



Constraint Networks

Techniques and Algorithms

Christophe Lecoutre

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Christophe Lecoutre

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Narendra Jussien

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Contents

Acknowledgements	11
Notation	13
Main Acronyms	19
List of Algorithms	21
Introduction	27
Chapter 1. Constraint Networks	39
1.1. Variables and constraints	39
1.2. Networks of variables and constraints	51
1.2.1. Basic definitions	51
1.2.2. Associated (hyper)graphs	56
1.2.3. Instantiations and solutions	59
1.3. Examples of constraint networks	65
1.3.1. Queens problem	65
1.3.2. Crossword problem	68
1.3.3. Sudoku problem	70
1.3.4. Edge-matching puzzles	72
1.4. Partial orders, decisions, nogoods and properties	74
1.4.1. Partial orders	74
1.4.2. Decisions and nogoods	78
1.4.3. Properties on values and variables	81
1.5. Data structures to represent constraint networks	86
1.5.1. Representation of finite domains	86
1.5.2. Representation of constraints	90

Chapter 2. Random and Structured Networks	93
2.1. Random constraint networks	94
2.1.1. Classical models	94
2.1.2. Models RB and RD	100
2.1.3. Random constraint networks in intension	103
2.1.4. Benchmarks	107
2.1.4.1. Random series	107
2.1.4.2. Random series containing a small structure	108
2.2. Structured constraint networks	109
2.2.1. Backbones and backdoors	109
2.2.2. Cores and cliques	112
2.2.3. Acyclicity	115
2.2.4. Small world structure and morphing technique	121
2.2.5. Benchmarks	122
2.2.5.1. Main series	123
2.2.5.2. Other series	127
PART ONE. INFERENCE	133
Chapter 3. Consistencies	137
3.1. Basic consistencies	138
3.2. Stability of consistencies	143
3.3. Domain-filtering consistencies	150
3.4. Higher-order consistencies	162
3.4.1. Taking the right path	165
3.4.2. Relation-based consistencies	170
3.5. Global consistency	173
3.5.1. Identifying global consistency	173
3.5.2. Toward tractability	178
3.5.2.1. Relational CSP classes	179
3.5.2.2. Structural CSP classes	181
3.5.2.3. Hybrid CSP classes	183
3.6. Caveats about node, arc and path consistencies	184
Chapter 4. Generic GAC Algorithms	185
4.1. Coarse-grained propagation schemes	186
4.1.1. Arc-oriented propagation scheme	187
4.1.2. Variable-oriented propagation scheme	190
4.1.3. Applying forward checking	195
4.2. Iterating over valid tuples	197
4.3. GAC3 and GAC2001	200
4.4. More about general-purpose GAC algorithms	205

4.4.1. Important properties	205
4.4.1.1. Incrementality	205
4.4.1.2. Multi-directionality	207
4.4.1.3. Substitutability	207
4.4.2. Overview	209
4.5. Improving the efficiency of generic GAC algorithms	214
4.5.1. Exploiting cardinality of conflict sets	215
4.5.2. Exploiting residues	219
4.5.2.1. Algorithm GAC3 ^{rm}	219
4.5.2.2. Complexity issues	221
4.5.2.3. Residues within MAC	223
4.5.3. Exploiting bitwise operations	224
4.5.3.1. Binary representation	227
4.5.3.2. Algorithms AC3 ^{bit} and AC3 ^{bit+rm}	231
4.6. Experimental results	233
4.7. Discussion	236
Chapter 5. Generalized Arc Consistency for Table Constraints	239
5.1. Classical schemes	240
5.1.1. Positive table constraints	240
5.1.2. GAC-valid scheme	241
5.1.3. GAC-allowed scheme	242
5.1.4. Illustration	243
5.2. Indexing-based approaches	244
5.2.1. NextIn indexing	244
5.2.2. NextDiff indexing	249
5.3. Compression-based approaches	253
5.3.1. Tries	253
5.3.2. Multi-valued decision diagrams	257
5.3.3. Compressed tables	261
5.3.4. Deterministic finite automata	263
5.4. GAC-valid+allowed scheme	264
5.4.1. Using binary search	265
5.4.2. Using tries	267
5.5. Simple tabular reduction	269
5.5.1. Original algorithm	269
5.5.2. Optimizing STR	274
5.5.3. Relationship with GAC4	278
5.6. GAC for negative table constraints	279
5.6.1. Negative table constraints	279
5.6.2. GAC-valid scheme	279
5.6.3. GAC-valid+forbidden scheme	280
5.6.4. Compressed tuples and MDDs	281

5.7. Experimental results	283
5.8. Conclusion	286
Chapter 6. Singleton Arc Consistency	287
6.1. SAC1 and SAC2	289
6.2. SAC-Opt and SAC-SDS	290
6.3. SAC3	292
6.4. SAC3+	296
6.5. Illustration	299
6.6. Weaker and stronger forms of SAC	300
6.6.1. Existential SAC	300
6.6.2. Weak k -singleton arc consistency	311
6.7. Experimental results	313
6.8. Conclusion	316
Chapter 7. Path and Dual Consistency	319
7.1. Qualitative study	321
7.2. Enforcing (conservative) path consistency	331
7.2.1. Algorithms PC8 and PC2001	331
7.2.2. Algorithms CPC8 and CPC2001	334
7.3. Enforcing strong (conservative) dual consistency	336
7.3.1. Algorithm sCDC1	336
7.3.2. Algorithm sDC2	339
7.3.3. Illustration	343
7.3.4. Discussion	343
7.3.4.1. Comparison with CPC	343
7.3.4.2. Path consistency by dual consistency	348
7.4. Experimental results	348
7.4.1. With CDC algorithms	349
7.4.2. With DC algorithms	349
7.5. Conclusion	353
PART TWO. SEARCH	355
Chapter 8. Backtrack Search	359
8.1. General description	361
8.2. Maintaining (generalized) arc consistency	367
8.3. Classical look-ahead and look-back schemes	370
8.3.1. A general backtracking algorithm	371
8.3.2. The kernel of the algorithm	373
8.3.3. Dealing with constraint propagation	373
8.3.4. Closely related algorithms	377

8.4. Illustrations	378
8.5. The role of explanations	383
Chapter 9. Guiding Search toward Conflicts	391
9.1. Search-guiding heuristics	392
9.1.1. Classical variable ordering heuristics	393
9.1.2. Value ordering heuristics	397
9.2. Adaptive heuristics	398
9.2.1. Using constraint weighting	399
9.2.2. Using impacts	404
9.3. Strength of constraint weighting	405
9.3.1. Boosting systematic search	405
9.3.2. Identifying small unsatisfiable cores	408
9.3.3. Questioning heuristic policies	412
9.3.4. Statistical analysis and extensions	413
9.4. Guiding search to culprit decisions	415
9.4.1. Nogood identification through testing-sets	416
9.4.2. Reasoning from the last conflict	418
9.4.3. Generalized reasoning from the last conflict	424
9.4.4. Experimental results	427
9.5. Conclusion	427
Chapter 10. Restarts and Nogood Recording	431
10.1. Restarting search	432
10.1.1. Heavy-tailed behavior	433
10.1.2. Restart strategies	434
10.2. Nogood recording from restarts	436
10.2.1. Reduced nld-nogoods	436
10.2.2. Extracting nogoods	439
10.3. Managing standard nogoods	441
10.3.1. Nogood constraints	441
10.3.2. Watched decisions	443
10.3.3. Making inferences	447
10.3.4. Complexity analysis	449
10.4. Minimizing nogoods	450
10.4.1. Minimal ϕ -nogoods	450
10.4.2. Minimization techniques	451
10.4.3. Complexity analysis	452
10.5. Experimental results	454
10.6. Conclusion	457
Chapter 11. State-based Reasoning	459

11.1. Inconsistent partial states	460
11.1.1. Definitions	460
11.1.2. Pruning the search tree	464
11.2. Learning from explanations and failed values	470
11.2.1. Learning generalized nogoods	470
11.2.2. Reasoning from failed values	473
11.3. Reducing elementary inconsistent partial states	476
11.3.1. E-eliminable variables	477
11.3.2. Proof-based extraction	481
11.3.3. Justification-based extraction	483
11.4. Equivalence detection	487
11.5. Experimental results	492
11.6. Conclusion	494
Chapter 12. Symmetry Breaking	495
Christophe LECOUTRE, Sébastien TABARY	
12.1. Group theory	496
12.2. Symmetries on constraint networks	499
12.3. Symmetry-breaking methods	503
12.3.1. Symmetry-breaking constraints	504
12.3.2. Dynamic symmetry breaking	506
12.4. Automatic symmetry detection	508
12.5. Lightweight detection of variable symmetries	511
12.5.1. Locally symmetric variables	512
12.5.2. Computing normal forms of predicate expressions	516
12.5.3. Constructing lsv-graphs	518
12.6. A GAC algorithm for lexicographic ordering constraints	520
12.7. Experimental results	527
Appendices	531
A. Mathematical Background	531
A.1. Sets, relations, graphs and trees	531
A.2. Complexity	536
B. XML Representation of Constraint Networks	541
Bibliography	547
Index	571

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Contents

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Chapter 1. Constraint Networks	39
1.1. Variables and constraints	39
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1.2.2. Associated (hyper)graphs	56
1.2.3. Instantiations and solutions	59
1.3. Examples of constraint networks	65
1.3.1. Queens problem	65
1.3.2. Crossword problem	68
1.3.3. Sudoku problem	70
1.3.4. Edge-matching puzzles	72
1.4. Partial orders, decisions, nogoods and properties	74
1.4.1. Partial orders	74
1.4.2. Decisions and nogoods	78
1.4.3. Properties on values and variables	81
1.5. Data structures to represent constraint networks	86
1.5.1. Representation of finite domains	86
1.5.2. Representation of constraints	90

Chapter 2. Random and Structured Networks	93
2.1. Random constraint networks	94
2.1.1. Classical models	94
2.1.2. Models RB and RD	100
2.1.3. Random constraint networks in intension	103
2.1.4. Benchmarks	107
2.1.4.1. Random series	107
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2.2. Structured constraint networks	109
2.2.1. Backbones and backdoors	109
2.2.2. Cores and cliques	112
2.2.3. Acyclicity	115
2.2.4. Small world structure and morphing technique	121
2.2.5. Benchmarks	122
2.2.5.1. Main series	123
2.2.5.2. Other series	127
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3.4.1. Taking the right path	165
3.4.2. Relation-based consistencies	170
3.5. Global consistency	173
3.5.1. Identifying global consistency	173
3.5.2. Toward tractability	178
3.5.2.1. Relational CSP classes	179
3.5.2.2. Structural CSP classes	181
3.5.2.3. Hybrid CSP classes	183
3.6. Caveats about node, arc and path consistencies	184
Chapter 4. Generic GAC Algorithms	185
4.1. Coarse-grained propagation schemes	186
4.1.1. Arc-oriented propagation scheme	187
4.1.2. Variable-oriented propagation scheme	190
4.1.3. Applying forward checking	195
4.2. Iterating over valid tuples	197
4.3. GAC3 and GAC2001	200
4.4. More about general-purpose GAC algorithms	205

4.4.1. Important properties	205
4.4.1.1. Incrementality	205
4.4.1.2. Multi-directionality	207
4.4.1.3. Substitutability	207
4.4.2. Overview	209
4.5. Improving the efficiency of generic GAC algorithms	214
4.5.1. Exploiting cardinality of conflict sets	215
4.5.2. Exploiting residues	219
4.5.2.1. Algorithm GAC3 ^{rm}	219
4.5.2.2. Complexity issues	221
4.5.2.3. Residues within MAC	223
4.5.3. Exploiting bitwise operations	224
4.5.3.1. Binary representation	227
4.5.3.2. Algorithms AC3 ^{bit} and AC3 ^{bit+rm}	231
4.6. Experimental results	233
4.7. Discussion	236

Chapter 5. Generalized Arc Consistency for Table Constraints 239

5.1. Classical schemes	240
5.1.1. Positive table constraints	240
5.1.2. GAC-valid scheme	241
5.1.3. GAC-allowed scheme	242
5.1.4. Illustration	243
5.2. Indexing-based approaches	244
5.2.1. NextIn indexing	244
5.2.2. NextDiff indexing	249
5.3. Compression-based approaches	253
5.3.1. Tries	253
5.3.2. Multi-valued decision diagrams	257
5.3.3. Compressed tables	261
5.3.4. Deterministic finite automata	263
5.4. GAC-valid+allowed scheme	264
5.4.1. Using binary search	265
5.4.2. Using tries	267
5.5. Simple tabular reduction	269
5.5.1. Original algorithm	269
5.5.2. Optimizing STR	274
5.5.3. Relationship with GAC4	278
5.6. GAC for negative table constraints	279
5.6.1. Negative table constraints	279
5.6.2. GAC-valid scheme	279
5.6.3. GAC-valid+forbidden scheme	280
5.6.4. Compressed tuples and MDDs	281

5.7. Experimental results	283
5.8. Conclusion	286
Chapter 6. Singleton Arc Consistency	287
6.1. SAC1 and SAC2	289
6.2. SAC-Opt and SAC-SDS	290
6.3. SAC3	292
6.4. SAC3+	296
6.5. Illustration	299
6.6. Weaker and stronger forms of SAC	300
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6.6.2. Weak k -singleton arc consistency	311
6.7. Experimental results	313
6.8. Conclusion	316
Chapter 7. Path and Dual Consistency	319
7.1. Qualitative study	321
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7.2.1. Algorithms PC8 and PC2001	331
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7.3.1. Algorithm sCDC1	336
7.3.2. Algorithm sDC2	339
7.3.3. Illustration	343
7.3.4. Discussion	343
7.3.4.1. Comparison with CPC	343
7.3.4.2. Path consistency by dual consistency	348
7.4. Experimental results	348
7.4.1. With CDC algorithms	349
7.4.2. With DC algorithms	349
7.5. Conclusion	353
PART TWO. SEARCH	355
Chapter 8. Backtrack Search	359
8.1. General description	361
8.2. Maintaining (generalized) arc consistency	367
8.3. Classical look-ahead and look-back schemes	370
8.3.1. A general backtracking algorithm	371
8.3.2. The kernel of the algorithm	373
8.3.3. Dealing with constraint propagation	373
8.3.4. Closely related algorithms	377