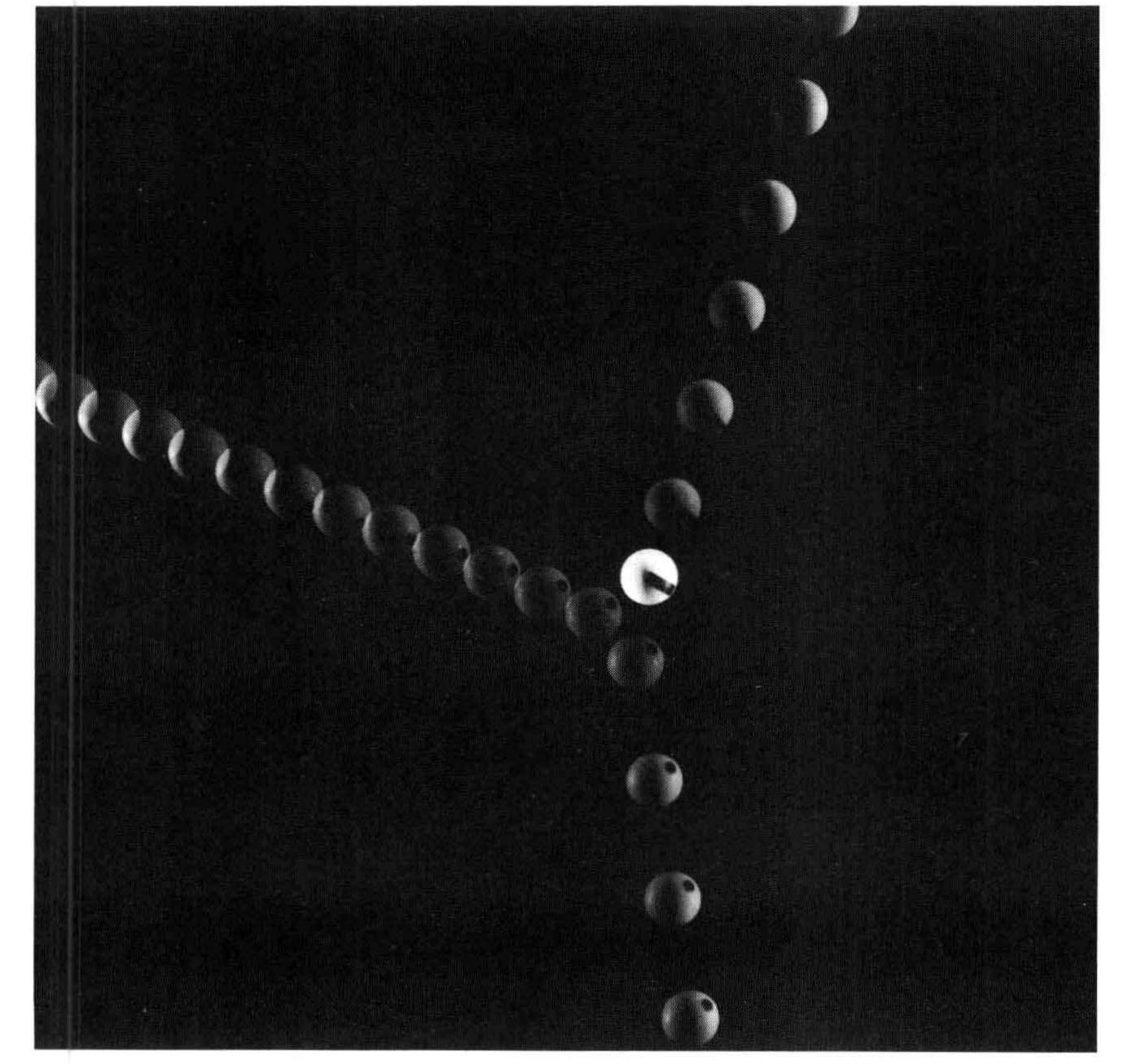
# Waves versus particles

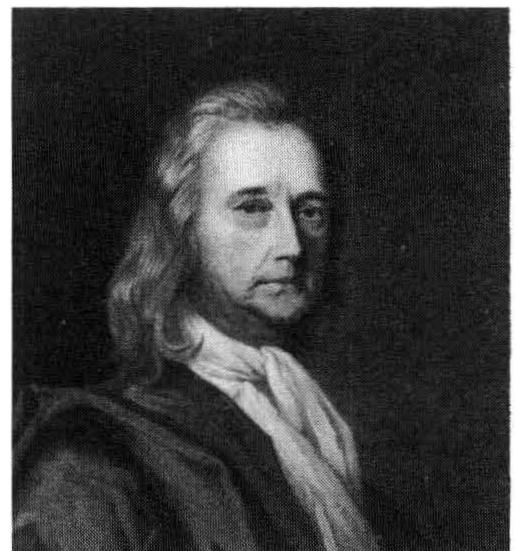
. . . I think I can safely say that nobody understands quantum mechanics.

Richard Feynman

#### Science and experiment

Science is a special kind of explanation of the things we see around us. It starts with a problem and curiosity. Something strikes the scientist as odd. It doesn't fit in with the usual explanation. Maybe harder thinking or more careful observation will resolve the problem. If it remains a puzzle, it stimulates the scientist's imagination. Perhaps a completely new way of looking at things is needed? Scientists are perpetually trying to find better explanations – better in the sense that any new explanation must not only explain the new puzzle, but also be consistent with all of the previous explanations that still work well. The hallmark of any scientific explanation or 'theory' is that it must be able to make successful predictions. In other words, any decent theory must be able to say what will happen in any given set of circumstances. Thus, any new theory will only become generally accepted by the scientific community if it is able, not only to explain the observations that scientists have already made, but also to foretell the results of new, as yet unperformed, experiments. This rigorous testing of new





Isaac Newton (1642–1727)
published his book Optics in
1704 that explained the rainbow
and put forward the 'corpuscular'
theory of light. In his 1687 book
Mathematical Principles of
Natural Philosophy Newton set
down the principles of mechanics
and gravity that guided science
until the mid 19th century.

Fig. 1.1 A multi-flash photograph of a billiard ball collision. The motions of the balls can be calculated using Newton's laws but we have a good feel for what will happen from watching snooker on television or playing ourselves.

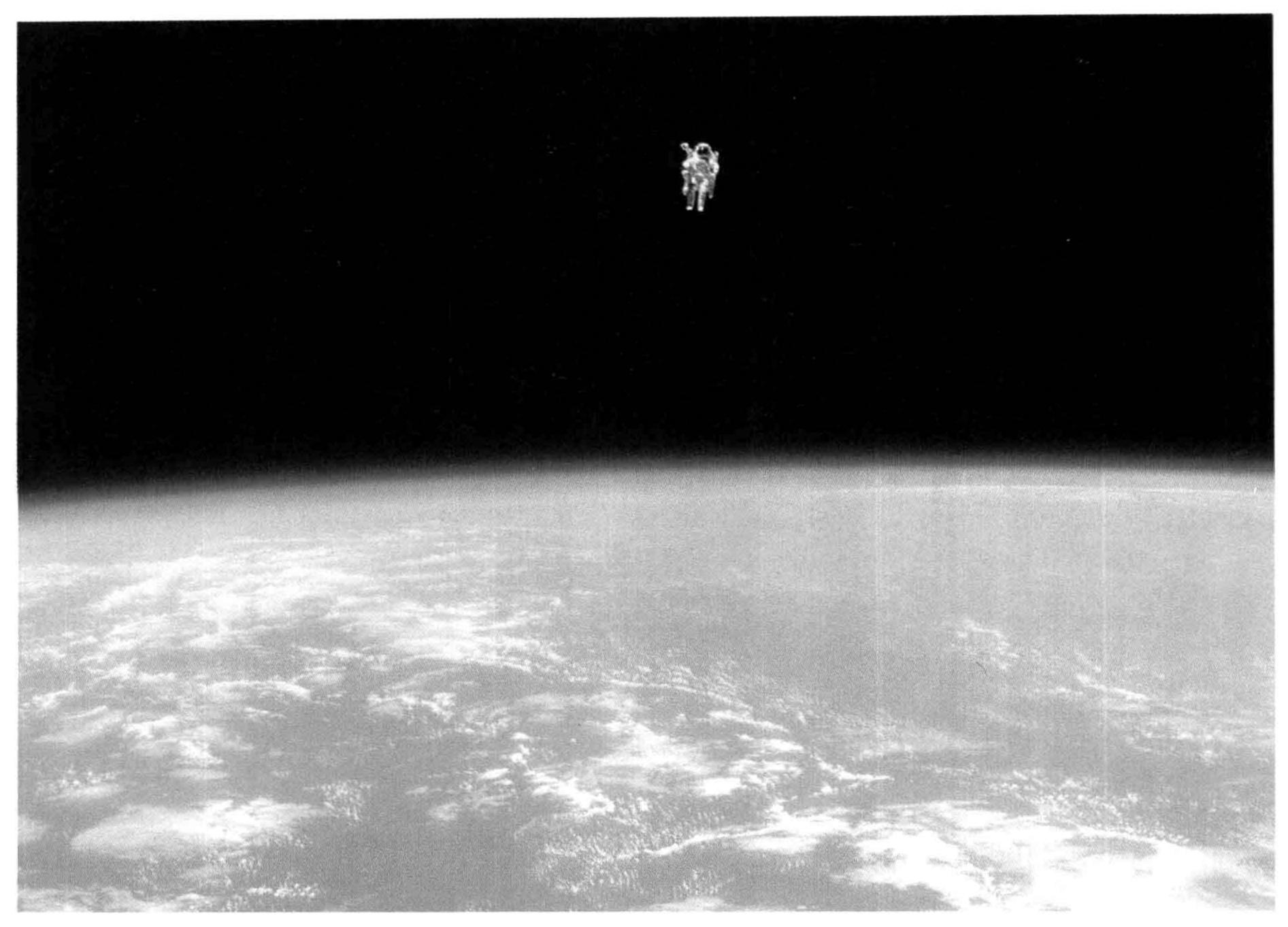


Fig. 1.2 Astronaut Bruce
McCandless floats in space
during the first untethered space
walk on February 7th, 1984.
The astronaut is essentially an
independent space-craft in orbit
near the shuttle. McCandless
commented 'Well that may have
been one small step for Neil
[Armstrong] but it's a heck of a
big leap for me!'

scientific ideas is the key feature that distinguishes science from other fields of intellectual endeavour – such as history or even economics – or from a pseudoscience such as astrology.

In the 17th century Isaac Newton and several other great scientists developed a wonderfully successful explanation of the way things move. This whole theoretical framework is called 'classical mechanics', and its scope encompasses the motion of everything from billiard balls to planets. Newton's explanation of motion in terms of forces, momentum and acceleration is encapsulated in his 'laws of motion'. These principles are incorporated into so many of our machines and toys that classical mechanics is familiar from our everyday experience. We all know what to expect in the collision of two billiard balls. Perhaps the most spectacular application of classical mechanics is in the exploration of space. Nowadays it surprises no one that the astronaut and the space shuttle float side by side and neither falls dramatically to Earth. A hundred years ago it was not so 'obvious', and in Jules Verne's famous story A Trip Around The Moon the passengers of the spacecraft were amazed to find the body of a dog that died on takeoff, and which they had jettisoned outside the craft, floating side by side with them all the way to the Moon. Today, you may not know how Newton's theory works in detail but you can see that it works. It is part of our daily experience.

All this brings us to the problem most of us have in coming to terms with 'quantum mechanics'. It is just this. At the very small distances involved in the study of atoms and molecules, things do *not* behave in a familiar way. Classical mechanics is inadequate and an entirely new explanation is needed. Quantum mechanics is that new explanation, and it is cunningly

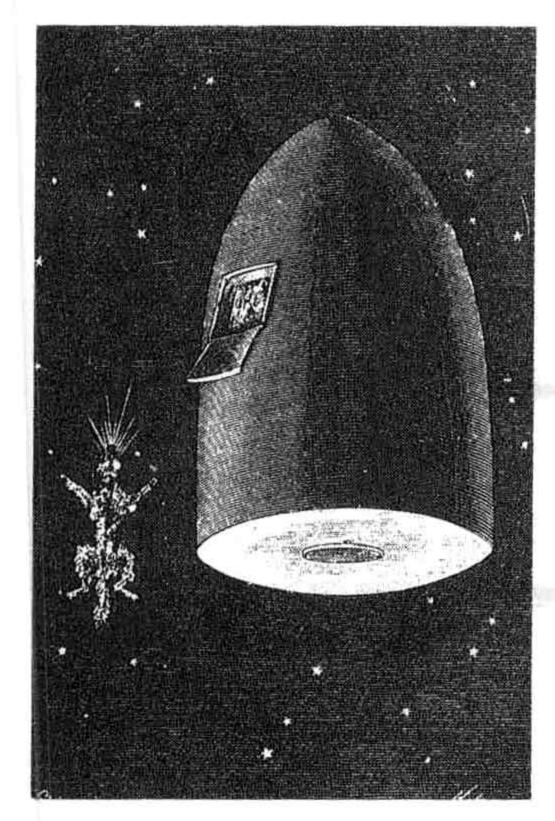


Fig. 1.3 In the story by Jules Verne A Trip Around The Moon, published in 1865, the dog 'Satellite' died on take-off and was jettisoned from the space-ship. Much to the surprise of the occupants the dog's body floated along with them all the way to the Moon!



Thomas Young (1773–1829) was an infant prodigy who could read at age two. During his youth he learnt to speak a dozen languages. He is best remembered for his work on vision and for establishing the wave theory of light. However, he was also the first to make progress on deciphering the hieroglyphic language of the ancient Egyptians.

constructed so that it not only works in the quantum domain of very short length scales, but also so that, for larger distances, its predictions are identical with those of Newton. An atom is a typical quantum thing – it cannot be understood from the standpoint of classical physics. One popular visualization of an atom imagines electrons orbiting the nucleus of the atom much in the way planets orbit the Sun in the solar system. In fact, for negatively charged electrons in orbit round a positively charged nucleus, this simple model is unstable! According to classical physics the electrons would spiral into the centre and the atom would collapse. This nice and comforting model of the atom simply cannot account for even the existence of real atoms, let alone predict their expected behaviour. It is important to be aware at the outset that there is no simple picture that can accurately describe the behaviour of electrons in atoms. This is the first hurdle faced by the newcomer to the quantum domain: the inescapable and unpalatable fact that behaviour of quantum objects is totally unlike anything you have ever seen.

How can we convince you that quantum mechanics is both necessary and useful? Well, a physicist, just like a good detective, sifts through the evidence and remembers the old maxim of Sherlock Holmes that 'when you have excluded the impossible, whatever remains, however improbable, must be the truth'. Nonetheless, it was not without much reluctance that 20th century physicists became convinced that the whole magnificent edifice of classical physics was not 'almost right' for describing the behaviour of atoms, but had, instead, to be radically rebuilt. Nowhere was the confusion generated by this painful realization more evident than in their attempts to understand the nature of light.

#### Light and quantum mechanics

It was way back in the 17th century that Isaac Newton suggested that light should be regarded as a stream of particles, rather like bullets from a machine gun. Such was Newton's reputation that this view persisted, apart from some isolated pockets of opposition, until the 19th century. It was then that Thomas Young and others conclusively showed that the particle picture of light must be wrong. Instead, they favoured the idea that light was a kind of wave motion. One property of waves that is familiar to us is that of 'interference', to use the physicists' term for what happens when two waves collide. For example, in fig. 1.4 we show the 'interference' patterns produced by two sources of water waves on the surface of the water. Using his famous 'double-slit' apparatus to make two sources of light, Thomas Young had observed similar interference patterns using light.

Alas, physicists were not able to congratulate themselves for long. Experiments at the end of the 19th century revealed effects that were inexplicable by a wave theory of light. The most famous of such experiments concerns the so-called 'photo-electric' effect. Ultra-violet light shone onto a negatively charged metal caused it to lose its charge, while shining visible light on the metal had no effect. This puzzle was first explained by Albert Einstein in the same year that he invented the 'theory of relativity' for which he later became famous. His explanation of the photo-electric effect resurrected the particle view of light. The discharging of the metal was caused by electrons being knocked out of the metal by light energy concentrated into individual little 'bundles' of energy, which we now call 'photons'. According to Einstein's theory, ultra-violet photons have more energy than visible-light ones, and so no matter how much visible light you shine on the metal, none of the photons have enough energy to kick out an electron.

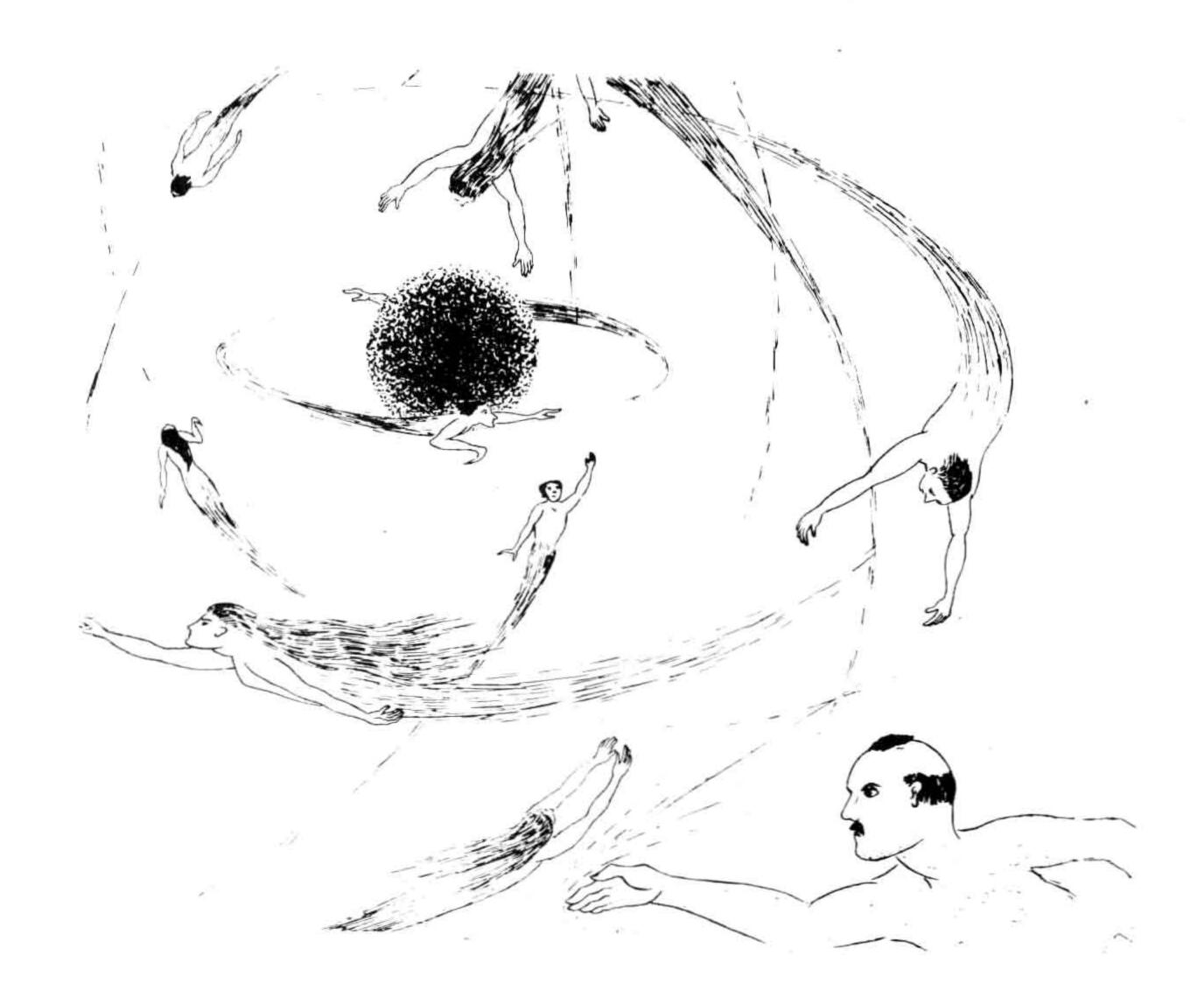
After several decades of confusion in physics, a way out of this mire was found in the 1920s with the emergence of quantum mechanics, pioneered

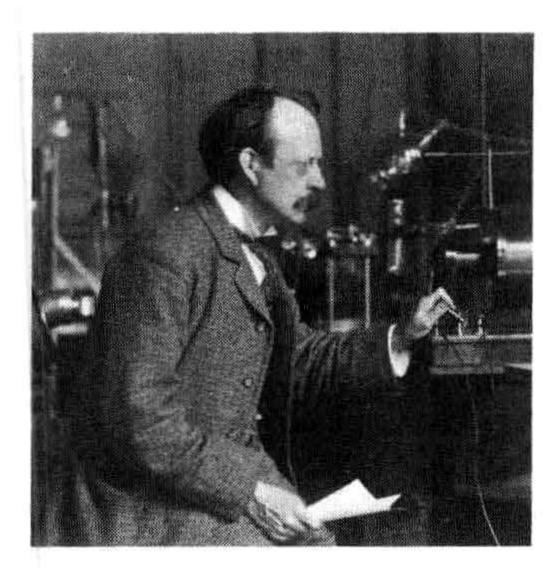
Fig. 1.4 Photograph of the interference pattern produced by two vibrating sources in water.



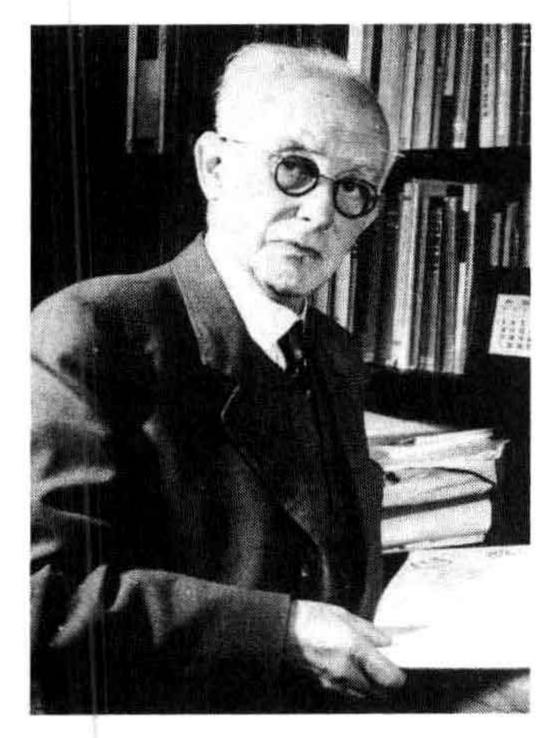
by physicists such as Heisenberg, Schrödinger and Dirac. This theory is able to provide a successful explanation of the paradoxical nature of light, atoms and much else besides. But there is a price to pay for this success. We must abandon all hope of being able to describe the motion of things at atomic scales in terms of everyday concepts like waves or particles. A 'photon' does not behave like anything anyone has ever seen. This does not, however, mean that quantum mechanics is full of vague ideas and lacks predictive power. On the contrary, quantum mechanics is the only theory capable of making definite and successful predictions for systems of atomic sizes or smaller in much the same way that classical mechanics makes predictions

Fig. 1.5 George Gamow's rather whimsical view of the planetary model of the atom in Mr Tompkins Explores The Atom.





J. J. Thomson (1856–1940) measured the charge-to-mass ratio of the electron thus establishing it as a new elementary particle. He was awarded the Nobel Prize in 1906.



Max Born (1882–1970)
was awarded the 1954 Nobel
Prize as a very belated
recognition of his work on the
probability interpretation of
quantum mechanics. Born left
Germany when Hitler came to
power and was Professor of
Natural Philosophy in
Edinburgh from 1936 until his
retirement in 1953.

for the behaviour of billiard balls, rockets and planets. The difficulty with quantum things such as the photon is that, unlike billiard balls, their motion cannot be visualized in any accurate pictorial way. All we can do is summarize our lack of a picture by saying that a photon behaves in an essentially quantum mechanical way.

There is one sense in which Nature has been kind to us. Viewed from the perspective of classical physics, photons and electrons seem like very different kinds of objects. Remarkably, in the quantum domain both photons and electrons, and indeed all quantum objects, behave in the same strange quantum mechanical way. This is at least some compensation for our inability to picture quantum things! There is a curious little irony in the history of our attempts to understand the nature of electrons. In 1897 J. J. Thomson measured the charge-to-mass ratio of the electron and established the electron as a new elementary particle of Nature. Thirty years later, his son, G. P. Thomson, and also Davisson and Germer in the USA, performed a beautiful series of experiments which conclusively revealed that electrons also behave like waves. The historian Max Jammer wrote: 'One may feel inclined to say that Thomson, the father, was awarded the Nobel Prize for having shown that the electron is a particle, and Thomson, the son, for having shown that the electron is a wave'.

Our intention in this book is to impress even the most skeptical reader with the enormous range and diversity of the successful predictions of quantum mechanics. The apparently absurd ideas of de Broglie, Schrödinger and Heisenberg have now led to whole new technologies, the very existence of which depends on the discoveries of these pioneers of quantum mechanics. The modern electronics industry, with its silicon chip technology, is all based on the quantum theory of materials called semiconductors. Likewise, all the multitude of applications of lasers are possible only because of our understanding, at the fundamental quantum level, of a mechanism for radiation of light from atoms first identified by Einstein in 1916. Moreover, understanding how large numbers of quantum objects behave when packed tightly together leads to an understanding of all the different types of matter ranging from 'superconductors' to 'neutron stars'. In addition, although originally invented to solve fundamental problems concerned with the existence of atoms, quantum mechanics was found to apply with equal success to the tiny nucleus at the heart of the atom, and this has led to an understanding of radioactivity and nuclear reactions. As everyone knows, this has been a mixed blessing. Not only do we now know what makes the stars shine, but we also know how to destroy all of civilization with the awesome power of nuclear weapons.

But before we can explain how quantum mechanics made all these things possible, we must first attempt to describe the strange quantum mechanical behaviour of objects at atomic distance scales. This task is clearly difficult given the absence of any accurate analogy for the mathematical description of quantum behaviour. However, we can make progress if we use a mixture of analogy and contrast. Young's original 'double-slit' experiment used a screen with two slits in it to make two sources of light which could interfere and produce his famous 'interference fringes' – alternating light and dark lines. We shall describe the results of similar 'double-slit' experiments carried out using bullets, water waves and electrons. By comparing and contrasting the results obtained with the three different materials we shall be able to give you some idea of the essential features of quantum mechanical behaviour. Quantum mechanics textbooks contain detailed discussion of many types of experiments, but this double-slit experiment is sufficient to reveal all the mystery of quantum mechanics. All of the problems and paradoxes of quantum physics can be demonstrated in this single experiment.

A word of warning before we begin. To avoid running into a frustrating psychological cul-de-sac, try to be content with mere acceptance of the observed experimental facts. Try not to ask the question 'but how can it be like that?' As Richard Feynman says 'nobody understands quantum mechanics'. All we can give you is an account of the way Nature appears to work. Nobody knows more than that.

#### The double-slit experiment

This section may be rather hard going first time through. If so, just glance at the pictures and pass on quickly to the next chapter!

#### WITH BULLETS

Source: a wobbly machine gun that, as it fires, spreads the bullets out into a cone, all with the same speed but random directions.

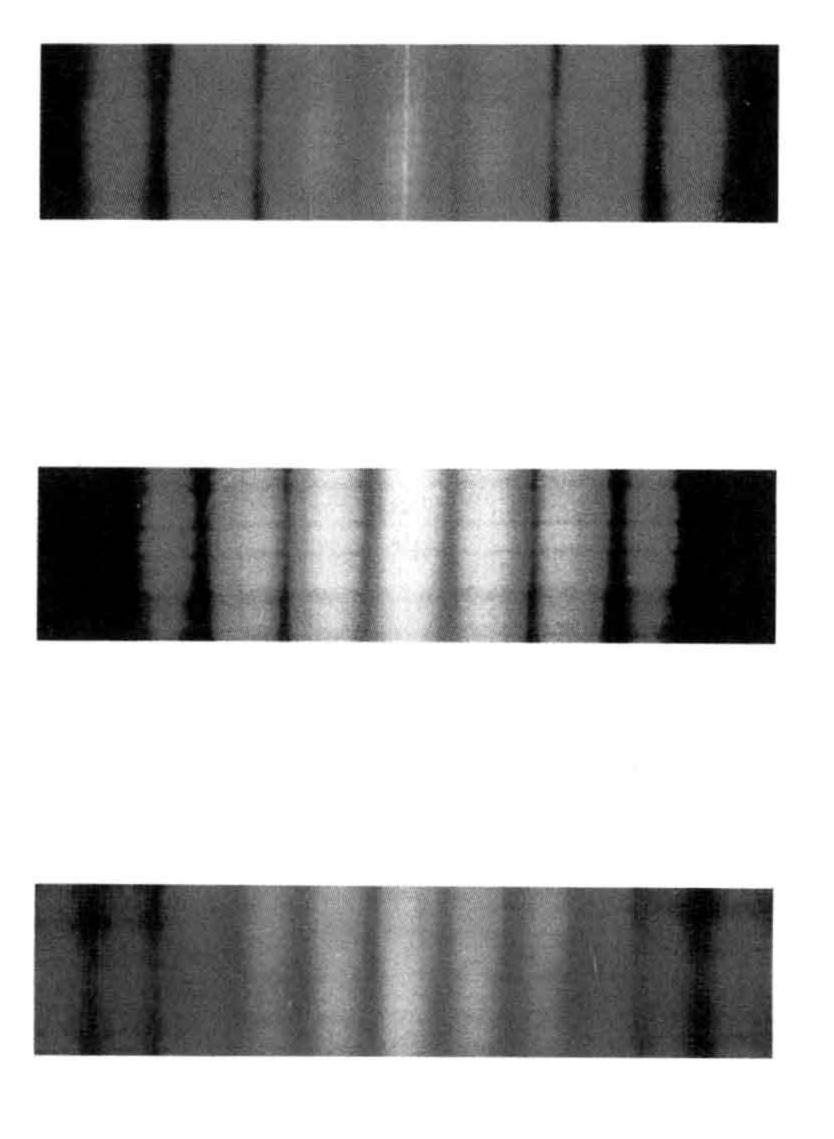
Screen: armour plate with two parallel slits in it.

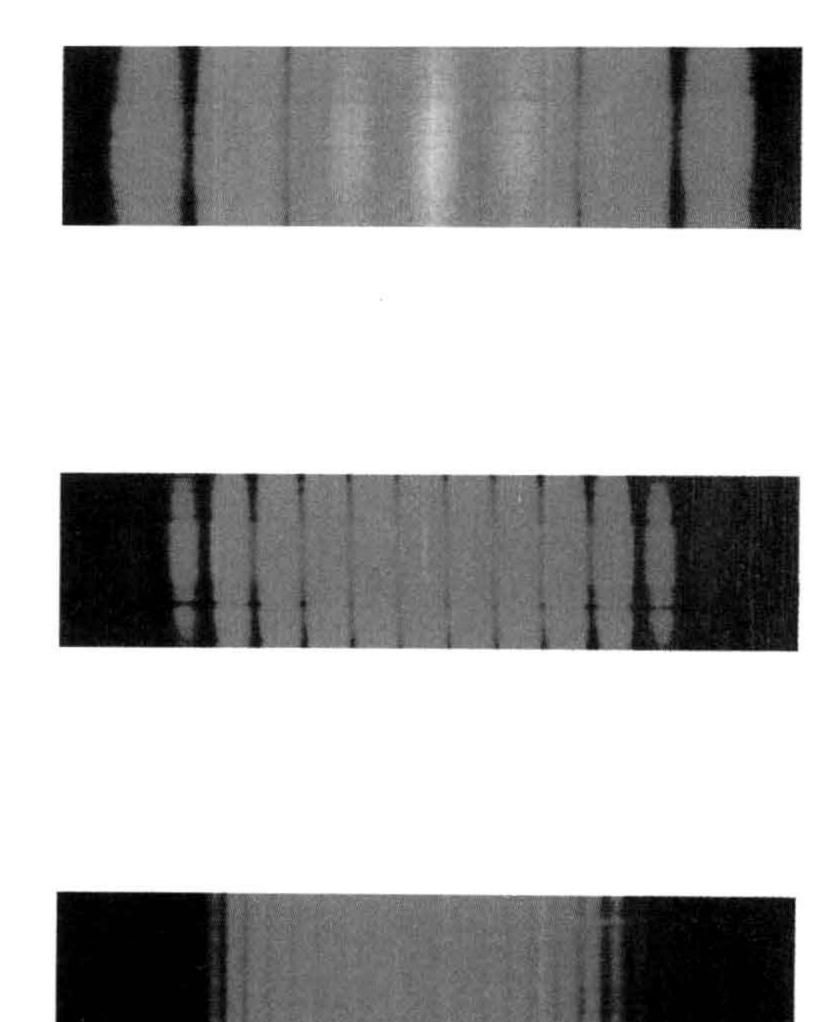
Detector: small boxes of sand to collect the bullets.

Results: the gun fires at a fixed rate and we can count the number of bullets that arrive in any given box in a given period of time. The bullets that go through the slits can either go straight through or else bounce off one of the edges, but must always end up in one of the boxes. The bullets we are using are made of a tough enough metal so that they never break up — we can never have half a bullet in a box. Moreover, no two bullets ever arrive at the same time — we have only one gun, and each bullet is a single identifiable 'lump'.

If we let the experiment run for an hour and then count the bullets in each of the boxes, we can see how the 'probability of arrival' of a bullet varies with the position of the detector box. The total number of bullets arriving at any given position is clearly the sum of the number of bullets going through slit 1 plus the number going through slit 2. How this 'probability of arrival' varies with position of the sand boxes is shown in fig. 1.7. We shall label this result,

Fig. 1.6 Double-slit interference patterns for light, usually taken as demonstrating that light is a wave motion. In the left hand pictures, as the wavelength of the light is decreased and the colour changes from red to blue, the interference fringes become closer together. On the right, for red light, the decrease in the fringe separation is caused by increasing the separation of the slits.





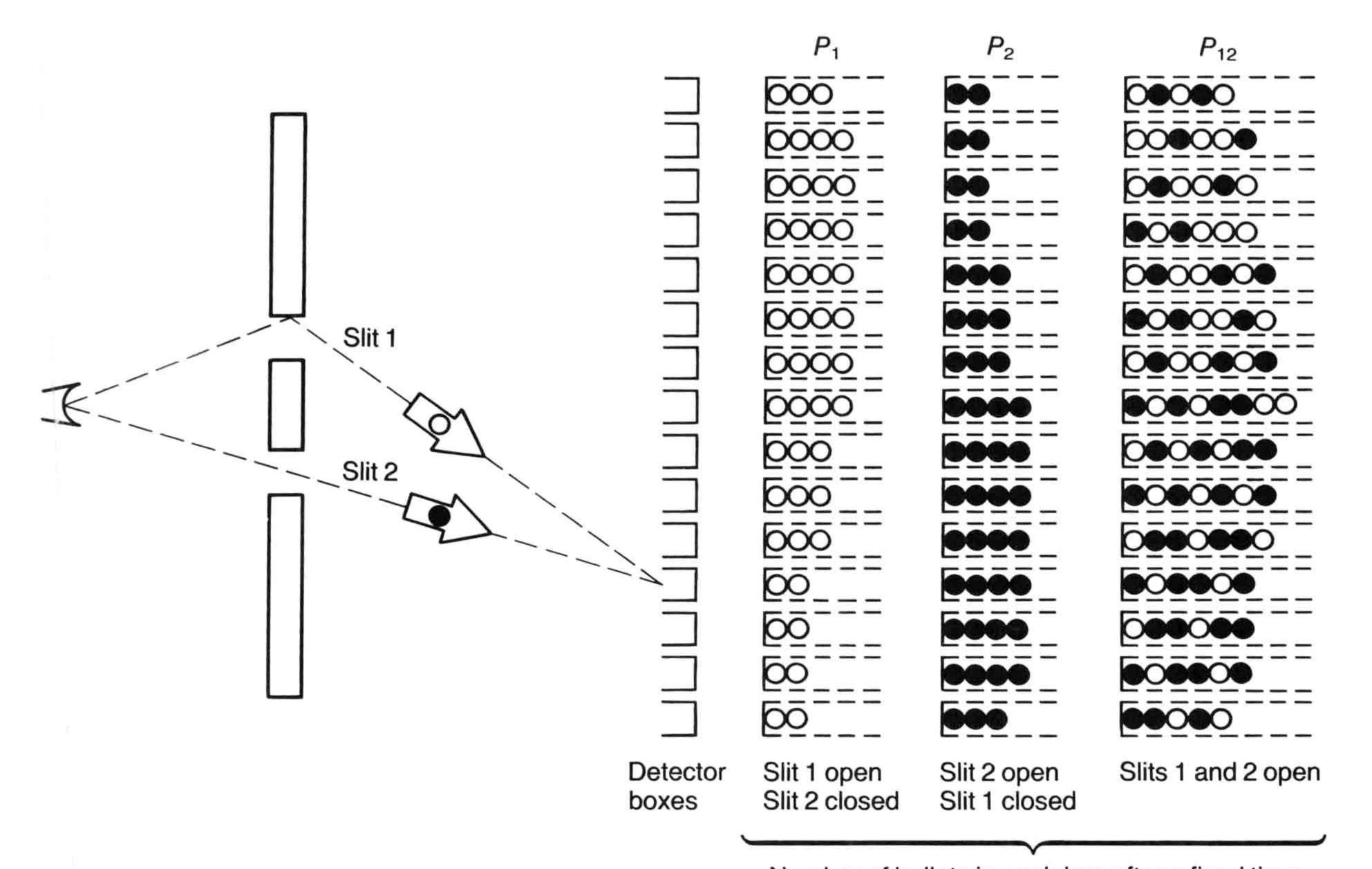


Fig. 1.7 A diagram of a double-slit experiment with bullets. The experimental set-up is shown on the left of the figure and the results of three different experiments indicated on the right. We have shown bullets that pass through slit 1 as open circles and bullets through slit 2 as black circles. The column labelled P<sub>1</sub> shows the distribution of bullets arriving at the detector boxes when slit 2 is closed and

only slit 1 is open. Column P<sub>2</sub> shows a similar distribution obtained with slit 1 closed and slit 2 open. As can be seen, the maximum number of bullets appears in the boxes directly in line with the slit that is left open. The result obtained with both slits open is shown in the column labelled P<sub>12</sub>. It is now a matter of chance through which slit a bullet will come and this is shown by the scrambled mixture

Number of bullets in each box after a fixed time

of black and white bullets collected in each box. The important point to notice is that the total obtained in each box when both slits are open is just the sum of the numbers obtained when only one or other of the slits is open. This is obvious in the case of bullets since we know that bullets must pass through one of the slits to reach the detector boxes.

the probability of arrival of bullets when both slits are open,  $P_{12}$ . We also show in fig. 1.7 the results obtained with slit 2 closed, which we call  $P_1$ , and those obtained with slit 1 closed, which we call  $P_2$ . Looking at the figures, it is evident that the curve labelled  $P_{12}$  is obtained by adding curves  $P_1$  and  $P_2$ . We can write this mathematically as the equation

$$P_{12} = P_1 + P_2$$

For reasons that will become apparent in a moment, we call this result the case of *no interference*.

#### WITH WATER WAVES

Source: a stone dropped into a large pool of water.

Screen: a jetty with two gaps in it.

Detector: a line of small floating buoys whose jiggling up and down gives a