

**ALM 35**

Advanced Lectures in Mathematics

# The Legacy of Bernhard Riemann After One Hundred and Fifty Years

黎曼的学术遗产：150 年巡礼

Editors: Lizhen Ji • Frans Oort • Shing-Tung Yau



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## ALM

The launch of this Advanced Lectures in Mathematics series is aimed at keeping mathematicians informed of the latest developments in mathematics, as well as to aid in the learning of new mathematical topics by students all over the world. Each volume consists of either an expository monograph or a collection of significant introductions to important topics. This series emphasizes the history and sources of motivation for the topics under discussion, and also gives an overview of the current status of research in each particular field. These volumes are the first source to which people will turn in order to learn new subjects and to discover the latest results of many cutting-edge fields in mathematics.

### The Legacy of Bernhard Riemann After One Hundred and Fifty Years 黎曼的学术遗产：150 年巡礼

黎曼 (1826—1866) 对数学有着独特的创造力和广博的洞察力，同时对数学的处理富有强有力技巧和远见的思想。在他短暂的一生中，黎曼对数学几乎所有的分支都有着广泛的影响。时至 150 年后的今日，我们仍然要尝试理解在数学后续发展中的黎曼的工作以及他的数学思想。

我们邀请了一批具有前沿思想的专家撰写了相关的综述和展望性质的文章，希望这些专题也会是黎曼所感兴趣的。

在本文集中，我们讨论了若干历史细节，并且着重介绍了黎曼所开创的数学领域的现代发展。本书对于希望了解黎曼深邃的数学思想及其对现代数学影响的读者具有重要的参考价值。



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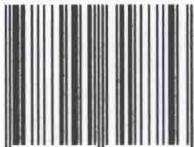


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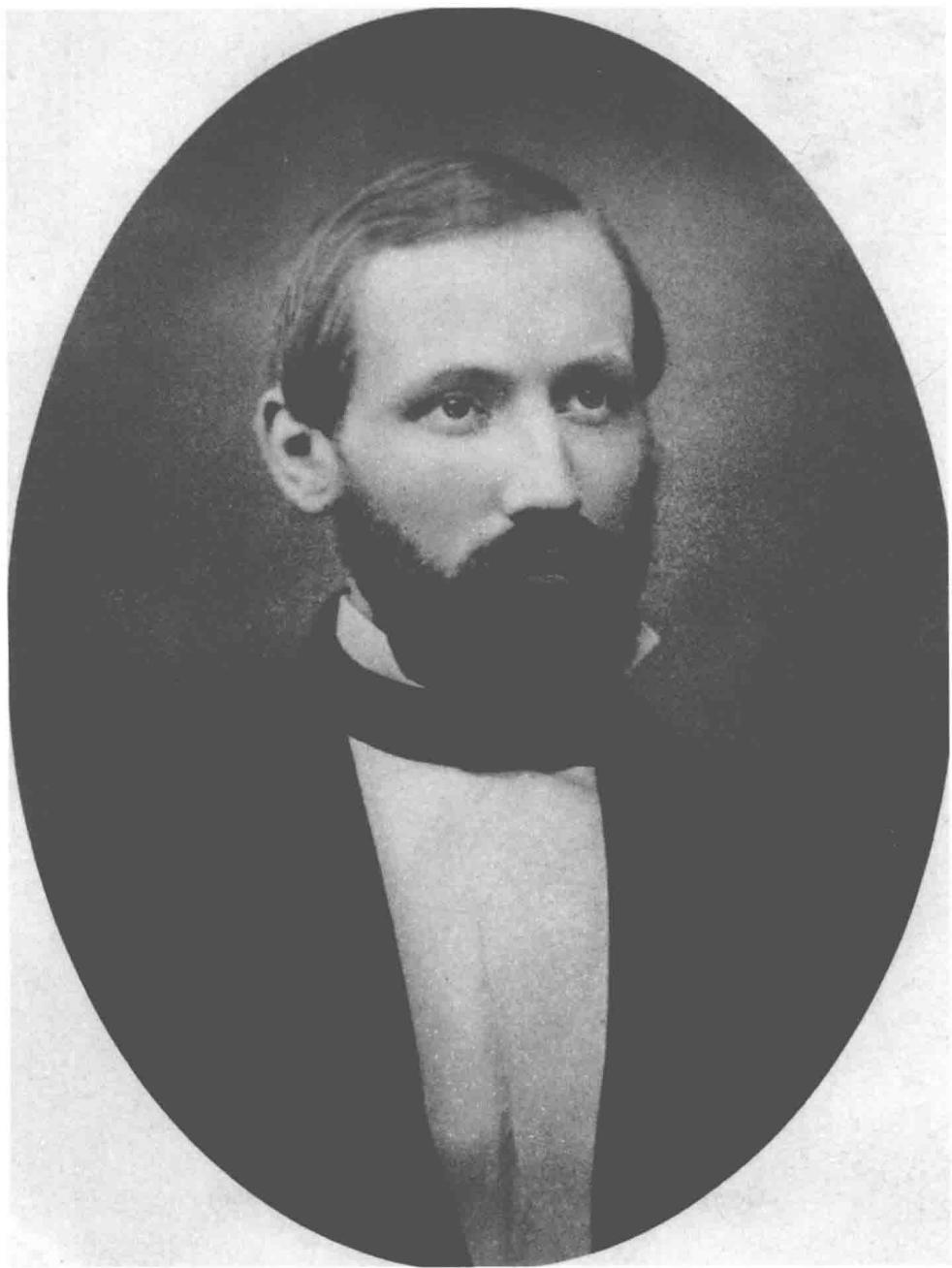
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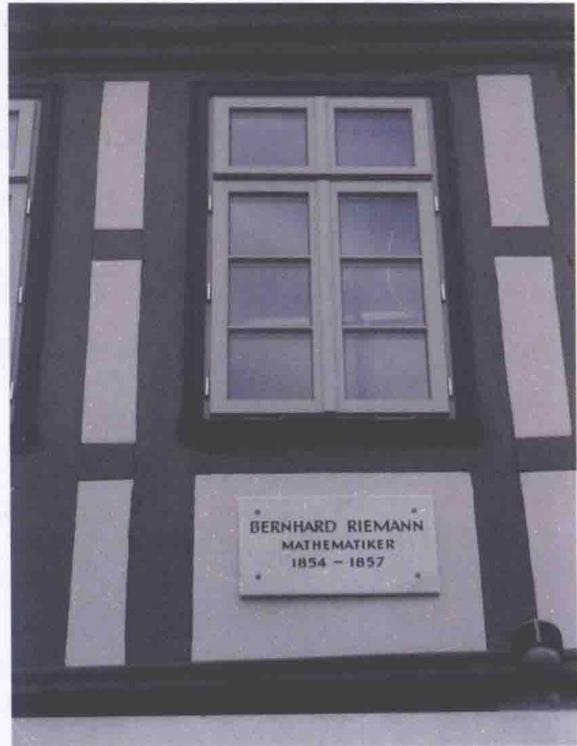
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Georg Friedrich Bernhard Riemann (1826—1866)



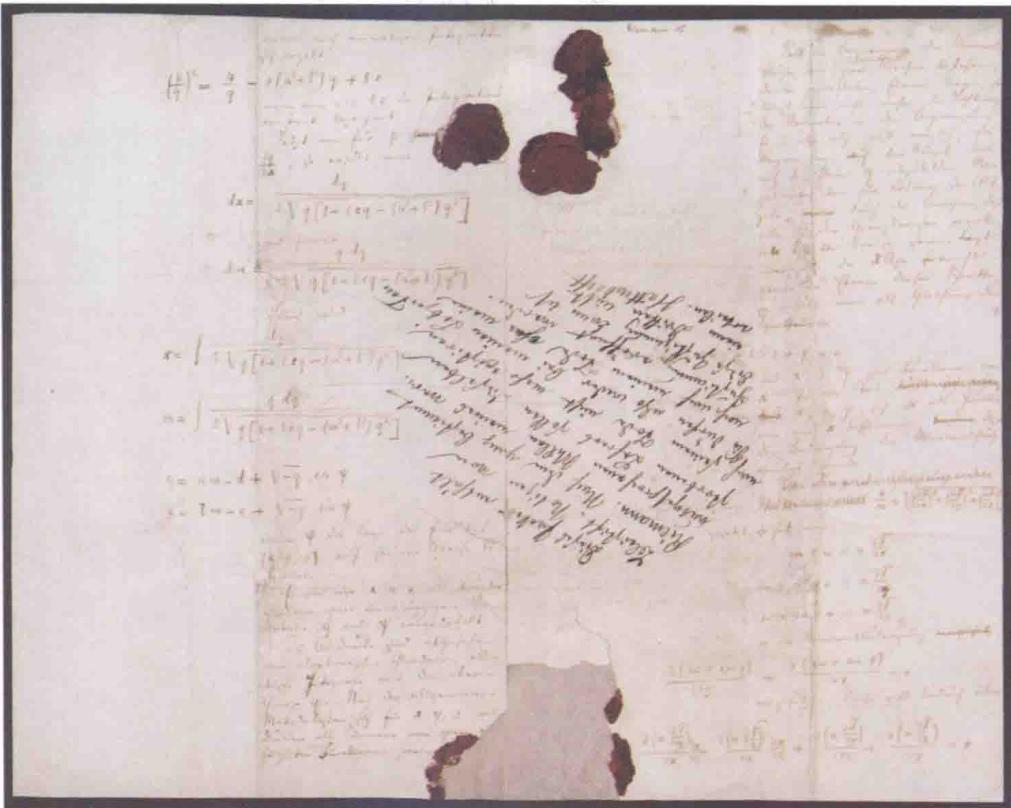
Bernhard Riemann as a student



Bernhard Riemann's former residence in Göttingen (1854—1857)



Bernhard Riemann's tombstone in Biganzolo, Italy



Über die Fläche vom kleinsten Inhalt bei gegebener Begrenzung (On the surface of least area with a given boundary) / Bernhard Riemann, Richard Dedekind, Heinrich Weber, Karl Hattendorff

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Handwritten mathematical notes by Bernhard Riemann, likely related to his work on the distribution of prime numbers. The notes include complex equations, summations, and calculations involving primes.

Über die Anzahl der Primzahlen unter einer gegebenen Grösse ; Monatsberichte der Berliner, Akademie, November 1859 (On the number of primes less than a given magnitude) / Bernhard Riemann

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