

# Physical Chemistry of Polymer Solutions

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*Theoretical Background*

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*Theoretical Background*

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# Physical Chemistry of Polymer Solutions

*Theoretical Background*

*To our families*

## Preface

Numerous physical properties of polymer solutions are known to be significantly different from those of low molecular weight solutions. The most probable explanation of this obvious discrepancy is the large molar volume ratio of solute to solvent together with the large number of consecutive segments that constitute each single molecule of the polymer chains present as solute. Thorough understanding of the physical chemistry of polymer solutions requires some prior mathematical background in its students. In the original literature, detailed mathematical derivations of the equations are universally omitted for the sake of space-saving and simplicity, which physicists hold in high regard. However, students learning about the physical chemistry of polymer solutions are predominantly chemists, whose physical and mathematical skills are not necessarily high. In textbooks of polymer science only extremely rough schemes of the theories and then the final equations are shown. As a consequence, the student cannot unaided learn the details of the theory in which he or she is interested from the existing textbooks. Without a full understanding of the theory, one cannot analyze actual experimental data to obtain more basic and realistic physical quantities. In particular, if one intends to apply the theories in industry, accurate understanding and ability to modify the theory as one wishes are essential.

This book is mainly concerned with building a narrow but secure ladder which polymer chemists or engineers can climb from the primary level to an advanced level without great difficulty (but by no means easily, either). The need for a book of this kind has been widely recognized for many years among polymer scientists, including ourselves, but as far as we know, no such book has yet been published. We believe that this book is, in that sense, unique and unparalleled. The Japanese manuscript was originally written by Kamide and nearly completed in the 1970s and 1980s, but was not published. Based on these Japanese manuscripts, Kamide gave a series of intensive lecture courses to students at universities of Kanazawa, Hokkaido, Okayama, Osaka City, Essex, Bristol and Bradford. The manuscripts, thanks to this valuable experience, have been repeatedly revised and improved. The English edition was originally planned for the beginning of the 1990s and the Japanese manuscript was drastically reduced to fit the size requested by the publishers and transformed in part into an English version for this book. Although this attempt was interrupted

by Kamide's illness, the book has now materialized through the enthusiastic cooperation of Professor Dobashi.

This book describes some fundamentally important topics, carefully chosen, covering subjects from thermodynamics to molecular weight and its distribution effects. For help in self-education the book adopts a "Questions and Answers" format. The mathematical derivation of each equation is shown in detail. For further reading, some original references are also given. The contents of the book frankly belong to the realm of classical physical chemistry, because it does not treat areas newly developed during the 1980s and 1990s, such as scaling theory and spectroscopic theory (e.g., NMR). To add chapters about these areas is beyond both our ability and the planned size of the book. The reader should not, however, consider that all the theories covered by the book have already been firmly established. Any topic, although apparently mature and fully grown, may start to develop quickly again due to the advent of a new motive force, hitherto unknown, and so may present important new unsolved problems. We experienced this in the study of phase separation and critical phenomena of multicomponent polymer solutions during the 1970s and 1980s (see, for example, K. Kamide, "Thermodynamics of Polymer Solutions : Phase Separation and Critical Phenomena", Elsevier, 1990). Throwing out "old" and jumping after "new" techniques seems an inevitable fashion intrinsic as inherent to polymer science as to other disciplines of applied science, but is not always unconditionally reasonable.

The authors will be very gratified if this book proves a help not only to students at universities and to industrial researchers, who may be studying the physical chemistry of polymer solutions and dissatisfied with existing books, but also, as a reference book, to technologists intending to apply the physical chemistry of polymer solutions to industrial practice (but not as a simple quality control method, like a fully automated analytical instrument!) and to educators teaching this or related subjects. We should like to thank Professor J. Eric McIntyre of University of Leeds, UK for his meticulous reading of the manuscript, and for important suggestions. The authors would like to offer thanks to Dr Masatoshi Saito, Dr Shigenobu Matsuda, Dr Hironobu Shirataki, Dr Kunio Hisatani, Professor Yukio Miyazaki, Mr Kazuishi Sato, Mr Yuji Ito and Mr Katsunari Yasuda of the Fundamental Research Laboratory of Fibers and Fiber-Forming Polymers, Asahi Chemical Industry Company, Ltd., Takatsuki, Osaka, Japan, who cooperated in the downsizing of the Japanese

manuscript and in re-examination of the mathematical derivations in the text although the authors are, of course, responsible for any possible errors and mistakes in the book.

The authors have a pleasure of thanking Professor Motozo Kaneko of Hokkaido University for sending us complete solution of <<Problem 8-45>> and also wish to acknowledge the support of typing the manuscript to Mr Takashi Sato of Gunma University.

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## Glossary

A: Area

$A_i$  :  $i$  th virial coefficient

$a_i$  : activity of  $i$  th component

B: enthalpy per contact area

b: bond length

$b_m$  :  $m$  th cluster integral

C: weight concentration

$c_p$  : specific heat under constant pressure

$c_v$  : specific heat under constant volume

c: velocity of light in vacuum

D: diffusion coefficient

D: electric displacement

d: diameter

$d'Q$ : differential heat

$d'W$ : differential work

E: energy

**E**: electric field

**E**: unit tensor

e: strain

**e**: strain tensor

F: Helmholtz free energy

$F_N$ : N -body partition function

**F**: force

$f_D$ : friction coefficient for translational motion

$f_s$ : friction coefficient for sedimentation

$f_n(n)dn$ : number fraction of the polymer with the degree of polymerization  $n$

$f_w(n)dn$ : weight fraction of the polymer with the degree of polymerization  $n$

G: Gibbs free energy

$\bar{G}$ : mean molar Gibbs free energy

$G_{ij}$  : differential of Gibbs free energy with respect to the mole fractions of  $i$  th component and  $j$  th component

g: velocity gradient

g: Huggins' free energy correction factor

$g_n(M)$ : number distribution of molecular weight

$g_w(M)$ : weight distribution of molecular weight  
 $H$ : enthalpy  
 $H_i$  : partial molar enthalpy of  $i$  th component  
 $h$ : Planck constant  
 $h$ : inhomogeneity parameter for molecular weight distribution  
 $I_0$ : incident light intensity  
 $I_s$ : scattered light intensity  
 $I_t$ : transmitted light intensity  
 $I$ : inertia moment  
 $J$ : extensive thermodynamic variable  
 $j$ : flow by diffusion  
 $K$ : tension  
 $K$ : Flory constant  
 $K$ : optical constant  
 $k$ : Boltzmann constant  
 $k$ : degree of connectivity  
 $k$ : reaction constant  
 $k'$ : Huggins' constant  
 $L$ : camera length  
 $L_0$ : molar heat of vaporization  
 $l$ : segment length  
 $l$ : position vector of segment  
 $M$ : molecular weight  
 $M_n$ : number-average molecular weight  
 $M_w$ : weight-average molecular weight  
 $M_z$ : z-average molecular weight  
 $M_v$ : viscosity-average molecular weight  
 $M_{SD}$ : sedimentation-diffusion-average molecular weight  
 $m_0$ : molecular weight of segment  
 $N$ : number of moles of molecule  
 $N$ : number of molecules  
 $N_A$ : Avogadro's number  
 $n$ : association number  
 $n$ : segment number or chain length  
 $n$ : number density  
 $n_m$ : medium for the degree of polymerization

- $n_n$ : number-average degree of polymerization  
 $n_w$ : weight-average degree of polymerization  
 $n_z$ : z-average degree of polymerization  
 $n_v$ : viscosity-average degree of polymerization  
 $n_r$ : refractive index of solution  
 $n_r^0$ : refractive index of solvent  
 $P$ : pressure  
 $\mathbf{P}$ : polarization  
 $\mathbf{P}$ : stress tensor  
 $P_i$ : vapor pressure of  $i$  th component  
 $P_i^0$ : vapor pressure of  $i$  th component before mixing  
 $P(q)$ : particle scattering factor  
 $p$ : probability that condensation reaction occurs  
 $p_i$ : coefficient for concentration dependence of  $\chi$   
 $Q$ : heat  
 $q_1$ : molecular partition function  
 $q_{w,z}$ : correction factor for molecular weight distribution  
 $\mathbf{q}$ : wave vector  
 $R$ : gas constant  
 $R$ : phase volume ratio  
 $R$ : Reynolds number  
 $R$ : end-to-end distance  
 $\mathbf{R}$ : vector connecting one end to another end of polymer  
 $R_e$ : radius of hydrodynamically identical sphere  
 $R_\theta$ : Rayleigh ratio  
 $R_\theta'$ : reduced scattered light intensity  
 $\langle R^2 \rangle$ : mean square end-to-end distance  
 $\langle R^2 \rangle_0$ : mean square end-to-end distance of unperturbed chain  
 $r$ : radius  
 $S$ : entropy  
 $S$ : radius of gyration  
 $S^0$ : entropy before mixing  
 $S_i$ : partial molar entropy of  $i$  th component  
 $\langle S^2 \rangle$ : mean square radius of gyration  
 $\mathbf{s}$ : scattering vector  
 $s_0$ : sedimentation coefficient at infinite dilution

- $T$ : absolute temperature  
 $T$ : matrix for rotational mapping  
 $T_c$ : critical solution temperature  
 $t$ : time  
 $U$ : internal energy  
 $u$ : potential energy  
 $V$ : volume  
 $V'$ : volume of dilute phase  
 $V''$ : volume of concentrated phase  
 $V_i$ : partial molar volume of  $i$  th component  
 $V_0$ : molar volume  
 $V_0^0$ : molar volume of solvent  
 $V^0$ : molar volume before mixing  
 $v_0$ : volume of molecule  
 $\bar{v}$ : partial specific volume  
 $\mathbf{v}$ : velocity  
 $W$ : work  
 $w$ : interchange energy  
 $w$ : mean force potential between solute molecules  
 $w_i$ : weight fraction of  $i$  th component  
 $X$ : parameter for free draining  
 $x$ : distance from rotational axis to surface  
 $x_i$ : mole fraction of  $i$  th component  
 $x_c$ : critical mole fraction  
 $Y$ : increment of refractive index  
 $Z$ : partition function for canonical ensemble  
 $Z$ : excluded volume parameter  
 $z$ : coordination number  
  
 $\alpha$ : expansion factor  
 $\alpha$ : polarizability  
 $\alpha$ : volumetric thermal expansion coefficient at constant pressure  
 $\alpha_R$ : end distance expansion factor  
 $\alpha_S$ : radius expansion factor  
 $\alpha_\eta$ : viscosity expansion factor

$\beta$  : effective two-body excluded volume per segment pair or two-body cluster integral  
 $\gamma$  : shear modulus  
 $\gamma_i$  : activity coefficient of  $i$  th component  
 $\Delta G_{\text{mix}}$ : Gibbs free energy of mixing  
 $\Delta G_v$ : Gibbs free energy of mixing per unit volume  
 $\Delta H_{\text{mix}}$ : enthalpy of mixing  
 $\Delta Q_{\text{mix}}$ : heat of mixing  
 $\Delta S_{\text{mix}}$ : entropy of mixing  
 $\Delta V_{\text{mix}}$ : mixing volume  
 $\Delta \mu_i$ : chemical potential of mixing of  $i$  th component  
 (superscripts id and E denote the quantities for ideal solution and excess quantities)  
 $\delta C$ : fluctuation of concentration  
 $\epsilon$  : dielectric constant  
 $\epsilon$  : interaction energy  
 $\zeta$  : friction coefficient between fluid and small particle  
 $\eta$ : viscosity coefficient  
 $\eta_0$ : viscosity coefficient of solvent  
 $\eta_r$ : relative viscosity  
 $\eta_{sp}$ : specific viscosity  
 $[\eta]$ : intrinsic viscosity or limiting viscosity number  
 $\eta_v$  : volumetric viscosity  
 $\Xi$  : partition function for grand canonical ensemble  
 $\xi$  : partition function for semi-grand canonical ensemble  
 $\theta$ : Flory temperature  
 $\theta$ : bond angle  
 $\kappa$  : isothermal compressibility  
 $\kappa$  : enthalpy parameter  
 $\kappa$  : elastic modulus  
 $\kappa_0$  : Flory enthalpy parameter at infinite dilution  
 $\lambda$ : Lamé constant  
 $\lambda$  : absolute activity  
 $\lambda_0$  : wave length of incident light  
 $\mu$  : permeability  
 $\mu$  : Lamé constant

$\mu$  : moment for molecular weight distribution

$\mu_i$  : chemical potential of  $i$  th component

$\mu'_i$  : chemical potential of  $i$  th component in vapor phase

$\mu^0$  : chemical potential before mixing

$\mu_0^0$  : chemical potential of solvent

$\mu_{ij}$  : differential of chemical potential with respect to mole fractions of  $i$  th component and  $j$  th component

$\Pi$ : osmotic pressure

$\rho$ : density

$\rho_s$ : weight fraction of polymer partitioned in dilute phase

$\rho_p$  : weight fraction of polymer partitioned in concentrated phase

$\sigma$  : partition coefficient

$\sigma$ : stress

$\sigma$ : steric hindrance factor

$\sigma$ : standard deviation

$\tau$ : turbidity

$\tau$ : delay time

$\Phi$ : Flory-Fox viscosity coefficient

$\phi_0$ : volume fraction of solvent

$\phi_1$ : volume fraction of solute

$\phi_n$ : volume fraction of polymer with the degree of polymerization  $n$

$\chi$ : thermodynamic interaction parameter

$\Psi$  : penetrating function

$\psi$ : entropy parameter

$\psi_0$ : Flory entropy parameter at infinite dilution

$\Omega$ : partition function for microcanonical ensemble

$\omega$ : vibration number

$\omega$ : angular velocity of molecular chain

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