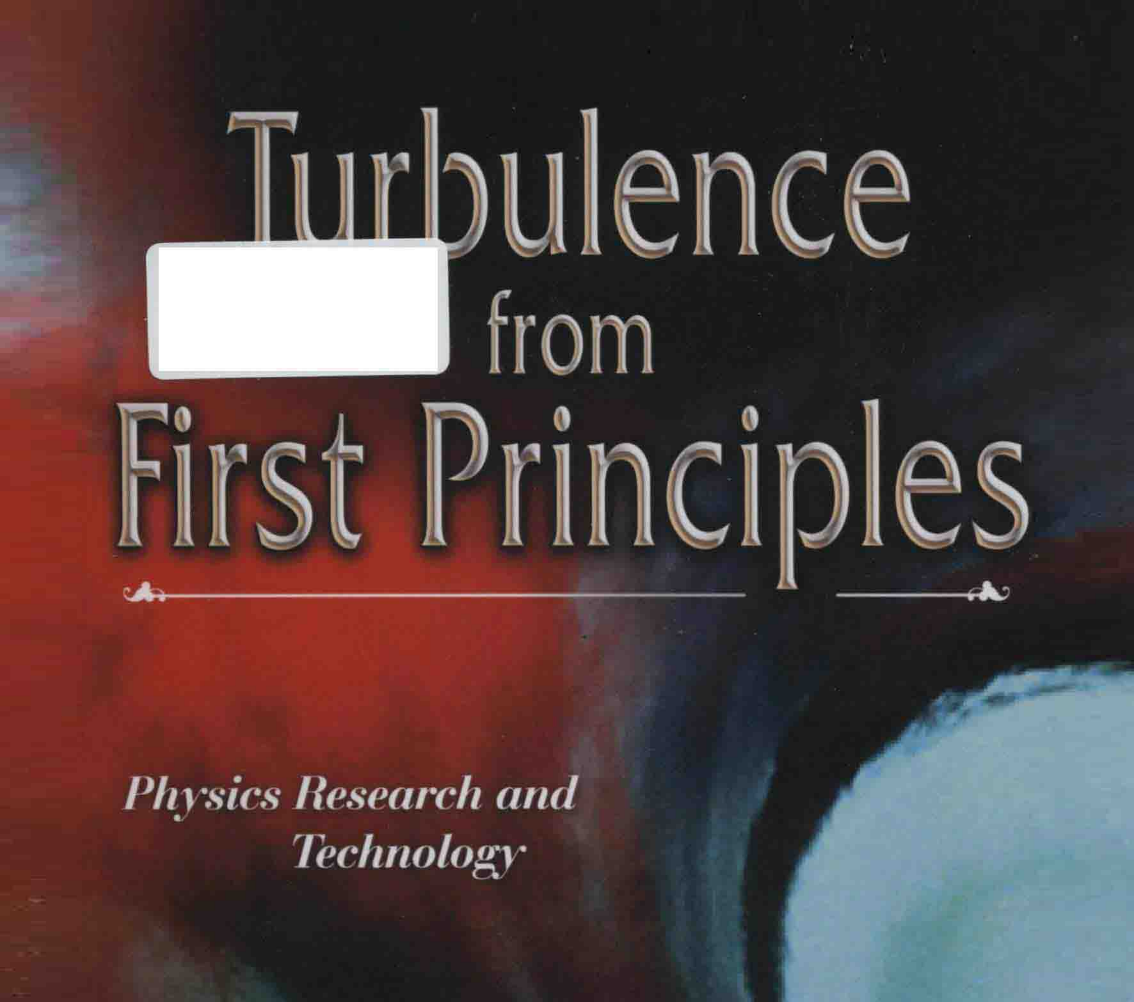

Turbulence from First Principles

Physics Research and Technology

Michail Zak

Novinka

PHYSICS RESEARCH AND TECHNOLOGY

TURBULENCE FROM FIRST PRINCIPLES

MICHAIL ZAK

New York

LIBRARY OF CONGRESS CATALOGING-IN-PUBLICATION DATA

Zak, Michail, 1932-
Turbulence from first principles / Michail Zak.
pages cm
Includes bibliographical references and index.
ISBN 978-1-62808-468-9 (soft cover)
1. Turbulence. 2. Turbulence--Mathematical models. I. Title.
QA913.Z35 2013
532'.05270151--dc23

2013025326

Published by Nova Science Publishers, Inc. † New York

PHYSICS RESEARCH AND TECHNOLOGY

TURBULENCE FROM FIRST PRINCIPLES

PHYSICS RESEARCH AND TECHNOLOGY

Additional books in this series can be found on Nova's website
under the Series tab.

Additional e-books in this series can be found on Nova's website
under the e-book tab.

To my family

"When everything goes to hell, the people who stand by you without flinching—they are your family."

Jim Butcher

PREFACE

Turbulence is the most important unsolved problem of classical physics.
Richard Feynman

This book presents a non-traditional approach to theory of turbulence. Its objective is to prove that Newtonian mechanics is fully equipped for description of turbulent motions without help of experimentally obtained closures. Turbulence is one of the most fundamental problems in theoretical physics that is still unsolved. The term "unsolved " here means that turbulence cannot be properly *formulated,* i.e. reduced to standard mathematical procedure such as solving differential equations. In other words, it is not just a computational problem: prior to computations, a consistent mathematical model must be found. Although applicability of the Navier-Stokes equations as a model for fluid mechanics is not in question, the instability of their solutions for flows with supercritical Reynolds numbers raises a more general question: is Newtonian mechanics complete?

The problem of turbulence (stressed later by the discovery of chaos) demonstrated that the Newton's world is far more complex than those represented by classical models. It appears that the Lagrangian or Hamiltonian formulations do not suggest any tools for treating postinstability motions, and this is a major flaw of the classical approach to Newtonian mechanics. The explanation of that limitation is proposed in this book: the classical formalism based upon the Newton's laws exploits additional mathematical restrictions (such as space–time differentiability, and the Lipchitz conditions) that are not required by the Newton's laws. The only purpose for these restrictions is to apply a powerful technique of classical mathematical analysis. However, in many cases such restrictions are incompatible with physical reality, and the most obvious case of such incompatibility is the Euler's model of inviscid fluid in which absence of shear stresses are not compensated by a release of additional degrees of freedom as required by the principles of mechanics.

Chapter 1 presents a brief review of standard mathematical approach to fluids that includes inviscid/viscose and incompressible/compressible models. Omitting mathematical details, attention is concentrated on inconsistencies and paradoxes that limit the boundary of applicability of these models. The main objective of this Chapter is to prepare a reader to a revision of the Euler/Navier-Stokes equations for describing turbulent motions.

Chapter 2 introduces and illustrates the Stabilization Principle that provides a strategy for modeling post instability behavior in dynamics, including turbulence and chaos. It starts with investigation of different types of instability in fluids with the objective to demonstrate that stability is not a physical invariant since it depends upon the frame to which the motion of fluid is referred, upon the class of functions in which the governing equations are derived, ect. The application of the Stabilization Principle to the Navier-Stokes equations is illustrated by closure of the Reynolds equations for the Poiseuille flow.

In Chapter 3, it has been demonstrated that according to the principle of release of constraints, absence of shear stresses in the Euler equations must be compensated by additional degrees of freedom, and that led to a Reynolds-type enlarged Euler equations (EE equations) with a doublevalued velocity field that *do not require any closures*. In the first part of this Chapter, the theory is applied to turbulent mixing and illustrated by propagation of mixing zone triggered by a tangential jump of velocity. A comparison of the proposed solution with the Prandtl's solution is performed and discussed. In the second part of the Chapter, a semi-viscous version of the Navier-Stokes equations is introduced. The *model does not require any closures* since the number of equations is equal to the number of unknowns. Special attention is paid to transition from laminar to turbulent state. The analytical solution for this transition demonstrates the turbulent mean velocity profile that deviates from the laminar one.

Chapter 4 is devoted to Lagrangian turbulence and Chaos. The Lagrangian turbulence is defined as postinstability motion of individualized trajectories of a fluid generated by a laminar flow. The formulation of L-turbulence is reduced to a system of three nonlinear ODE describing kinematics of transition from Euler's to Lagrange's frames of reference. It has been demonstrated that the complexity of this ODE is equivalent to that of the simplest chaotic systems like a Lorentz attractor. Applications of the

Stabilizaion Principle to Lagrangian turbulence with generalization to the Navier-Stokes equations and n-body problems as well as a computational strategy are discussed.

Chapter 5 presents a revision of the mathematical formalism of fluid dynamics, and in particular, some physical inconsistencies (infinite time of approaching equilibrium and fully deterministic solutions of the Navier-Stokes equations). As shown there, these inconsistencies can be removed by relaxing the Lipchitz conditions, i.e., the boundedness of the derivatives in the constitutive equations. Physically such a modification can be interpreted as an incorporation of an infinitesimal static friction in the constitutive law. A modified version of the Navier-Stokes equations is introduced, discussed, and illustrated by examples. It is demonstrated that all the new effects in the modified model emerge within vanishingly small neighborhoods of equilibrium states that are the only domains where the governing equations are different from the classical equations.

The accessible presentation of this book makes it eminently suitable for graduate students, researchers and engineers in the areas of fluid mechanics, general physics and applied mathematics.

CONTENTS

Chapter 1

MATHEMATICAL MODELS OF FLUIDS

It would be better for the true physics if
there were no mathematicians in the world.
Daniel Bernoulli

ABSTRACT

This Chapter presents a brief review of standard mathematical approach to fluids that includes inviscid/viscose and incompressible/ compressible models. Omitting mathematical details, attention is concentrated on inconsistencies and paradoxes that limit the boundary of applicability of these models. The main objective of this Chapter is to prepare a reader to a revision of the Euler/Navier-Stokes equations for describing turbulent motions.

1. DEFINITIONS

Motion of a fluid is considered uniquely defined if its velocity field $\mathbf{v}(x,t)$ can be found at any point x of the volume it occupies at any instant of time t. Besides of that, the parameters describing the state of the fluid – the density $\rho(x,t)$, the pressure $p(x,t)$ and the temperature $T(x,t)$ must be defined as well.

Mathematical description of fluid motions required appropriate mathematical models that take into account only the most important physical property of a chosen phenomenon since the more specific the model the more transparent the connections with mathematics and reality. In our brief review we consider the models in the order of their complexity: inviscid incompressible fluid, inviscid compressible fluid, and viscous fluid. But it does not mean that the more complex model is always covers all the properties of the less complex one. For instance, shock waves are usually studied in invicsid rather than in viscous model since there their study is simpler and more transparent; the condition of incompressibility representing a global constrain is not always convenient and it is replaced by artificial compressibility, etc.

The governing equations describing the change of the state variables listed above can be derived from the Newton Laws, or from the Variational Principles, but in all cases, prior to derivations, the following condition is imposed: *all the functions describing the state variables must be twice-differentiable with respect to time t and space variables x.* It should be emphasized that this limitation is not required by physics: it is required by mathematicians in order to reduce the model to a well-established mathematical formalism of differential equations. The consequences of this compromise are the central point of this book.

2. Model of Inviscid Incompressible Fluid

Inviscid fluid, by definition, does not have friction forces, while normal stress is always directed inside of the selected volume. Therefore, the stress tensor is spherical, Figure 1.

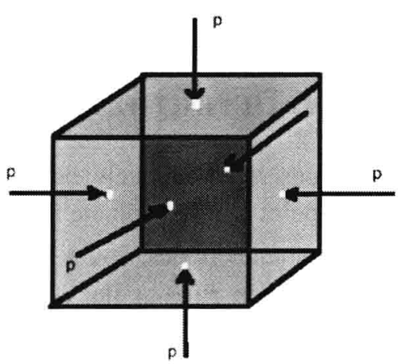

Figure 1. Spherical pressure tensor.

$$T = -\frac{1}{3} pE, \quad p < 0 \tag{1.1}$$

The mathematical formulation of incompressibility is represented as

$$\nabla \cdot \mathbf{v} = 0 \tag{1.2}$$

Then the equation of motion follows from the second Newton's law is

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\nabla\mathbf{v} = -\frac{1}{\rho}\nabla p + \mathbf{F} \tag{1.3}$$

where **F** is external force per unit mass.

Eqs. (1.2) and (1.3) form a closed system of PDE to be solved subject to specific initial and boundary conditions.

The boundary conditions at a surface separating two different flows are

$$\mathbf{v}_1 \cdot \mathbf{n} = \mathbf{v}_2 \cdot \mathbf{n} \text{ (the free-slip condition)} \tag{1.4}$$

$$p_1 = p_2 \tag{1.5}$$

in which **n** is the normal to this surface, and $\mathbf{v}_1, \mathbf{v}_2$ are velocities at this surface.

If the surface is represented by a rigid wall, Eq. (1.5) should be eliminated.

Let us turn to a particular case when the external force has a potential U

$$\mathbf{F} = -\nabla U \tag{1.6}$$

and the motion starts from rest. Then its velocity has a potential φ as well

$$\mathbf{v} = \nabla\varphi \tag{1.7}$$

and therefore, its vortex vector $\mathbf{\Omega}$ is zero

$$\mathbf{\Omega} = \nabla \times \mathbf{v} = 0 \tag{1.8}$$

In this particular case, the system (1.2) and (1.3) is reduced to the Cauchy integral

$$\frac{\partial\varphi}{\partial t}+\frac{1}{2}v^2+U+\frac{p}{\rho}=\Phi(t) \qquad (1.9)$$

in which $\Phi(t)$ is an arbitrary function, and the Laplace equation

$$\Delta\varphi=0 \qquad (1.10)$$

Eq. (1.10), subject to appropriate initial and boundary conditions, uniquely defines the velocity potential φ, and therefore, the velocity field **v**. Then the pressure p is found from Eq. (1.9), while the arbitrary function $\Phi(t)$ is defined if the function $p\ (t)$ is known in one point of the space.

Thus, on the first sight, the potential model (1.9), (1.10) seems perfect: it is reduced to a well behaved Laplace equation, and uniqueness of solution subject to appropriate boundary conditions is guaranteed. However more detailed analysis discovers paradoxes and inconsistencies of the model. The most damaging, zero-drag paradox was introduced by D'Alembert in 1752, [1]. As demonstrated in Figure 1, the potential flow around a circular cylinder is symmetric, and it cannot generate any drag. In addition to that, exact solutions predict infinite velocity at the sharp edges of boundaries, Figure 2. The absurdity of the exact solution alerted scientists and undermined their confidence in the model.

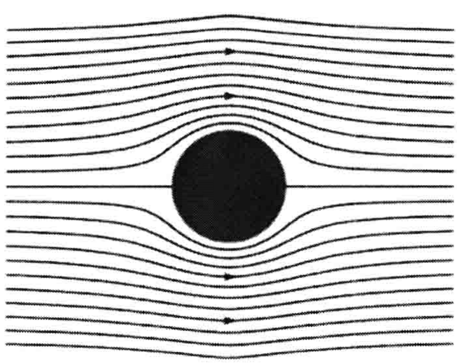

Figure 2. Streamlines for the potential flow around a circular cylinder in a uniform onflow.