

Numerical Calculation of Lubrication
—Methods and Programs

# 润滑数值计算

——方法与程序

黄平 著



清华大学出版社



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### 内容简介

本书针对 Reynolds 方程、能量方程和弹性变形方程结合计算方法介绍实用计算程序。前 9 章主要介绍了不同工况下的 Reynolds 方程的数值计算与程序,如:滑块不可压稳态润滑、径向滑动轴承润滑、动载轴承润滑、气体润滑和脂润滑等数值计算方法与程序。第 10、11 章结合能量方程的求解介绍了热流体润滑的数值计算方法与程序。第 12~20 章主要介绍了弹流润滑数值计算与程序,方法是通过对弹性变形方程的计算,结合与 Reynolds 方程以及能量方程的联立求解能量方程等考虑等温、热和脂润滑弹流润滑计算的方法和程序。第 21~24 章介绍了作者针对工程润滑问题开发的一些程序,包括:微型电机人字沟轴承润滑计算程序及其优化设计、磁盘磁头薄膜气体稳态润滑计算及优化设计程序。

本书可供高等学校的相关专业的教师、硕士生、博士生,以及工程技术人员和科研人员使用。

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## Preface

Lubrication calculation is the most successful area of tribology, using mathematical methods to obtain numerical solutions. Due to the development of computer science in the recent half a century, it has made remarkable achievements.

However, most books on tribology mainly introduce theories or calculation methods of lubrication. They rarely give and discuss numerical calculation programs. Not only is this inconvenient for research or the production of practical lubrication, but also many similar programming tasks have to be carried out repeatedly. Furthermore, because of the limitations of lubrication theory, some numerical calculation programs may give mistaken solutions, leading to wrong conclusions.

This book is different from previous theoretical books or monographs on lubrication; it mainly introduces the numerical calculation programs of lubrication. This is the main feature of the book. Some of the programs have been used for many years in lubrication calculations and research by the author.

Due to the complexity of lubrication problems, the book mainly focuses on how to numerically solve the Reynolds equation, energy equation, elastic deformation equation and their combinations. The analyzed lubrication problems include line, surface and point contacts, which correspond to thrust bearings, journal bearings and rolling contact bearings. Furthermore, the working conditions include incompressible, compressible, nonthermal, thermal, isoviscosity, variable viscosity, Newtonian fluid, non-Newtonian fluid (only grease), rigid and elastic deformation situations.

The book is divided into four parts, covering 24 chapters:

The first part (Chapters 1–9) is about the solution of the Reynolds equation, which is the basic technique for the numerical analysis of lubrication. The contents include the boundary and connection conditions of the Reynolds equation, discretizing the Reynolds equation, numerical methods and programs of slider lubrication, numerical methods and programs of journal bearing lubrication, numerical methods and programs of dynamic bearing lubrication, numerical methods and programs of gas lubrication (especial magnetic hard disk/head) and so on. In this part, we also discuss the rheology of lubricants. As an example, grease lubrication is discussed, which is a kind of non-Newtonian fluid.

The second part of the book (Chapters 10 and 11) is on temperature calculation. First, we give a discrete form of the energy equation and the temperature–viscosity

ii Preface

equation. Then, combining the Reynolds equation and the energy equation, we give numerical methods and programs of thermal hydrodynamic lubrication.

Elastohydrodynamic lubrication (EHL) is a difficult topic in lubrication calculation, because of its poor convergence. In the third part (Chapters 12–20), calculations of elastic deformation equations for line and point contacts are given first. Then, combined with the Reynolds equation, the pressure–viscosity equation and the elastic deformation equation, calculation programs of EHL are introduced in detail. Furthermore, combined with the energy equation, numerical methods and programs of thermal EHL are given. We also give numerical methods and programs of EHL and thermal EHL for grease in this part, and we consider the rheological effect.

Finally, in the last part of the book (Chapters 21–24), we introduce some programs developed for practical lubrication design. These programs include a lubrication calculation package and its optimized design package for the herringbone groove bearing of a micro motor and a calculation program and balancing attitude program of ultra thin gas lubrication for magnetic hard disk/head design. Because these packages and programs have some special requirements, pre-treatment and post-treatment have been added for easy usage in engineering. Although the basic theories of these contents are introduced at the front of the book, more details about the function and usages of the packages and programs can be found on the Wiley Companion Website: www.wiley.com/go/huang/lubrication.

The reason why we provide all source codes and an attached source code disc for all the programs is that most users need not repeat programming tasks even if they have well mastered the principles of lubrication. Especially, those who are not familiar with lubrication analysis can directly use the programs to carry out lubrication calculation. If some users have enough lubrication knowledge, they can use the programs or need only rewrite the pre-assignment or data sentences to input the different parameters to solve their own lubrication problems more easily. This will bring great convenience for researchers and technical staff in this field.

The book is mainly aimed at teachers, post-graduate students and doctoral students at colleges and universities. It can also be used as a reference book for technical personnel and research staff in engineering.

I would like to thank all of my post-graduate students who participated in the program writing and debugging and the book writing. Among them, I thank Li Ping for Chapters 2 and 3, Sun Zhonghua for Chapters 5 and 6, Niu Rongjun for Chapter 8, Wang Qiliang, Glenn and Liu Ping for Chapters 10 and 11, Wang Yazhen for Chapters 12–14, Yu Mei for Chapters 15 and 19, Lai Tianmao for Chapters 16 and 20, Yao Huaping for Chapters 21 and 22, and Wang Hongzhi for Chapters 23 and 24.

Huang Ping South China University of Technology 31 August, 2012

# Contents

Par	t 1	NUMERICAL METHOD FOR REYNOLDS EQUATION	1
1	Rey	nolds Equation and its Discrete Form	3
	1.1	General Reynolds Equation and Its Boundary Conditions	3
		1.1.1 Reynolds Equation	3
		1.1.2 Definite Condition	3
		1.1.3 Computation of Lubrication Performances	5
	1.2	Reynolds Equations for Some Special Working Conditions	6
		1.2.1 Slider and Thrust Bearing	6
		1.2.2 Journal Bearing	7
		1.2.3 Hydrostatic Lubrication	8
		1.2.4 Squeeze Bearing	9
		1.2.5 Dynamic Bearing	9
		1.2.6 Gas Bearing	10
	1.3	Finite Difference Method of Reynolds Equation	10
		1.3.1 Discretization of Equation	11
		1.3.2 Difference Form of Reynolds Equation	12
		1.3.3 Iteration of Differential Equation	13
		1.3.4 Iteration Convergence Condition	13
2	Nun	nerical Method and Program for Incompressible and Steady	
-		rication of One-dimensional Slider	17
	2.1	Basic Equations	17
		2.1.1 Reynolds Equation	17
		2.1.2 Boundary Conditions	18
		2.1.3 Continuity Equation	18
	2.2	Numerical Method for Incompressible and Steady Lubrication	10
	2.2	of One-dimensional Slider	18
		2.2.1 Discrete Reynolds Equation	19
		Zizii Ziboroto Heyrtotti Lyttuttott	17

	2.3		lation Program for Incompressible and Steady Lubrication	
		of On	e-dimensional Slider	20
		2.3.1	Introduction	20
		2.3.2	Calculation Diagram	21
			Calculation Program	21
			Calculation Results	24
3	Nun	erical I	Method and Program for Incompressible and Steady	
	Lub	rication	of Two-dimensional Slider	25
	3.1	Basic	Equations	25
	3.2		ete Reynolds Equation	26
	3.3		lation Program for Incompressible and Steady Lubrication	
			o-dimensional Slider	27
		3.3.1	Introduction	27
			Calculation Diagram	27
			Calculation Program	28
			Calculation Results	31
4	Nun	nerical I	Method and Program for Incompressible and Steady	
			of Journal Bearing	33
	4.1		Equations	33
			Axis Position and Clearance Shape	33
			Reynolds Equation	34
	4.2		rical Method for Incompressible and Steady Lubrication	
			irnal Bearing	35
			Dimensionless Reynolds Equation	35
			Discrete Form of Reynolds Equation	36
	4.3		lation Program for Incompressible and Steady Lubrication	50
	1.5		irnal Bearing	37
			Calculation Diagram	
			Calculation Program	
			Calculation Results	40
5	Nun	nerical l	Method and Program for Incompressible	
			brication	41
	5.1		Equation 100 but seld a securious Equation 100 but seld as a securior 100 b	41
	5.2		rical Method and Program for Rectangular Plane Squeeze	42
		5.2.1	Basic Equations	42
		5.2.2	Numerical Method	42
		5.2.3	Calculation Diagram	43
		5.2.4	Calculation Program	44
		5.2.5	O	47
		0.20	CHICKERULUI ACCINIU	6.7

Contents

	5.3	Nume	rical Method and Program for Disc Squeeze	47
		5.3.1	Basic Equations	47
		5.3.2	Numerical Method	48
		5.3.3	Calculation Diagram	48
		5.3.4	Calculation Program	49
		5.3.5	Calculation Results	52
	5.4	Nume	rical Method and Program for Journal Bearing Squeeze	52
		5.4.1	Basic Equations	52
		5.4.2	Numerical Method	54
		5.4.3	Calculation Diagram	54
		5.4.4	Calculation Program	55
		5.4.5	Calculation Results	60
6	Nun	nerical I	Method and Program for Dynamic Bearing	61
	6.1	Basic	Equations	61
	6.2	Nume	rical Method for Trace of Journal Center	65
		6.2.1	Introduction	65
		6.2.2	Calculation Steps	66
	6.3	Calcul	lation Program for Dynamic Journal Bearing	67
		6.3.1	Introduction	67
		6.3.2	Calculation Diagram	67
		6.3.3	Calculation Program	68
		6.3.4	Calculation Results	82
7	Nun	nerical I	Method and Program for Gas Lubrication	85
	7.1	Basic	Equations Equations	85
		7.1.1	General Reynolds Equation of Gas Lubrication	85
	7.2	Nume	rical Method of Gas Lubrication	86
		7.2.1	Basic Equations of Steady and Isothermal Gas Lubrication	86
		7.2.2	Numerical Method	87
	7.3	Calcul	lation Program for Gas Lubrication	88
		7.3.1	Calculation Program and Solutions of One-Dimensional Gas Lubrication	88
		7.3.2	Numerical Program and Solutions of Two-Dimensional Gas Lubrication	91
		733	Numerical Program and Solutions of Journal Bearing	91
		7.3.3	Gas Lubrication	94
8	Nun		Method and Program for Rarefied Gas Lubrication	97
	8.1	Basic	Equations The High Photograph of the Equation of the High Photograph	97
	8.2		rical Method of Rarefied Gas Lubrication	99
		8.2.1	Rarefied Gas Lubrication Model	99
		8.2.2	Treatment of the Ultra-Thin Gas Film Lubrication Equation	100

vi Contents

	8.3	Discretization and Iteration of Modified Reynolds Equation	101
	0.3	8.3.1 Discrete Equation	101
		8.3.2 Iteration Method	101
	8.4	Calculation Program for Rarefied Gas Lubrication of Slider	102
	0.4	8.4.1 Procedures Introduction	102
		8.4.2 Calculation Diagram	102
		8.4.3 Calculation Program	102
		8.4.4 Calculation Results	106
9	Num	nerical Method and Program for One-dimensional	
		ase Lubrication	107
	9.1	Basic Equations	107
		9.1.1 Introduction	107
		9.1.2 Constitutive Equations of Grease	108
		9.1.3 Reynolds Equation	109
	9.2	Numerical Method of One-Dimensional Grease Lubrication	109
	9.3	Calculation Program of One-Dimensional Grease Lubrication	110
		9.3.1 Calculation Diagram	110
		9.3.2 Calculation Program	111
		9.3.3 Calculation Results	113
Par	t 2	NUMERICAL METHOD FOR ENERGY EQUATION	115
10	Ener	gy Equation and its Discrete Form	117
	10.1	Basic Equations	117
		10.1.1 Simplified Energy Equation	118
		10.1.2 Boundary Conditions	118
		10.1.3 Numerical Method	119
	10.2	Influence of Temperature on Lubricant Performance	120
		10.2.1 Viscosity–Temperature Equation	120
		10.2.2 Density–Temperature Equation	120
	10.3	Numerical Method for Thermal Hydrodynamic Lubrication 10.3.1 Methods and Program for One-dimensional Thermal	121
		Hydrodynamic Lubrication	121
		10.3.2 Numerical Method and Program for Two-dimensional Thermal Hydrodynamic Lubrication	124
11	Num	nerical Method and Program for Incompressible and Steady	
		mal Hydrodynamic Lubrication of Journal Bearing	131
	11.1	Basic Equations	131
		11.1.1 Reynolds Equation	131
		11.1.2 Energy Equation	132

		11.1.3 Viscosity–Temperature Equation	132
	11.2		132
		11.2.1 Discrete Reynolas Equation	132
		11.2.2 Discrete Energy Equation	133
		11.2.3 Temperature–Viscosity Equation	133
	11.3		133
		11.3.1 Calculation Diagram	133
		11.3.2 Calculation Program	134
		11.3.3 Calculation Results	138
Pai	rt 3	NUMERICAL METHOD FOR ELASTIC	
		DEFORMATION AND THERMAL	
		ELASTOHYDRODYNAMIC LUBRICATION	141
12	Nun	nerical Method and Program for Elastic Deformation and	
	Visc	osity-Pressure Equation	143
	12.1	Basic Equations of Elastic Deformation	143
		12.1.1 Film Thickness Equation	143
		12.1.2 Elastic Deformation Equation	143
	12.2	Numerical Methods and Programs of Elastic Deformation	145
		12.2.1 Numerical Method and Program of Elastic Deformation	
		Equation in Line Contact	145
		12.2.2 Numerical Method and Program of Elastic Deformation	
		Equation in Point Contact	148
	12.3	Viscosity–Pressure and Density–Pressure Equations	155
		12.3.1 Viscosity–Pressure Relationship	155
		12.3.2 Viscosity–Pressure–Temperature Relationship	156
		12.3.3 Density–Pressure Relationship	156
13	Nun	nerical Method and Program for EHL in Line Contact	159
	13.1	Basic Equations	159
	13.2	Numerical Method	160
		13.2.1 Dimensionless Equations	160
		13.2.2 Discrete Equations	161
		13.2.3 Iterative Method	162
		13.2.4 Selection of Iterative Methods	163
		13.2.5 Relaxation Factors	164
	13.3		164
		13.3.1 Calculation Diagram	164
		13.3.2 Calculation Program	165
		13 3 3 Calculation Results another and brack 1.	171

14		erical Method and Program for EHL in Point Contact	173
	14.1	Basic Equations	173
	14.2	Numerical Method	174
		14.2.1 Dimensionless Equations	174
		14.2.2 Discrete Equations	175
	14.3	Calculation Program	176
		14.3.1 Calculation Diagram	176
		14.3.2 Calculation Program	177
		14.3.3 Calculation Results	186
15	Num	erical Method and Program for Grease EHL in Line Contact	187
	15.1	Basic Equations	187
		15.1.1 Reynolds Equation	187
		15.1.2 Film Thickness Equation	187
		15.1.3 Viscosity–Pressure Equation	188
		15.1.4 Density–Pressure Equation	188
	15.2	Numerical Method	188
		15.2.1 Dimensionless Equations	188
		15.2.2 Discrete Equations	189
	15.3	Coloulation Programs	189
		15.3.1 Calculating Diagram	189
		15 3 2 Calculation Program	190
		15.3.3 Calculation Results	199
16	Num	erical Method and Program for Grease EHL in Point Contact	201
	16.1	Basic Equations	201
		16.1.1 Reynolds Equation	201
		16.1.2 Film Thickness Equation	201
		16.1.3 Elastic Deformation Equation	202
		16.1.4 Viscosity–Pressure Equation	202
		16.1.5 Density Equation	202
	16.2	Numerical Method	202
		16.2.1 Dimensionless Equations	202
		16.2.2 Discrete Equations	203
	16.3	Calculation Program	204
		16.2.1 Calculation Diagram	204
		1622 61 1 .: 2	205
		16.3.2 Calculation Program 16.3.3 Calculation Results	214
17	Num	erical Method and Program for Thermal EHL in Line Contact	215
		Pasic Equations	215
		17.1.1 Reynolds Equation	215

Contents

		17.1.2 France Equation	215			
		17.1.2 Energy Equation	216			
		17.1.3 Film Thickness Equation	216			
		17.1.4 Elastic Deformation Equation	216			
		17.1.5 Roelands Viscosity—Pressure—Temperature Equation	217			
	170	17.1.6 Density—Pressure—Temperature Equation	217			
	17.2	Numerical Method	217			
		17.2.1 Dimensionless Equations	218			
	172	17.2.2 Discrete Equations Calculation Program	220			
	17.3		220			
		17.3.1 Calculation Diagram of Multigrid Method	221			
		17.3.2 Calculation Diagram of Temperature	222			
		17.3.3 Calculation Program 17.3.4 Calculation Results	236			
		17.5.4 Calculation Results	230			
18	Num	erical Method and Program for Thermal EHL in Point Contact	237			
	18.1	Basic Equations	237			
		18.1.1 Reynolds Equation	237			
		18.1.2 Energy Equation	237			
		18.1.3 Film Thickness Equation	238			
		18.1.4 Elastic Deformation Equation	238			
		18.1.5 Roelands Viscosity–Pressure–Temperature Equation	239			
		18.1.6 Density–Pressure–Temperature Equation	239			
	18.2	Numerical Method	239			
		18.2.1 Dimensionless Equations	239			
		18.2.2 Discrete Equations	241			
	18.3	Calculation Program	242			
		18.3.1 Calculation Diagram	242			
		18.3.2 Calculation Program	242			
		18.3.3 Calculation Results	261			
19	Num	erical Method and Program for Thermal Grease				
	EHL in Line Contact 26					
		Basic Equations	263			
		19.1.1 Reynolds Equation	263			
		19.1.2 Energy Equation	264			
		19.1.3 Film Thickness Equation	264			
		19.1.4 Elastic Deformation Equation	265			
		19.1.5 Viscosity–Pressure–Temperature Equation	265			
		19.1.6 Density–Pressure–Temperature Equation	265			
	19.2	Numerical Method	265			
		19.2.1 Dimensionless Equations	265			
		19.2.2 Discrete Equations	267			

X		Contents

	19.3	Calculation Program	268
		19.3.1 Calculation Diagram	268
		19.3.2 Calculation Program	268
		19.3.3 Calculation Results	287
20	Num	erical Method and Program for Thermal Grease EHL	
		pint Contact	289
	20.1	Basic Equations	289
		20.1.1 Reynolds Equation	289
		20.1.2 Energy Equation	290
		20.1.3 Film Thickness Equation	290
		20.1.4 Elastic Deformation Equation	291
		20.1.5 Roelands Viscosity-Pressure-Temperature Equation	291
		20.1.6 Density-Pressure-Temperature Equation	291
	20.2	Numerical Method	291
		20.2.1 Dimensionless Equations	291
		20.2.2 Discrete Equations	293
	20.3		294
		20.3.1 Calculation Diagram	294
		20.3.2 Calculation Program	295
		20.3.3 Calculation Results	310
Par	t 4	CALCULATION PROGRAMS FOR LUBRICATION	
		ANALYSIS IN ENGINEERING	311
21	Lubr	rication Calculation Program for Herringbone Grooved Journal	
	Bear	ing of Micro Motor	313
	21.1	Basic Theory of Lubrication Calculation of Herringbone	
		Groove Bearing	313
		21.1.1 Journal Center Position and Film Thickness	313
		21.1.2 Reynolds Equation	314
		21.1.3 Boundary Conditions	315
		21.1.4 Flux Calculation	316
		21.1.5 Temperature Calculation	316
	21.2	Program for Performance Calculation	318
		21.2.1 Lubrication Performances	318
		21.2.2 Calculation Program	318
	21.3		326
	21.4		332
		21.4.1 Package Contents	332
		21.4.2 Program Installation	332
		21.4.3 Program Operation	333

22	Lubi	rication Optimization Program of Herringbone Grooved	
	Jour	nal Bearing of Micro Motor	337
	22.1	Method of Optimization Calculation	337
		22.1.1 Requirements of Parameter Optimization	337
		22.1.2 Optimization Model	337
		22.1.3 Optimization Methods and Steps	338
	22.2	Program Layout of Optimization Calculation	338
		22.2.1 Optimization Program Diagram	338
		22.2.2 Calculation Program	339
		22.2.3 Parameters in Program	352
	22.3	Optimization Calculation Examples	352
		22.3.1 Example 1: Optimization Calculation for Static Load 22.3.2 Example 2: Optimization Calculation for Static Flux	352
		(Eccentricity Ratio ε is Constant)	354
		22.3.3 Example 3: Optimization Calculation for Static Flux (Load W is Constant)	354
		22.3.4 Example 4: Optimization Calculation for Dynamic Load 22.3.5 Example 5: Optimization Calculation for Dynamic Flux	354
		(Eccentricity $\varepsilon$ is Constant)	354
		22.3.6 Example 6: Optimization Calculation for Dynamic	
		Flux (Load W is Constant)	355
	22.4	Instructions for HBOA Software Package	355
		22.4.1 Program Package	355
		22.4.2 Program Execution	356
23		ulation Program for Gas Lubrication of Hard Disk/Head	2/1
		tra Thin Film	361
	23.1		361
		23.1.1 Basic Equations	361
		23.1.2 Gas Film Thickness	362
	22.2	23.1.3 Poiseuille Flow Rate	362
	23.2	Discrete Equation and Special Treatments	363
		23.2.1 Iterative Scheme Considering High Bearing Numbers	363
	22.2	23.2.2 Abrupt Changes between Steps on ABS	364
	23.3		364
		23.3.1 Calculation Diagram	364
		23.3.2 Calculation Program 23.3.3 Calculation Results	366
		25.5.5 Calculation Results	371
24	Calc	ulation Program of Flight Attitude of Magnetic Head	373
	24.1	Search Strategy for Flight Attitude	373
	24.2	Calculation Program	375

**		
XII		Contents

	2/21	Program Introduction	375
		Calculation Diagnam	MARKET ARE SHEET AND ADDRESS.
		Calculation Diagram	376
		Calculation Program	376
	24.2.4	Calculation Results	386
D - C			200
Reference	es		389
Index			391

# Part One Numerical Method for Reynolds Equation

Fant One

Identifical Method

Solvenstion

Identification