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# Phased Array Antennas

SECOND EDITION

*Robert C. Hansen*

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# Phased Array Antennas

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Second Edition

**R. C. HANSEN**

*Consulting Engineer R. C. Hansen, Inc.*  
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# **Phased Array Antennas**

This book is dedicated to those who made Microwave Scanning  
Antennas possible:

Nicolas A. Begovich  
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Arthur A. Oliner  
Jack F. Ramsay  
Joseph A. Vitale

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# Preface to the First Edition

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Although array antennas have many decades of history, the last two decades have experienced a maturation, both in the understanding and design of arrays, and in the use of large sophisticated arrays. Radars utilizing electronic scanning arrays are in common use, from airport surveillance to missile detection and tracking; names of U.S. military systems, such as Aegis, Patriot, and Pave Paws, are well known. This book is a comprehensive treatment of all aspects of phased arrays; much has changed since the only other such work, *Microwave Scanning Antennas*, appeared in 1966. Most noteworthy has been the parallel development of inexpensive computer power and the theoretical understanding of nearly all aspects of phased array design. Design algorithms suitable for computers are emphasized here, with numerical tips and short algorithms sprinkled throughout the chapters. The work is prepared from the dual viewpoint of a design engineer and an antenna array analyst.

Chapter 2, on basic array characteristics, which covers grating lobes, quantization lobes, bandwidth, and directivity follows an introductory chapter. Highly efficient linear aperture and array synthesis techniques, including sum and difference patterns, are covered in Chapter 3. Chapter 4 treats synthesis of planar arrays. Array elements are covered in Chapter 5 and include not only the classic dipoles and slots, but TEM horns and patches. In Chapter 6, feeds for linear and planar arrays, both fixed beam and scanning, are examined; photonic time delay and feeders are included. Array performance is strongly affected by mutual impedance. Chapter 7 investigates ways of calculating this for various arrays elements, including an extensive treatment of ways of calculating array performance with mutual effects included. Among these are unit cell, spectral moment method, finite impedance matrix, and scattering techniques. Finite arrays are examined in Chapter 8, including the recently developed Gibbbsian models. Next, Chapter 9 is an extensive view of superdirective arrays; the implications of high-temperature superconductors for antennas is an important feature. Multiple-beam arrays, as opposed to multiple-beam reflector feeds, are treated in Chapter 10.

Included are one- and two-dimensional Butler and Rotman lenses, and the practical meaning of beam orthogonality. Conformal arrays, ranging from ring arrays to arrays on cones, are covered next; much previously unpublished material is included in this chapter. Finally, Chapter 12 discusses array diagnostics, waveguide simulators in depth, and array tolerances. Extensive references to the archival literature are used in each chapter to offer additional sources of data.

ROBERT C. HANSEN

Tarzana, CA



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# Preface to the Second Edition

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Several specialized types of phased arrays have attracted attention since the first edition. Connected dipole arrays offer wide bandwidth compared to a conventional array; these are discussed in detail in Chapter 12. (The old Chapter 12 is now Chapter 15). Reflectarrays provide reduced fabrication costs compared to a phased array. And retrodirective arrays offer interesting capabilities for data links. Both of these technologies are the subject of Chapter 13. The combination of reflectors and arrays is addressed in Chapter 14, both for focal plane arrays, including coma correction, and near-field Cassegrainian and Gregorian antennas.

Updates and additions have been made to existing sections: time delay deployment options for corporate fed arrays; fundamental limitations on Artificial Magnetic Conductors; Substrate Integrated Waveguide to replace rectangular waveguide; antennas for 60 GHz and beyond; impedances matching capabilities and limitations including Bode criterion limitations; elaboration of Scan Impedance and Scan Element Pattern calculations and measurements; and finally comments on completely overlapped sub-arrays.

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