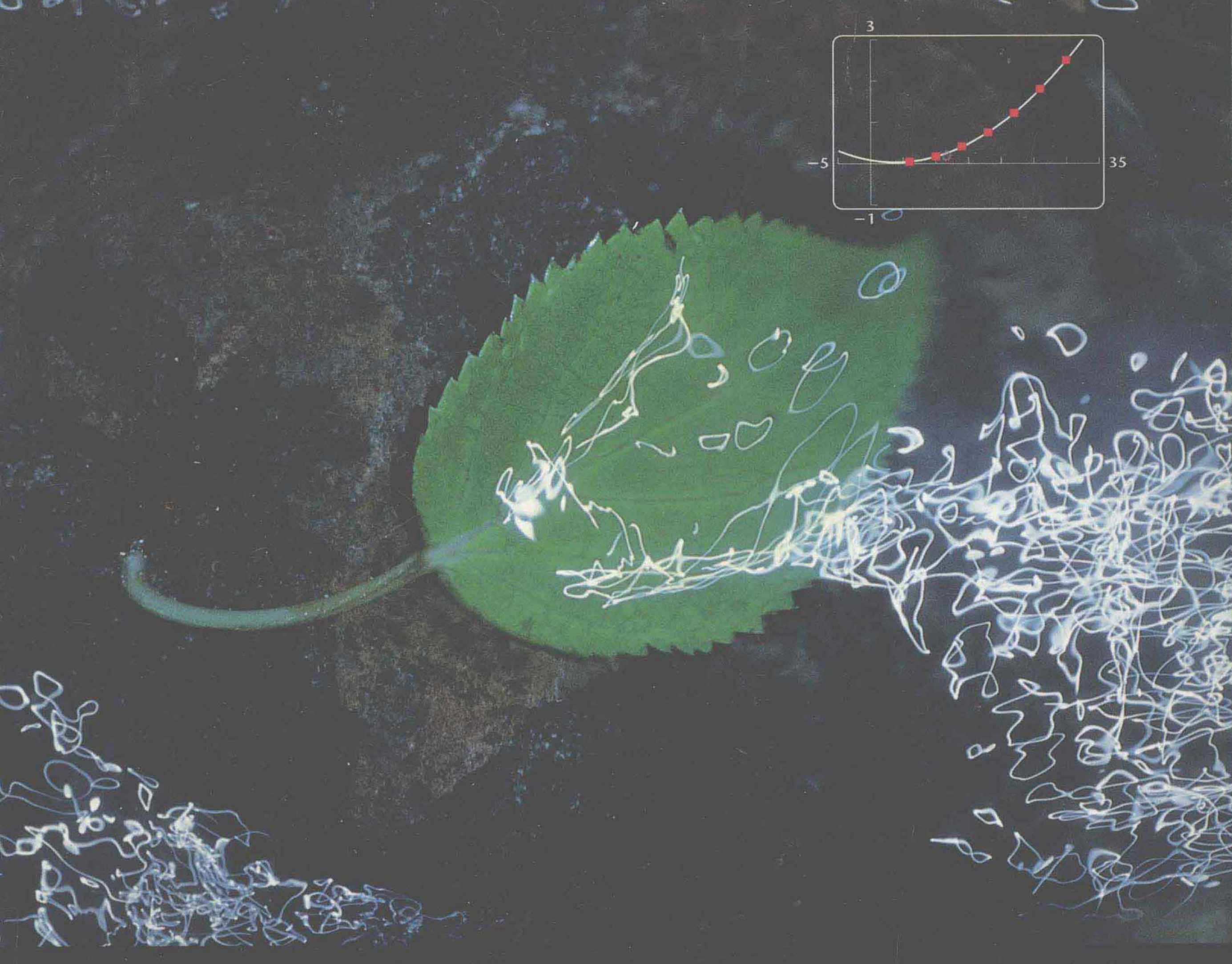
PRECALCULUS

GRAPHS & MODELS



BITTINGER • BEECHER • ELLENBOGEN • PENNA

Precalculus

GRAPHS AND MODELS

Marvin L. Bittinger
Indiana University–Purdue University at Indianapolis

Judith A. Beecher
Indiana University–Purdue University at Indianapolis

David Ellenbogen
St. Michael's College

Judith A. Penna
Indiana University–Purdue University at Indianapolis

ADDISON-WESLEY

An imprint of Addison Wesley Longman, Inc.

Reading, Massachusetts • Menlo Park, California • New York • Harlow, England
Don Mills, Ontario • Sydney • Mexico City • Madrid • Amsterdam

Sponsoring Editor	Jason Jordan
Project Manager	Kari Heen
Managing Editor	Karen Guardino
Text Design	Geri Davis/The Davis Group, Inc.
Art, Design, Editorial, and Production Services	Martha Morong/Quadrata, Inc. Geri Davis/The Davis Group, Inc.
Marketing Manager	Michelle Babinec
Illustrators	Scientific Illustrators, J. A. K. Graphics, Parrot Graphics, and Gail Hayes
Compositor	The Beacon Group
Cover Designer	Karen Rappaport
Cover Photograph	Takeshi Odawara/PHOTONICA

Photo Credits

134, PF Sports Images **140 (left),** SuperStock **309,** SuperStock **314,** The Topps Company, Inc. **366,** Darryl Jones **371,** courtesy of Goss Graphics Systems **702,** © Mary Dyer

2, Graph on number of births: From THE NEW YORK TIMES, 2/12/95, p. A1. Copyright © 1995 by The New York Times Company. Reprinted with permission.

Library of Congress Cataloging-in-Publication Data

Algebra & trigonometry: graphs & models/Marvin L. Bittinger . . . [et al.].
p. cm.
Includes index.
ISBN 0-201-84888-0
ISBN 0-201-69442-5 (Precalculus)
1. Algebra. 2. Algebra—Graphic methods. 3. Trigonometry.
4. Trigonometry—Graphic methods. I. Bittinger, Marvin L.
QA154.2.A3124 1996
512′ .13—dc20 96-21889
CIP

5 6 7 8 9 10 – RNV – 01 00 99 98

Preface

Precalculus: Graphs and Models covers college-level algebra and trigonometry and is appropriate for a one- or two-term course in precalculus mathematics. The approach of this text is more interactive than most other precalculus texts. Our goal is to enhance the learning process through the use of technology and to provide as much support and help for students as possible in their study of algebra and trigonometry. A course in intermediate algebra is a prerequisite for the text, although Chapter R provides sufficient review to unify the diverse backgrounds of most students.

Content Features

- **Integrated Technology** The technology of the graphing calculator is completely integrated throughout the text to provide a visual means of increasing understanding. In this text, we use the term "grapher" to refer to all graphing calculator technology. The use of the grapher is woven throughout the exposition, the exercise sets, and the testing program without sacrificing algebraic and trigonometric skills. We use the grapher technology to enhance, not to replace, the students' mathematical skills and to alleviate the tedium associated with certain procedures. It is assumed that each student is required to have a grapher (or at least access to one) while enrolled in this course.

- **Learning to Use the Technology** To minimize the need for valuable class time to teach students how to use a grapher, we have provided several features that shorten the learning curve while increasing the students' knowledge of the fundamentals of a grapher. The first of these is the section entitled "Introduction to Graphs and Graphers" found at the beginning of the text. It introduces students to the basic functions of the grapher. The others are the *Graphing Calculator Manual* (see p. xiv) and the video series entitled *Graphing Calculator Instructional Videos* (see p. xiv). In addition, a set of programs has been included in the *Graphing Calculator Manual*. All of these features have been specifically written and produced for this text.

- **Interactive Discoveries** The grapher provides an exciting teaching opportunity in which a student can discover and further investigate mathematical concepts. This unique Interactive Discovery feature is used to introduce new topics and provides a vehicle for students to "see" a concept quickly. This feature reinforces the idea that grapher technology is an integral part of the course as well as an important learning tool. It invites the student to develop analytic and reasoning skills while taking an active role in the learning process. (See pp. 114, 226, and 392.)

- **Function Emphasis** The use of technology with its immediate visualization of a concept encourages the early presentation of functions. Graphing and functions are both introduced in the first section of Chapter 1. The study of the family of functions (linear, quadratic, higher-degree polynomial, rational, exponential, logarithmic, and trigonometric) has been enhanced and streamlined with the inclusion of the grapher. Applications with graphs are incorporated throughout to amplify and add relevance to the study of functions. (See pp. 84, 205, and 380.)

- **Variety in Approaches to Solutions** Skill in solving mathematical problems is expanded when a student is exposed to a variety of approaches to finding a solution. We have carefully incorporated three solution approaches throughout the text: algebraic, graphical, and numerical. Chapter openers illustrate an application with a concurrent grapher presentation of both a table and a graph (see pp. 171 and 521). The TABLE feature on a grapher provides a numerical display or check of the solution (see pp. 198 and 329).

  To highlight both the algebraic- and graphical-solution approaches in solving equations, we have used a two-column solution format in numerous examples (see pp. 184, 297, and 454). In the algebraic/graphical side-by-side features, both methods are presented together; each method provides a complete solution. This feature emphasizes that there is more than one way to obtain a result and illustrates the comparative efficiency and accuracy of the two methods.

- **Real-Data Applications** Throughout the writing process, we conducted an energetic search for real-data applications. The result of that effort is a variety of examples and exercises that connect the mathematical content with the real world. Source lines appear with most real-data applications and charts and graphs are frequently included. Many applications are drawn from the fields of health, business and economics, life and physical sciences, social science, and areas of general interest such as sports and daily life. We encourage students to "see" and interpret the mathematics that appears around them every day. (See pp. 134, 304, 371, and I-1.)

- **Regression** Using regression or curve fitting to model data is introduced in Chapter 1 with linear functions. This visual theme is continued with quadratic, cubic, quartic, higher-degree polynomial, exponential, logarithmic, logistic, and trigonometric functions. Although the theoretic aspects of curve fitting cannot be developed in this course, the power of the grapher is very apparent in this area as the technique is applied to real data. Students can quickly make the "what is this used for?" connection between real data and the extrapolated results of the curve fitting, thus giving them a better conceptual understanding of the material. (See pp. 129 and 312.)

- **Verifying Identities** Identities can be partially verified with a grapher using both the GRAPH and TABLE features. This use of the grapher is first seen in the Introduction to Graphs and Graphers and is continued in later chapters in discovery and verification of possible identities (see pp. 11, 287, and 413). This content feature allows a visual answer to such frequent questions as "Why isn't $(x + 2)^2$ equal to $x^2 + 4$?" This approach also provides a unique lead-in to the development of the properties of exponents and logarithms.

- **Optional Review Chapter** Chapter R provides an optional review of intermediate algebra. We purposely placed the Introduction to Graphs and Graphers before this chapter to allow the grapher to be used in the review. The incorporation of technology gives these topics a fresh perspective and sets the tone for the rest of the course (see pp. 41 and 60). Chapter R can also be used as a convenient source of information for a student who needs a quick review of a particular topic.

Pedagogical Features

- **Use of Color** The text uses full color in an extremely functional way, as seen in the design elements and artwork on nearly every page. The choice of color has been carried out in a methodical and precise manner so that its use carries a consistent meaning, which enhances the readability of the text for both student and instructor. (See pp. 93 and 397.)

- **Art Package** The text contains over 1500 art pieces including a new form of art called photorealism. Photorealism superimposes mathematics on a photograph and encourages students to "see mathematics" in familiar settings (see pp. 115 and 340). The exceptional situational art and statistical graphs throughout the text highlight the abundance of real-world applications while helping students visualize the mathematics (see pp. 205, 335, and 447). The design and use of color with the grapher windows exemplifies the impact that technology has in today's mathematical curriculum (see pp. 13, 159, and 448).

- **Annotated Examples** Over 770 examples fully prepare the student for the exercise sets. Learning is carefully guided with numerous color-coded art pieces and step-by-step annotations, with substitutions and annotations highlighted in red (see pp. 182 and 305). The basis for problem solving is a five-step process established early in the text to aid the student in strategically approaching and solving applications. (See pp. 72 and 527.)

- **Variety of Exercises** There are over 5500 exercises in this text. The exercise sets are enhanced not only by the inclusion of real-data applications with source lines, detailed art pieces, and technology windows that include both tables and graphs, but also by the following features.

  Technology Exercises Since the grapher is totally integrated in this text, exercise sets include both grapher and nongrapher exercises. In some cases, detailed instruction lines indicate the approach the student is expected to use. In others, the student is left to choose the approach that seems best, thereby encouraging critical thinking. (See pp. 194 and 302.)

  Skill Maintenance The exercises in this section have been specifically selected to review concepts previously taught in the text that are foundations for the material presented in the following section. They are chosen to prepare the student for the new concept(s) that will be covered next. (See p. 97.)

  Synthesis Exercises These exercises, which appear at the end of each exercise set, encourage critical thinking by requiring students to synthesize concepts from several sections or to take a concept a step further than in the regular exercises. (See p. 195.)

  Thinking and Writing Exercises for thinking and writing, at the beginning of the synthesis exercises, are denoted with a maze icon ◈. They encourage students to both consider and write about key mathematical ideas in the chapter. Many of these exercises are open-ended, making them particularly suitable for use in class discussions or as collaborative activities. (See p. 260.)

- **Chapter Openers** Each chapter opens with an application illustrated with both technology windows and situational art. The openers also include a table of contents listing section titles. (See pp. 83 and 249.)

- **Section Objectives** Content objectives are listed at the beginning of each section. These, together with subheadings throughout the section, provide a useful outline for both instructors and students. (See pp. 111 and 522.)

- **Highlighted Information** Important definitions, properties, and rules are displayed in screened boxes. Summaries and procedures are listed in color-outlined boxes. Both of these design features present and organize the material for efficient learning and review. (See pp. 116 and 123.)

- **Summary and Review** The Summary and Review at the end of each chapter provides an extensive set of review exercises along with a list of important properties and formulas covered in that chapter. This feature provides an excellent preparation for chapter tests and the final examination. Answers to all review exercises appear in the text along with section references that direct students to material to reexamine if they have difficulty with a particular exercise. (See pp. 318 and 404.)

Supplements for the Instructor

Instructor's Solutions Manual

The Instructor's Solutions Manual by Judith A. Penna contains worked-out solutions to all exercises in the exercise sets, including the thinking and writing exercises. It also includes a sample test with answers for each chapter and answers to the exercises in the appendixes. The sample tests are also included in the Student's Solutions Manual.

Printed Test Bank/Instructor's Manual

Prepared by Donna DeSpain, the Printed Test Bank/Instructor's Manual contains the following:

- 4 free-response test forms for each chapter, following the format and with the same level of difficulty as the tests in the Student's Solutions Manual.
- 2 multiple-choice test forms for each chapter.
- 6 alternate forms of the final examination, 4 with free-response questions and 2 with multiple-choice questions.
- Index to the Graphing Calculator Instructional Videos.

Testgen EQ

Testgen EQ is a computerized test generator that allows instructors to select test questions manually or randomly from selected topics or to use a ready-made test for each chapter. The test questions are algorithm-driven so that regenerated number values maintain problem types and provide a large number of test items in both multiple-choice and open-ended formats for one or more test forms. Test items can be viewed on screen, and the built-in question editor lets instructors modify existing questions or add new ones that include pictures, graphs, accurate math symbols, and variable text and numbers.

Additional features in the new Windows and Macintosh Test Generators allow the instructor to customize both the look and content of testbanks and tests. Test questions are easily transferred from the testbank to a test and can be sorted, searched, and displayed in various ways. Testgen EQ is free to adopters.

Course Management and Testing System

InterAct Math Plus for Windows and Macintosh (available from Addison Wesley Longman) combines course management and on-line testing with the features of the basic tutorial software (see "Supplements for the Student") to create an invaluable teaching resource. Consult your local Addison Wesley Longman sales consultant for details.

Supplements for the Student

Graphing Calculator Manual

The Graphing Calculator Manual by Judith A. Penna, with the assistance of John Garlow and Mike Rosenborg, contains keystroke level instruction for the Texas Instruments TI-82, TI-83, TI-85, and Hewlett Packard HP 38G graphing calculators. Modules for the Casio 9850, the Sharp 9200 and 9300, and the HP48G are available on request. Contact your local Addison Wesley Longman sales consultant for details.

Bundled free with every copy of the text, the Graphing Calculator Manual uses actual examples and exercises from *Algebra and Trigonometry: Graphs and Models* to help teach students to use their graphing calculator. The order of topics in the Graphing Calculator Manual mirrors that of the texts, providing a just-in-time mode of instruction.

Student's Solutions Manual

The Student's Solutions Manual by Judith A. Penna contains completely worked-out solutions with step-by-step annotations for all the odd-numbered exercises in the exercise sets in the text, with the exception of the thinking and writing exercises. It also includes a self-test with answers for each chapter and a final examination.

The Student's Solutions Manual can be purchased by your students from Addison Wesley Longman.

Graphing Calculator Instructional Videos

Designed and produced specifically for *Precalculus: Graphs and Models*, the Graphing Calculator Instructional Videos take students through procedures on the graphing calculator using content from the text. These videos include most topics covered in the Graphing Calculator Manual, as well as several additional topics.

Every video section uses actual text examples or odd-numbered exercises from the text to help motivate students while they learn to use the graphing calculator. In the videos, an instructor shows students keystroke level procedures that they will need to succeed with the grapher as they proceed through the course. The videos are correlated to the sections of the text.

A complete set of Graphing Calculator Instructional Videos is free to qualifying adopters.

InterAct Math Tutorial Software

InterAct Math Tutorial Software has been developed and designed by professional software engineers working closely with a team of experienced math educators.

InterAct Math Tutorial Software includes exercises that are linked with every objective in the textbook and require the same computational and problem-solving skills as their companion exercises in the text. Each exercise has an example and an interactive guided solution that are designed to involve students in the solution process and to help them identify precisely where they are having trouble. In addition, the software recognizes common student errors and provides students with appropriate customized feedback.

With its sophisticated answer recognition capabilities, InterAct Math Tutorial Software recognizes appropriate forms of the same answer for any kind of input. It also tracks student activity and scores for each section, which can then be printed out.

Available for both Windows and Macintosh computers, the software is free to qualifying adopters.

Acknowledgments

We wish to express our genuine appreciation to a number of people who contributed in special ways to the development of this textbook. Jason Jordan and Greg Tobin, our editors at Addison Wesley Longman, shared our vision and provided encouragement and motivation. In addition, the production and marketing departments of Addison Wesley Longman brought to the project their unsurpassed commitment to excellence. The unwavering support from the Higher Education Group has been a continuing source of strength for this author team. For this we are most grateful. Mike Rosenborg, Barbara Johnson, Patty Slipher, Irene Doo, and Larry Bittinger provided many constructive comments as well as accuracy checks to the manuscript.

Finally, Professor Bittinger would like to thank his MA 153 students at IUPUI for their productive response to parts of the manuscript that were class-tested in the spring of 1996 using the graphing calculator. This teaching approach resulted in the most satisfying class he has taught at IUPUI in 28 years. Further information regarding this class can be obtained from Professor Bittinger at his e-mail address, exponent@aol.com, or through his home page (see Web Connection that follows).

We would also like to thank the following reviewers for their invaluable contribution to the development of this text:

Sandra Beken, *Horry-Georgetown Technical College*
Diane Benjamin, *University of Wisconsin—Platteville*
Robert Bohac, *Northwest College*
Diane W. Burleson, *Central Piedmont Community College*
John W. Coburn, *St. Louis Community College at Florissant Valley*
Elaine N. Daniels, *Salve Regina University*
Donna DeSpain, *Benedictine University*
Eunice F. Everett, *Seminole Community College*
Rob Farinelli, *Community College of Allegheny County South Campus*
Betty P. Givan, *Eastern Kentucky University*
Allen R. Hesse, *Rochester Community College*

Heidi A. Howard, *Florida Community College at Jacksonville*
Steve Kahn, *Anne Arundel Community College*
Timothy A. Loughlin, *New York Institute of Technology*
Larry Luck, *Anoka-Ramsey Community College*
Joseph D. Mahoney, *Paducah Community College*
Peggy I. Miller, *University of Nebraska at Kearney*
John A. Oppelt, *Bellarmine College*
Tom Schaffter, *Fort Lewis College*
Eric Schulz, *Walla Walla Community College*
Judith D. Smalling, *St. Petersburg Junior College*
Craig M. Steenberg, *Lewis-Clark State College*
Kathryn C. Wetzel, *Amarillo College*
Kemble Yates, *Southern Oregon State College*

Web Connection

Students and instructors can obtain more information about books written by Professor Bittinger and his co-authors by contacting *Marv's Math Corner* on the Internet at the following address:

http://www.math.iupui.edu/~mbitting/

Included on this Web site is information about books and their supplements, as well as study tips and sample practice final examinations. Students and instructors are welcome to e-mail questions, comments, and constructive criticism.

M.L.B.
J.A.B.
D.J.E.
J.A.P.

- **Applied Chapter Openers:** Each chapter begins with a relevant application highlighting how concepts presented in the chapter can be put to use in the real world. These applications are accompanied by numerical tables, equations, and grapher windows to show students the many different ways in which problem situations can be examined.

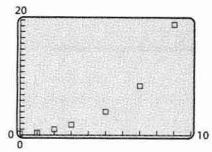
Exponential and Logarithmic Functions 3

APPLICATION

The number of cellular phones in this country is modeled by an exponential function, where y = the number of telephones, in millions, in the year x. Here $x = 0$ corresponds to 1985. (*Source:* Cellular Telecommunications Industry Association)

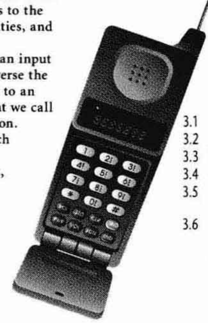
X	Y1
0	.2
1	.4
2	1.0
3	1.8
5	4.1
7	8.6
9	19.3

X = 5

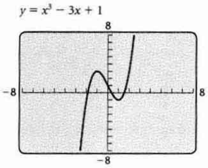


In this chapter, we will consider two kinds of closely related functions. The first, called *exponential functions,* are those that have a variable in the exponent. Such functions have many applications to the growth of populations, commodities, and investments.

Recall that a function takes an input to an output. Suppose we can reverse the process and take the output back to an input. That process produces what we call the *inverse* of the original function. Functions that are inverses of each other are closely related. The inverses of exponential functions, called *logarithmic functions,* or *logarithm functions,* are also important in many applications such as earthquake magnitude, sound level, and chemical pH.

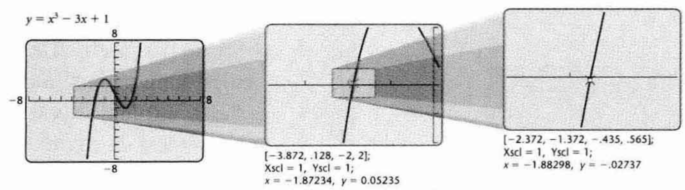


Equation S



Example 10
tions to thre

SOLUTION T
coordinates
the x-axis.
x-intercept
which $y = 0$
such x-value
To acqui
fast way to d
zoom-in feat
small sectio
amine the g
zoom in on the gr
sired accuracy is a



To find the solution to the nearest thousandth, we trace and zoom until the cursor's x-value just to the left of the intercept and the cursor's x-value just to the right of the intercept are the same, when rounded to the nearest thousandth. By using the TRACE and ZOOM features three more times, we find the solution to be about −1.879. In a similar manner, we find that the other solutions of the equation $x^3 - 3x + 1 = 0$ are about 0.347 and 1.532. If available, we might also use the SOLVE or ROOT features to approximate solutions.

There are many ways in which the ZOOM feature can be used. For example, we can zoom in for more precision, as in Example 10, or zoom out on a graph—say, to reveal more of its curvature—adjusting the factors of the zoom in any way we choose. In Example 10, we used zoom factors of 4. We may also be able to use a ZOOM-BOX feature to zoom in on a boxed region of our choosing. All such details can be found by consulting the manual for your particular grapher or the Graphing Calculator Manual that accompanies this book.

- **Grapher Integration:** The author team assumes that the student will use a grapher throughout the course and during homework or group sessions. Numerous grapher windows appear throughout the text, some in a unique ZOOM pattern.

- **Interactive Discoveries:** Throughout the exposition, students are directed to investigate new concepts before they are formally developed. This design invites students to be actively involved with the material in order to identify a mathematical pattern or form an intuitive understanding of a new topic.

Interactive Discovery

With a square viewing window (see the Introduction to Graphs and Graphers), graph the following equations:

$$y_1 = x, \quad y_2 = 2x, \quad y_3 = 5x, \quad \text{and} \quad y_4 = 10x.$$

What do you think the graph of $y = 128x$ will look like?

Clear the screen and graph the following equations:

$$y_1 = x, \quad y_2 = \tfrac{3}{4}x, \quad y_3 = 0.48x, \quad \text{and} \quad y_4 = \tfrac{3}{25}x.$$

What do you think the graph of $y = 0.000029x$ will look like?

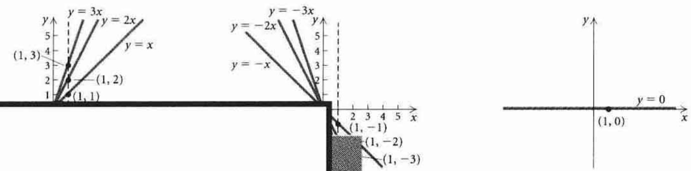
Again clear the screen and graph each set of equations:

$$y_1 = -x, \quad y_2 = -2x, \quad y_3 = -4x, \quad \text{and} \quad y_4 = -10x$$

and $$y_1 = -x, \quad y_2 = -\tfrac{2}{3}x, \quad y_3 = -0.35x, \quad \text{and} \quad y_4 = -\tfrac{1}{10}x.$$

From your observations, what do you think the graphs of $y = -200x$ and $y = -0.000017x$ will look like?

If a line slants up from left to right, the change in x and the change in y have the same sign, so the line has a positive slope. The larger the slope is, the steeper the line. If a line slants down from left to right, the change in x and the change in y are of opposite signs, so the line has a negative slope. The larger the absolute value of the slope, the steeper the line.



, or $y = 0$, as shown in the graph on the right s both the x-axis and a horizontal line.

ical Lines

l, the change in y for any two points is 0. ne has slope 0.

the change in x for any two points is 0. Thus *ned* because we cannot divide by 0.

e and an undefined slope are two very different

The Quadratic Formula

The solutions of $ax^2 + bx + c = 0$, $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example 3 Solve $3x^2 + 2x = 7$. Find exact solutions and approximate solutions rounded to the nearest thousandth.

We show both algebraic and graphical solutions. Note that only the algebraic approach yields the exact solutions.

ALGEBRAIC SOLUTION

After finding standard form, we are unable to factor, so we identify a, b, and c in order to use the quadratic formula:

$$3x^2 + 2x - 7 = 0;$$
$$a = 3, \quad b = 2, \quad c = -7.$$

We then use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{2^2 - 4(3)(-7)}}{2(3)} \quad \text{Substituting}$$
$$= \frac{-2 \pm \sqrt{4 + 84}}{6} = \frac{-2 \pm \sqrt{88}}{6}$$
$$= \frac{-2 \pm \sqrt{4 \cdot 22}}{6} = \frac{-2 \pm 2\sqrt{22}}{6}$$
$$= \frac{2}{2} \cdot \frac{-1 \pm \sqrt{22}}{3} = \frac{-1 \pm \sqrt{22}}{3}.$$

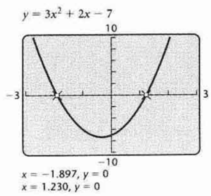
The exact solutions are

$$\frac{-1 - \sqrt{22}}{3} \quad \text{and} \quad \frac{-1 + \sqrt{22}}{3}.$$

Using the scientific keys on a grapher, we approximate the solutions to be −1.897 and 1.230.

GRAPHICAL SOLUTION

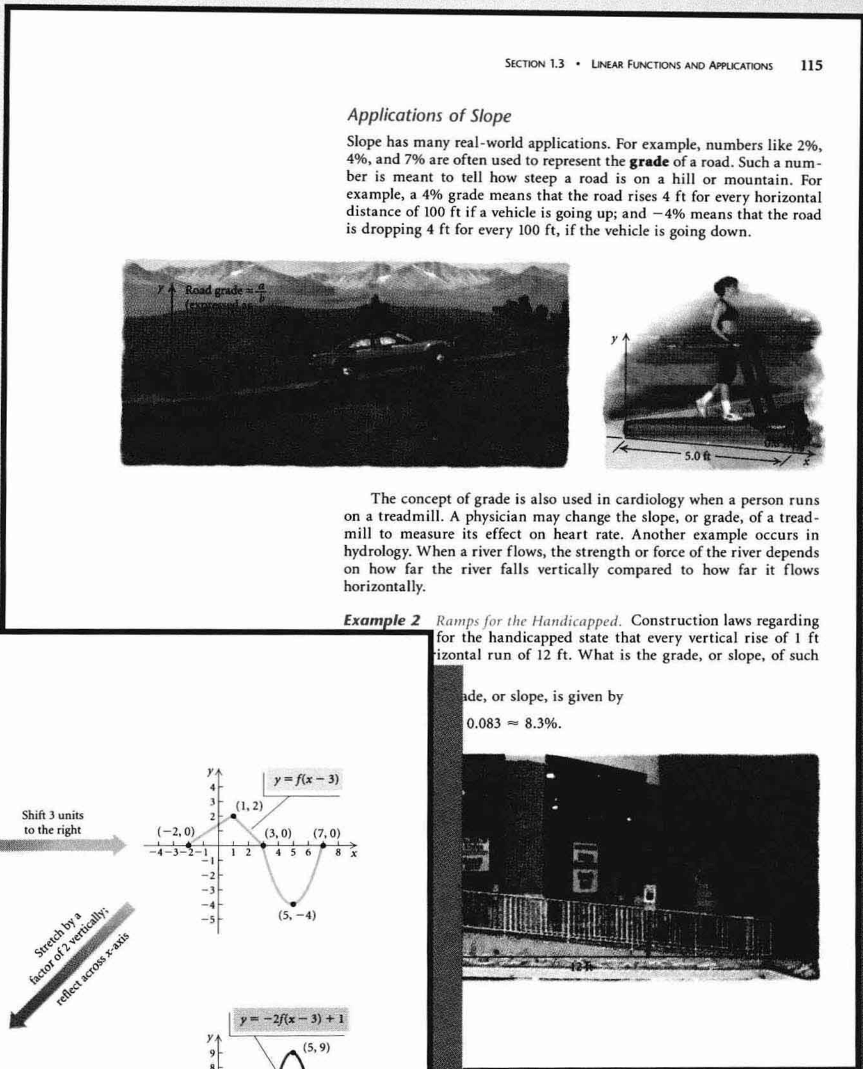
To solve $3x^2 + 2x = 7$, or $3x^2 + 2x - 7 = 0$, we first graph the function $f(x) = 3x^2 + 2x - 7$. Then we look for points where the graph crosses the x-axis. It appears that there are two possible zeros, one near −2 and one near 1. We can use TRACE and ZOOM to approximate these zeros, or we can use a SOLVE or POLY feature.



We get the approximate zeros −1.896805 and 1.2301386, or −1.897 and 1.230, rounded to three decimal places. The zeros of the function are the solutions of the equation $3x^2 + 2x = 7$.

- **Side-by-Side Algebraic and Graphical Solutions:** Many examples in the text are presented in a two-column format that shows simultaneous algebraic and graphical solution methods. This balanced approach allows students to compare the efficiency and appropriateness of each method.

- **Art:** Generous amounts of color-coded technical and situational art appear throughout the text to enhance understanding of an example or exercise, to interest students, and to aid in the visualization of concepts.



Applications of Slope

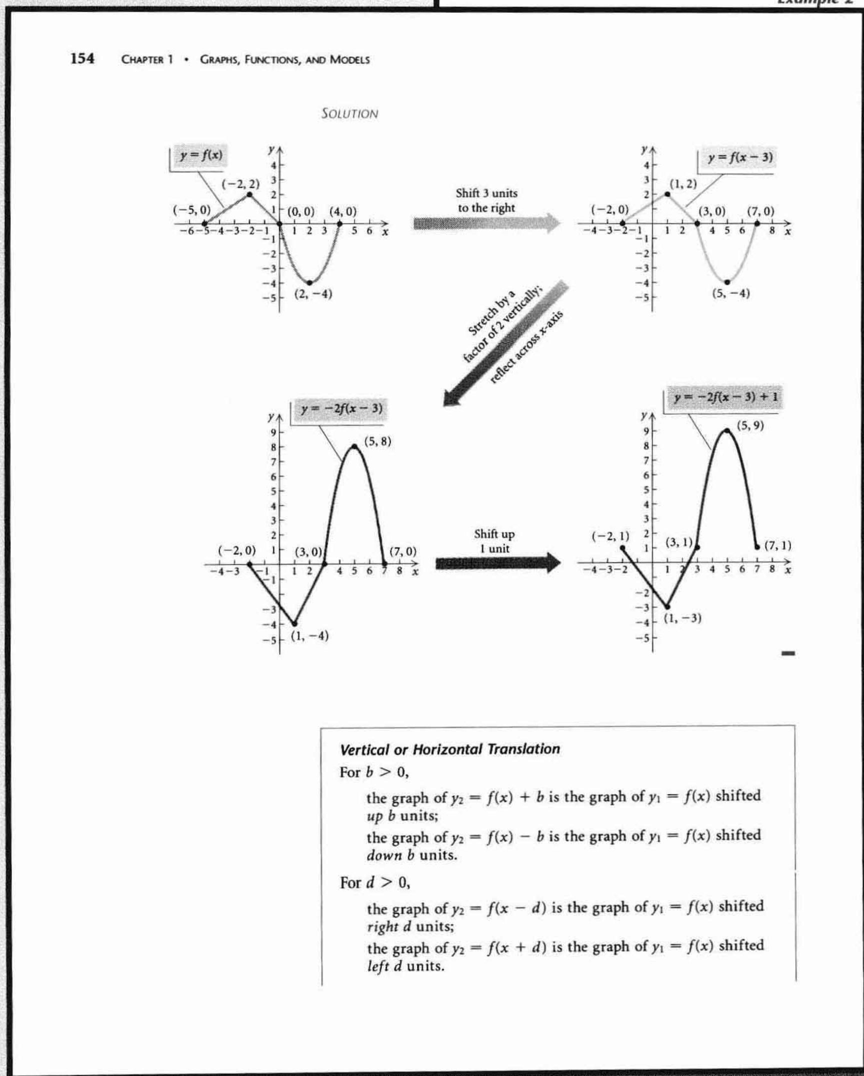
Slope has many real-world applications. For example, numbers like 2%, 4%, and 7% are often used to represent the **grade** of a road. Such a number is meant to tell how steep a road is on a hill or mountain. For example, a 4% grade means that the road rises 4 ft for every horizontal distance of 100 ft if a vehicle is going up; and −4% means that the road is dropping 4 ft for every 100 ft, if the vehicle is going down.

The concept of grade is also used in cardiology when a person runs on a treadmill. A physician may change the slope, or grade, of a treadmill to measure its effect on heart rate. Another example occurs in hydrology. When a river flows, the strength or force of the river depends on how far the river falls vertically compared to how far it flows horizontally.

Example 2 *Ramps for the Handicapped.* Construction laws regarding … for the handicapped state that every vertical rise of 1 ft … izontal run of 12 ft. What is the grade, or slope, of such …

…ade, or slope, is given by

… 0.083 ≈ 8.3%.



SOLUTION

Vertical or Horizontal Translation

For $b > 0$,

the graph of $y_2 = f(x) + b$ is the graph of $y_1 = f(x)$ shifted *up b* units;

the graph of $y_2 = f(x) - b$ is the graph of $y_1 = f(x)$ shifted *down b* units.

For $d > 0$,

the graph of $y_2 = f(x - d)$ is the graph of $y_1 = f(x)$ shifted *right d* units;

the graph of $y_2 = f(x + d)$ is the graph of $y_1 = f(x)$ shifted *left d* units.

- To assist students with understanding, **graphs with multiple curves** use different colors for each curve. Movement on the graph may be indicated by a gradual shift in color on an accompanying shift arrow.

- **Data Analysis and Modeling:** The author team highlights and reinforces the theme of data analysis throughout the text.

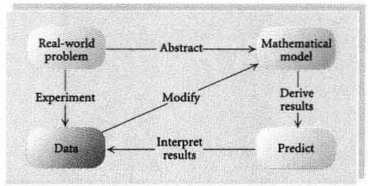
1.4 Data Analysis, Curve Fitting, and Linear Regression

- *Analyze a set of data to determine whether it can be modeled by a linear function.*
- *Fit a regression line to a set of data; then use the linear model to make predictions.*

Mathematical Models

When a real-world problem can be described in mathematical language, we have a **mathematical model**. For example, the natural numbers constitute a mathematical model for situations in which counting is essential. Situations in which algebra can be brought to bear often require the use of functions.

Mathematical models are abstracted from real-world situations. Procedures within the mathematical model then give results that allow one to predict what will happen in that real-world situation. If the predictions are inaccurate or the results of experimentation do not conform to the model, the model needs to be changed or discarded.

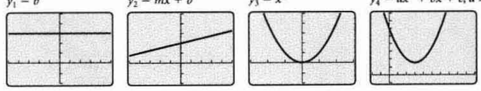


Mathematical modeling can be an ongoing process. For example, finding a mathematical model that will enable an accurate prediction of population growth is not a simple problem. Any population model that one might devise would need to be reshaped as further information is acquired.

Curve Fitting

We will develop and use many kinds of mathematical models in this text. In this chapter, we have considered many functions that can be used as models. Let's look at four of them.

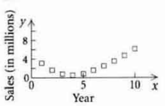
Constant function: $y_1 = b$ | Linear function: $y_2 = mx + b$ | Squaring function: $y_3 = x^2$ | Quadratic function: $y_4 = ax^2 + bx + c, a > 0$



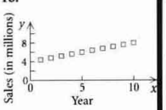
For the scatterplots and graphs in Exercises 17–25, determine which, if any, of the following functions migh[t] be used as a model for the data.

a) *Linear,* $f(x) = mx + b$
b) *Quadratic,* $f(x) = ax^2 + bx + c,\ a > 0$
c) *Quadratic,* $f(x) = ax^2 + bx + c,\ a < 0$
d) *Polynomial, not quadratic or linear*

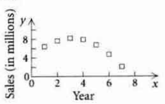
17.



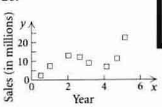
18.



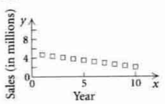
19.



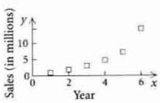
20.



21.

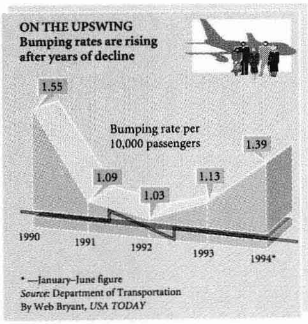


22.

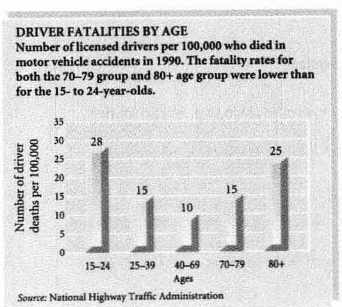


Data Center

23.



25.



- By fitting curves to data and discussing **mathematical models**, students develop an understanding of mathematical patterns as they are seen in the real world and gain insight into using models to make predictions.

- **Direction lines** in the exercise sets instruct students to use both algebraic and graphical methods to solve problems. Sometimes the student is asked to solve with a grapher and check algebraically. In other problems, the student is directed to solve algebraically and check the work with a grapher. Many exercises do not specify a solution method and allow the student to determine the method, thus encouraging critical thinking and analytical skills.

3.5 Exercise Set

Solve each exponential equation algebraically. Then check on a grapher.

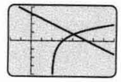
1. $3^x = 81$
2. $2^x = 32$
3. $2^{2x} = 8$
4. $3^{7x} = 27$
5. $2^x = 33$
6. $2^x = 40$
7. $5^{4x-7} = 125$
8. $4^{3x-5} = 16$
9. $27 = 3^{5x} \cdot 9^{x^2}$
10. $3^{x^2+4x} = \frac{1}{27}$
11. $84^x = 70$
12. $28^x = 10^{-3x}$
13. $e^t = 1000$
14. $e^{-t} = 0.04$
15. $e^{-0.03t} = 0.08$
16. $1000e^{0.09t} = 5000$
17. $3^x = 2^{x-1}$
18. $5^{x+2} = 4^{1-x}$
19. $(3.9)^x = 48$
20. $250 - (1.87)^x = 0$
21. $e^x + e^{-x} = 5$
22. $e^x - 6e^{-x} = 1$
23. $\dfrac{e^x + e^{-x}}{e^x - e^{-x}} = 3$
24. $\dfrac{5^x - 5^{-x}}{5^x + 5^{-x}} = 8$

Solve each logarithmic equation algebraically. Then check on a grapher.

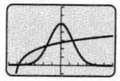
25. $\log_5 x = 4$
26. $\log_2 x = -3$
27. $\log x = -4$
28. $\log x = 1$
29. $\ln x = 1$
30. $\ln x = -2$
31. $\log_2 (10 + 3x) = 5$
32. $\log_5 (8 - 7x) = 3$
33. $\log x + \log (x - 9) = 1$
34. $\log_2 (x + 1) + \log_2 (x - 1) = 3$
35. $\log_8 (x + 1) - \log_8 x = 2$
36. $\log x - \log (x + 3) = -1$
37. $\log_4 (x + 3) + \log_4 (x - 3) = 2$
38. $\ln (x + 1) - \ln x = \ln 4$
39. $\log (2x + 1) - \log (x - 2) = 1$
40. $\log_5 (x + 4) + \log_5 (x - 4) = 2$

Use only a grapher. Find approximate solutions of each equation or approximate the point(s) of intersection of a pair of equations.

41. $e^{7.2x} = 14.009$
42. $0.082e^{0.05x} = 0.034$
43. $xe^{3x} - 1 = 3$
44. $5e^{5x} + 10 = 3x + 40$
45. $4 \ln (x + 3.4) = 2.5$
46. $\ln x^2 = -x^2$
47. $\log_8 x + \log_8 (x + 2) = 2$
48. $\log_3 x + 7 = 4 - \log_5 x$
49. $\log_5 (x + 7) - \log_5 (2x - 3) = 1$
50. $y = \ln 3x, \ y = 3x - 8$
51. $2.3x + 3.8y = 12.4, \ y = 1.1 \ln (x - 2.05)$



52. $y = 2.3 \ln (x + 10.7), \ y = 10e^{-0.07x^2}$



53. $y = 2.3 \ln (x + 10.7), \ y = 10e^{-0.007x^2}$

Skill Maintenance

54. Solve $K = \frac{1}{2}mv^2$ for v.

Solve.

55. $x^4 + 5x^2 = 36$
56. $t^{2/3} - 10 = 3t^{1/3}$

57. *Total Sales of Goodyear.* The following table shows factual data regarding total sales of The Goodyear Tire and Rubber Company.

YEAR, x	TOTAL SALES, y (IN MILLIONS)
1. 1991	$10,906.8
2. 1992	11,784.9
3. 1993	11,643.4
4. 1994	12,288.2

Source: The Goodyear Tire and Rubber Company Annual Report.

a) Use linear regression on a grapher to fit an equation $y = mx + b$, where $x = 1$ corresponds to 1991, to the data points. Predict total sales in 1999.

b) Use quadratic regression on a grapher to fit a quadratic equation $y = ax^2 + bx + c$ to the data points. Predict total sales in 1999.

Synthesis

58. ◆ In Example 3, we took the natural logarithm on both sides. What would have happened had we taken the common logarithm? Explain which seems best to you and why.

59. ◆ Explain how Exercises 29 and 30 could be solved using the graph of $f(x) = \ln x$.

3. Carry out.

ALGEBRAIC SOLUTION

We solve the equation:

$$x + 2x = 30$$
$$3x = 30$$
$$x = 10.$$

GRAPHICAL SOLU

Graph $y_1 = x$
point of inters

4. Check. When
the Australian
10 + 20, or 30 days

5. State. After one ye
10 vacation days and Australian employees get an average of 20 vacation days per year.

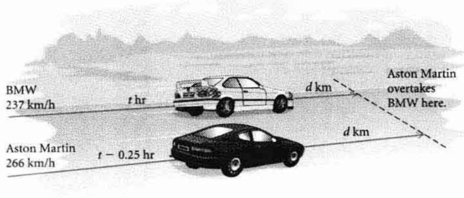
In some applications we need to use a formula that describes the relationship between variables. When a situation involves distance, speed, and time, for example, we need to recall the **distance formula**:

$d = rt$, where d = distance, r = rate (or speed), and t = time.

Example 2 *Speed.* A 1996 BMW M3 leaves a town on the Autobahn traveling at its top speed of 237 km/h. Fifteen minutes later, a 1995 Aston Martin DB7 leaves the same town and follows the same route at its top speed of 266 km/h. How long will it take the Aston Martin to overtake the BMW? (*Sources: Car and Driver*, August 1995 and February 1995)

SOLUTION

1. Familiarize. We make a drawing showing both the known and the unknown information. We let t = the time, in hours, that the BMW



- **Examples/Exercises:** The examples and exercises reflect the text's focus. Many use situational or grapher art and many also incorporate recent, source-based data to illustrate applications and concepts. An Index of Applications is included at the end of the text.