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INVERSE THEORY AND APPLICATIONS IN GEOPHYSICS

SECOND EDITION

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Inverse Theory and Applications in Geophysics

Second Edition

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Elsevier
Radarweg 29, PO Box 211, 1000 AE Amsterdam, Netherlands
The Boulevard, Langford Lane, Kidlington, Oxford OX5 1GB, UK
225 Wyman Street, Waltham, MA 02451, USA

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ISBN: 978-0-444-62674-5

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Cataloging-in-Publication Data
A catalog record for this book is available from the Library of Congress

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Printed in the United Kingdom

Inverse Theory and Applications in Geophysics

This book is dedicated to my wife Olga

Preface to the Second Edition

This book is a new edition of my original book entitled "Geophysical Inverse Theory and Regularization Problems," published in 2002.

In the original 2002 book, I laid down the mathematical foundations of geophysical inverse theory and presented different inversion and imaging techniques used in geophysical applications.

Over the last decade, significant progress has been made in the field of geophysical inversion. In this new edition, I have included additional material reflecting the recent developments in inversion theory and geophysical applications. The focus of the original publication was on providing a link between the methods used in gravity, electromagnetic, and seismic imaging and inversion. The major theme of the original book was also the need for regularization in the robust solution of inverse problems. The new edition is built on this theme of the original book related to systematic use of regularization theory and a unified approach to the inversion of different geophysical and physical data as well.

Several new chapters have been included in the new edition of the book. They are dedicated to the principles of multinary inversion, resolution analysis of regularized geophysical inversion, and joint inversion of multimodal data. Multinary inversion is a generalization of binary inversion to multiple physical properties. It can be applied to problems where the physical properties are best described by a finite number of possible values. The new chapter on resolution analysis provides a technique for appraisal of geophysical inverse images, which is extremely important in practical applications of the inversion. An overview of modern methods of joint inversion of multimodal data constitutes the subject of yet another new chapter included in the book. In particular, I discuss a new approach based on Gramian constraints, which makes it possible to consider in a unified way different types of properties of the model parameters playing an important role in the fusion of multimodal data.

This new edition includes also a concise presentation of methods based on the stochastic (probabilistic) approach to the inversion. They can be applied for solving the global minimization problem for the misfit functional with multiple local minima. These methods include the classical Monte Carlo methods, as well as the simulated annealing and genetic algorithms. I have also made quite a few corrections and modifications in other parts of the book, in order to include recent developments in different areas of geophysical modeling and inversion. They are related to the chapters on the migration of the potential fields, methods of numerical modeling of the electromagnetic fields, and full-waveform inversion of seismic field data.

Finally, I hope that this new edition of Inverse Theory and Applications will fill in the still existing gap between mathematical literature on inversion theory and practical applications in science and engineering.

Michael S. Zhdanov
Salt Lake City, Utah
January, 2015

Preface

Inverse solutions are key problems in many natural sciences. They form the basis of our understanding of the world surrounding us. Whenever we try to learn something about physical laws, the internal structure of the earth or the nature of the Universe, we collect data and try to extract the required information from these data. This is the actual solution of the inverse problem. In fact the observed data are predetermined by physical laws and by the structure of the earth or Universe. The method of predicting observed data for given sources within given media is usually referred to as the forward problem solution. The method of reconstructing the sources of some physical, geophysical, or other phenomenon, as well as the parameters of the corresponding media, from the observed data is referred to as the inverse problem solution.

In geophysics, the observed data are usually physical fields generated by natural or artificial sources and propagated through the earth. Geophysicists try to use these data to reconstruct the internal structure of the earth. This is a typical inverse problem solution.

Inversion of geophysical data is complicated by the fact that geophysical data are invariably contaminated by noise and are acquired at a limited number of observation points. Moreover, mathematical models are usually complicated, and yet at the same time are also simplifications of the true geophysical phenomena. As a result, the solutions are ambiguous and error-prone. The principal questions arising in geophysical inverse problems are about the existence, uniqueness, and stability of the solution. Methods of solution can be based on linearized and nonlinear inversion techniques and include different approaches, such as least-squares, gradient-type methods (including steepest-descent and conjugate-gradient), and others.

A central point of this book is the application of so-called "regularizing" algorithms for the solution of ill-posed inverse geophysical problems. These algorithms can use *a priori* geological and geophysical information about the earth's subsurface to reduce the ambiguity and increase the stability of the solution.

In mathematics, we have a classical definition of the ill-posed problem: a problem is ill-posed, according to Hadamard (1902), if the solution is not unique or if it is not a continuous function of the data (i.e., if to a small perturbation of data; there corresponds an arbitrarily large perturbation of the solution). Unfortunately, from the point of view of classical theory, all geophysical inverse problems are ill-posed, because their solutions are either nonunique or unstable. However, geophysicists solve this problem and obtain geologically reasonable results in one of two ways. The first is based on intuitive estimation of the possible solutions and selection of a geologically adequate model by the interpreter. The second is based on the application of different types of regularization algorithms, which allow automatic selection of the proper solution by the computer using *a priori* geological and geophysical information about the earth's structure. The most consistent approach to the construction of regularization algorithms has been developed in the works of Tikhonov and Arsenin (1977) (see also Strakhov, 1968, 1969; Lavrent'ev et al., 1986; Dmitriev, 1990). This approach gives a solid basis for the construction of effective inversion algorithms for different applications.

In the usual way, we describe the geophysical inverse problem by the operator equation:

$$A\mathbf{m} = \mathbf{d}, \quad \mathbf{m} \in M, \ \ \mathbf{d} \in D,$$

where D is the space of geophysical data and M is the space of the parameters of geological models; A is the operator of the forward problem that calculates the proper data $\mathbf{d} \in D$ for a given model $\mathbf{m} \in M$. The main idea of the regularization method consists of approximating the ill-posed problem with a family of well-posed problems A_α depending on a scalar regularization parameter α. The regularization must be such that as α vanishes, the procedures in the family A_α should approach the accurate procedure A. It is important to emphasize that regularization does not necessary mean "smoothing" of the solution. Regularization may include "smoothing," but the critical element of this approach is in selecting the appropriate solution from a class of models with the given properties. The main basis for regularization is an implementation of *a priori* information in the inversion procedure. The more information we have about the geological model, the more stable is the inversion. This information is used for the construction of the "regularized family" of well-posed problems A_α.

The main goal of this book is to present a detailed exposition of the methods of regularized solution of inverse problems based on the ideas of Tikhonov regularization, and to show different forms of their applications in both linear and nonlinear geophysical inversion techniques.

The book is arranged in five parts. Part I is an introduction to inversion theory. In this part, I formulate the typical geophysical forward and inverse problems and introduce the basic ideas of regularization. The foundations of regularization theory described here include: (1) definition of the sensitivity and resolution of geophysical methods, (2) formulation of well-posed and ill-posed problems, (3) development of regularizing operators and stabilizing functionals, (4) introduction of the Tikhonov parametric functional, and (5) elaboration of principles for determining the regularization parameter.

In Part II, I describe basic methods of solution of the linear inverse problem using regularization, paying special attention to iterative inversion methods. In particular, Chapter 4 deals with the classical minimal residual method and its generalizations based on different modifications of the Lanczos method. The important result of this chapter is that all iterative schemes, based on regularized minimal residual methods, always converge for any linear inverse problem. In Part II, I discuss the major techniques for regularized solution of nonlinear inverse problems using gradient-type methods of optimization. Thus, the first two parts outline the general ideas and methods of regularized inversion.

In the following parts, I describe the principles of the application of regularization methods in gravity and magnetic Part III, electromagnetic (Part IV), and seismic (Part V) inverse problems. The key connecting idea of these applied parts of the book is the analogy between the solutions of the forward and inverse problems in different geophysical fields. The material included in these parts emphasizes the mathematical similarity in constructing forward modeling operators, sensitivity matrices, and inversion algorithms for different physical fields. This similarity is based on the analogous structure of integral representations used in the solution of the forward and inverse problems. In the case of potential fields, integral representations provide a precise tool for linear modeling and inversion. In electromagnetic or seismic cases, these representations lead to rigorous integral equations, or to approximate but fast and accurate solutions, which help in constructing effective inversion methods.

The book also includes chapters related to the modern technology of geophysical imaging, based on seismic and electromagnetic migration. Geophysical field migration is treated as the first iteration in the iterative solution of the general inverse problem. It is also demonstrated that any inversion algorithm can be treated as an iterative migration of the residual fields obtained on each iteration. From this point of view, the difference between these two separate approaches to the interpretation of geophysical data—inversion and migration—becomes negligible.

In summary, this text is designed to demonstrate the close linkage between forward modeling and inversion methods for different geophysical fields. The mathematical tool of regularized inversion is the same for any geophysical data, even though the physical interpretation of the inversion procedure may be different. Thus, another primary goal of this book is to provide a unified approach to reconstructing the parameters of the media under examination from observed geophysical data of a different physical nature. It is impossible, of course, to cover in one book all the variety of modern methods of geophysical inversion. The selection of the material included in this book was governed by the primary goals outlined above. Note that each chapter in the book concludes with a list of references. A master bibliography is given at the end of the text, for convenience.

Portions of this book are based on the author's monograph "Integral Transforms in Geophysics" (1988), where the general idea of a unified approach to the mathematical theory of transformation and imaging of different geophysical fields was originally introduced. The corresponding sections of the book have been written using research results originated by the author in the Institute of Terrestrial Magnetism, Ionosphere and Radio-Wave Propagation (IZMIRAN) and later in the Geoelectromagnetic Research Institute of the Russian Academy of Sciences in 1980-1992. However, this text actually began as a set of lecture notes created for the course "Geophysical Inverse Theory," which I taught during the fall semester, 1992, and the spring semester, 1993, at the Colorado School of Mines. These notes resulted in a tutorial "Regularization in Inversion Theory," published in 1993 as Report #136 of the Center for Wave Phenomena (CWP), Colorado School of Mines. Over the years of teaching the "Inversion Theory and Applications" class at the University of Utah this set of notes was significantly expanded and improved.

In this book, I also present research results created by the author and his graduate students at the Consortium for Electromagnetic Modeling and Inversion (CEMI). CEMI is a research and educational program in applied geophysics based at the Department of Geology and Geophysics, University of Utah. It is supported by an industry consortium formed by many major petroleum and mining exploration companies. The general objectives of the Consortium are to develop forward and inverse solutions for gravity, magnetic, and electromagnetic methods of geophysics, and to provide interpretive insight through model studies (for additional information, please, see the CEMI web site at: http://www.mines.utah.edu/~wmcemi). The research goal is to improve the effectiveness of geophysical techniques in mining, petroleum, geothermal, and engineering applications. Progress in these fields requires the development of mathematically sophisticated methods which are aimed at the solution of practical geophysical problems. This philosophy is reflected in the current book which contains a mixture of basic mathematical material and advanced geophysical techniques, which, I hope, will fill a gap in the presently available literature on geophysical applications of mathematical inverse theory.

Some of the results contained in the book are based on research projects, which have been supported by grants and contracts from the National Science Foundation, the Department of Energy, the United States Geological Survey, and the Office of Naval Research. I am very grateful for the funding support provided by all these organizations.

It is a great pleasure for me to acknowledge those many people who have influenced my thinking and contributed to my learning of mathematics and geophysics. Among many, I must single out the unforgettable influence and encouragement given to me by Academician Andrei N. Tikhonov. During all his life he demonstrated in his research how the synthesis of advanced mathematical theory and practically oriented applications can generate exciting progress in science and technology. His ideas lie at the foundation of this book. I am also indebted to Academician Vladimir N. Strakhov of the Institute of the Physics of the Earth, Russian Academy of Sciences, whose contributions to mathematical geophysics are unsurpassed. The inspiring discussions with Professor Vladimir I. Dmitriev of Moscow State University on the geophysical aspects of regularization theory and integral equation methods were very helpful and important to me as well.

I also wish to thank Professor Frank Brown and other members of the University of Utah for providing stimulating support during the work on the book. I am thankful to all my past and present graduate students and research associates, who took this course in 1992-2001 and provided me with invaluable feedback and many constructive discussions and suggestions, which helped me to improve the text. While preparing the book, I received much assistance from Professor John Weaver of the University of Victoria, British Columbia, who touched every chapter and made the final version of the book much more readable and understandable. Thanks also go to Professor Robert Smith of the University of Utah, who read parts of the manuscript and made a number of useful suggestions and corrections.

Last, but foremost, I wish to dedicate this book to my wife, Olga Zhdanov, whose continuous patience, support, and unfailing love made this book a reality.

Michael S. Zhdanov
Salt Lake City, Utah
December, 2001

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