MILLER

HEEREN

HORNSBY

# MATHEMATICAL INEAC

EXPANDED NINTH EDITION

# Mathematical Ideas

#### EXPANDED NINTH EDITION

Charles D. Miller

Vern E. Heeren
American River College

John Hornsby
University of New Orleans

and

Margaret L. Morrow

Plattsburgh State University of New York for the chapter on Graph Theory

and

Iill Van Newenhizen

Lake Forest College for the chapter on Voting and Apportionment



Boston San Francisco New York
London Toronto Sydney Tokyo Singapore Madrid
Mexico City Munich Paris Cape Town Hong Kong Montreal

Sponsoring Editor: Bill Poole

Executive Project Manager: Christine O'Brien

Assistant Editor: Jennifer Kerber Managing Editor: Karen Guardino

Project Coordination: Elm Street Publishing Services, Inc.

Marketing Manager: Carter Fenton
Senior Prepress Supervisor: Caroline Fell
Manufacturing Buyer: Evelyn Beaton
Text & Cover Design: Barbara T. Atkinson

Cover Photo: Francisco Ontanon Nunez/Image Bank

For permission to use copyrighted material, grateful acknowledgment is made to the copyright holders on pages A-59-A-60, which are hereby made part of this copyright page.

#### Library of Congress Cataloging-in-Publication Data

Miller, Charles David

Mathematical ideas.—9th ed./Charles D. Miller, Vern E. Heeren, E. John Hornsby.

p. cm.

Includes bibliographical references and index.

ISBN 0-321-04324-3 (alk. paper), ISBN 0-321-07610-9 (expanded edition), ISBN 0-321-07602-8 (instructor's edition)

1. Mathematics. I. Heeren, Vern E. II. Hornsby, E. John. III. Title.

QA39.2.M55 2001

510-dc21

00-058658

Copyright © 2001 Addison Wesley Educational Publishers.

Reprinted with corrections, April 2001.

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, without the prior written permission of the publisher. Printed in the United States of America.

To my wife, Carole; our sons, Mark, Christopher, and Scott; their wives, Emily, Heather, and Aimee; and our grandsons,

David and Sean.

V. E. H.

#### To Anne Kelly

With an "urn-full" of gratitude for a decade of valued friendship and unparalleled editorial guidance.

You are the best.

E. J. H.

### Preface

Mathematical Ideas by Miller, Heeren, and Hornsby has enjoyed a long, illustrious history. We are proud to present this ninth edition. In the first preface, Chuck Miller and Vern Heeren wrote: "We believe strongly that mathematics can be very meaningful for the non-physical science major. In an attempt to convince the student of this, we have chosen topics that are interesting; we have attempted, whenever it is appropriate, to point out the practical applications of the ideas; we have included historical material; and we have written in a light and informal manner." Despite the fact that the table of contents has undergone extensive revision to reflect the changing trends in "liberal arts mathematics," we hope that the original intent of the text still is achieved.

Like its predecessors, this edition has been designed with a variety of students in mind. It is well-suited for several courses, including the aforementioned liberal arts audience, survey courses in mathematics, finite mathematics, and mathematics for prospective and in-service elementary and middle school teachers. Ample topics are included for a two-term course, yet the variety of topics and flexibility of sequence make the text suitable for shorter courses as well. Our main objectives continue to be to provide comprehensive coverage of topics, appropriate organization, clear exposition, an abundance of examples, and well-planned exercise sets with numerous applications.

We have worked diligently to incorporate the recommendations of the *Principles and Standards for School Mathematics*, prepared by the National Council of Teachers of Mathematics (NCTM), and the *Standards for Introductory College Mathematics Before Calculus*, presented by the American Mathematical Association of Two-Year Colleges (AMATYC).

#### Overview of the Chapters

- Chapter 1 (The Art of Problem Solving) introduces the student to inductive reasoning, pattern recognition, and problem-solving techniques. Many of the new problems are taken from the popular monthly calendars found in the NCTM publication *Mathematics Teacher*.
- Chapter 2 (The Basic Concepts of Set Theory) and Chapter 3 (Introduction to Logic) give brief overviews of set theory and elementary logic.
- Chapter 3 contains new material on logic puzzles. Instructors wishing to do so may cover Chapter 3 before Chapter 2.
- Chapter 4 (Numeration and Mathematical Systems) covers various types
  of numeration systems and group theory. (Modular Systems, formerly in
  this chapter, have been moved to Chapter 5.)

- Chapter 5 (Number Theory) presents an introduction to prime and composite numbers, the Fibonacci sequence, and magic squares, and includes updated information on new developments in the field.
- Chapter 6 (The Real Numbers and Their Representations) introduces some
  of the basic concepts of real numbers, their various forms of representation, and operations of arithmetic with them.
- Chapter 7 (The Basic Concepts of Algebra) and Chapter 8 (Graphs, Functions, and Systems of Equations and Inequalities) form the core of the text's algebra component, providing either an introduction to or a review of elementary algebra. These chapters include many new applications.
- Chapter 9 (Geometry) introduces the student to the basic concepts of Euclidean geometry and provides several enrichment sections as well.
- Chapter 10 (Counting Methods) focuses on elementary counting techniques, in preparation for the chapter to follow.
- Chapter 11 (Probability) covers the fundamentals of probability, odds, and expected value. We have added more basic exercises at the request of reviewers.
- Chapter 12 (Statistics) gives a brief introduction to the various topics of elementary statistics. We have included new applied exercises with reallife data.
- Chapter 13 (Consumer Mathematics) provides the student with the basics of mathematics of finance and includes a new section on investing, with an emphasis on mutual funds.

The following chapters are available in the new Expanded Ninth Edition of this text:

- Chapter 14 (Graph Theory) covers the concepts of graph theory and their applications.
- Chapter 15 (Voting and Apportionment) introduces the possibilities and impossibilities of voting and apportionment.

#### **Course Outline Considerations**

For the most part, the chapters in the text are independent and may be covered in the order chosen by the instructor. The few exceptions are as follows. Chapter 6 contains some material dependent on the ideas found in Chapter 5. Chapter 6 should be covered before Chapter 7 if student background so dictates. Likewise, Chapters 7 and 8 form an algebraic "package" and should be covered in sequential order. A thorough understanding of Chapter 11 depends on knowledge of Chapter 10 material, although probability can be covered without learning extensive counting methods by avoiding the more difficult exercises. The latter part of Chapter 12, on inferential statistics, requires an understanding of probability (Chapter 11).

#### Features of the Ninth Edition

**Enhanced: Varied Exercise Sets** We continue to present a variety of exercises including drill, conceptual, and applied problems. Approximately 25% of the exercises are new or revised. We have updated the exercises that focus on real-life data and

have added titles to them for easy identification. Chapters 6–8, 12, and 13 are especially rich in these new applications. We continue to use graphs, tables, and charts when appropriate. Most sections include a few challenging exercises that require students to extend the ideas presented in the section. To address the issue of writing across the curriculum, most exercise sets include some exercises that require the student to answer by writing a few sentences.

Enhanced: Margin Notes This popular feature has become a hallmark of this text and has been retained and updated where appropriate. New notes have been written as well. Interspersed throughout the text, they deal with various subjects such as lives of mathematicians, historical vignettes, philatelic and numismatic reproductions, anecdotes, newspaper and magazine articles, and current research in mathematics. We hope that users continue to enjoy the margin notes as much as we enjoy researching and composing them.

Enhanced: Collaborative Investigations In response to instructor requests, we continue to address the issue of cooperative learning in the end-of-chapter feature "Collaborative Investigation," introduced for the first time in the eighth edition. Some investigations have been updated; others have been completely revamped.

**Problem Solving** Special paragraphs labeled "problem solving" relate the discussion of problem-solving strategies to techniques learned earlier.

*Optional Graphing Technology* We continue to provide sample graphing calculator screens (generated in this edition by a TI-83 calculator) to show how technology can be used to support results found analytically. It is not essential, however, that a student have a graphing calculator to study from this text; the technology component is optional.

*Flexibility* Some topics in the first six chapters require a basic knowledge of equation solving. Depending on the background of the students, the instructor may omit topics that require this skill. On the other hand, the two algebra chapters (Chapters 7 and 8) provide an excellent overview of algebra, and because of the flexibility of the text, they may be covered at almost any time.

**Chapter Openers** Each chapter opens with an application of a chapter topic. These help students see the relevance of mathematics in our world.

**Art Program** We continue to use full color, and the art program remains one of the highlights of the book.

**Examples** Numerous, carefully selected examples illustrate the concepts and skills introduced.

For Further Thought These entries encourage students to share among themselves their reasoning processes to gain a deeper understanding of key mathematical concepts.

Emphasis on Real Data in the Form of Graphs, Charts, Tables Using magazines, newspapers, and the Internet as sources, we have included many new applications from daily life.

Chapter Tests Each chapter concludes with a chapter test so students can check their mastery of the material.

#### Supplements for the Student

Study Guide and Solutions Manual ISBN 0-321-07606-0 This for-sale manual contains solutions to all odd-numbered section, Extension, and appendix exercises,

as well as solutions to all Chapter Test exercises. In addition, chapter summaries review key points in the text, provide extra examples, and enumerate major topic objectives.

*Videotape Series* ISBN 0-321-07607-9 Videotapes presenting the text's topics are available. A qualified college-level mathematics instructor presents worked-out examples and uses visual aids to reinforce the mathematical concepts.

Guide to CLAST Mathematical Competency ISBN 0-673-98438-9 This special for-sale study guide for use in Florida offers help in preparing for the College Level Academic Skills Test (CLAST). It includes a review of arithmetic, a CLAST pretest, supplementary exercises, and a CLAST posttest.

Web Site The Web site for this text (http://www.awl.com/mhh) provides additional resources for both students and instructors.

InterAct Math Tutorial Software Windows/Macintosh (Dual Platform CD-ROM) ISBN 0-321-07666-4 InterAct Math Tutorial Software has been developed and designed by professional software engineers working closely with a team of experienced math educators. InterAct Math Tutorial Software includes exercises that are linked with every objective in the textbook and require the same computational and problem-solving skills as their companion exercises in the text. Each exercise has an example and an interactive guided solution designed to involve students in the solution process and to help them identify precisely the source of their confusion. The software recognizes common student errors and provides students with appropriate customized feedback. With its sophisticated answer recognition capabilities, InterAct Math Tutorial Software recognizes appropriate forms of the same answer for any kind of input. It also tracks student activity and scores for each section, which can then be printed out. The software is free to qualifying adopters or can be bundled with books for sale to students.

InterAct MathXL Software Text bundled with 12-month registration coupon ISBN 0-201-71412-4 (ISBN 0-201-72448-0 for the Expanded Edition) InterAct MathXL provides diagnostic testing and tutorial help online using InterAct Math Tutorial Software. Students can take chapter tests correlated to this textbook, receive individualized study plans based on those test results, work practice problems for areas in which they need improvement, receive tutorial instruction in those topics, and take further tests to gauge their progress. With InterAct MathXL, instructors can track students' test results, study plans, and practice work. The software is free when the access code is bundled with a new Addison-Wesley text.

**Digital Video Tutor** ISBN 0-321-08634-1 The videotape series for this text is provided on CD-ROM, making it easy and convenient for students to watch video segments from a computer at home or on campus. This complete video set, now affordable and portable for your students, is ideal for distance learning or extra instruction.

Addison Wesley Longman Math Tutor Center (http://www.awl.com/tutorcenter). Live one-on-one tutoring is available to students who purchase this text. Qualified mathematics tutors are available from 5 P.M. to midnight Eastern Standard Time Sunday through Thursday to answer students' questions on any problem with an answer at the back of the text. Students can contact the Tutor Center via toll-free phone, fax, e-mail, or Internet. This service is provided free when a Tutor Center

registration number is bundled with a new Addison-Wesley text. Consult your Addison Wesley Longman representative for details.

#### For the Instructor

*Instructor's Edition* ISBN 0-321-07602-8 This special edition of the text includes an answer section consisting of answers to all text exercises.

*Instructor's Solutions Manual* ISBN 0-321-07604-4 This manual contains solutions to all section, Extension, Chapter Test, and appendix exercises.

Instructor's Testing Manual ISBN 0-321-07603-6 This manual includes test forms paralleling the text's Chapter Tests.

TestGen-EQ with QuizMaster-EQ Windows/Macintosh (Dual-Platform CD-ROM) ISBN 0-321-07667-2. TestGen-EQ's friendly graphical interface enables instructors to easily view, edit, and add questions, transfer questions to tests, and print tests in a variety of fonts and forms. Search and sort features let the instructor quickly locate questions and arrange them in a preferred order. Six question formats are available, including short-answer, true-false, multiple-choice, essay, matching, and bimodal formats. A built-in question editor gives the user power to create graphs, import graphics, insert mathematical symbols and templates, and plug in variable numbers or text. Computerized testbanks include algorithmically defined problems organized according to each textbook. An "Export to HTML" feature lets instructors create practice tests for the Web. QuizMaster-EQ enables instructors to create and save tests using TestGen-EQ so students can take them for practice or for a grade on a computer network. Instructors can set preferences for how and when tests are administered. QuizMaster-EQ automatically grades the exams, stores results on disk, and allows the instructor to view or print a variety of reports for individual students, classes, or courses. Consult your Addison Wesley Longman sales representative for details.

InterAct Math Plus Software InterAct Math Plus combines course management and on-line testing with the features of the basic InterAct Math tutorial software to create an invaluable teaching resource. Consult your Addison Wesley Longman representative for details.

#### **Acknowledgments**

We wish to thank the following instructors who reviewed portions of the manuscript and contributed ideas for the Ninth Edition:

H. Achepohl, College of DuPage
Shahrokh Ahmadi, Northern
Virginia Community College
Richard Andrews, Florida A&M
University
Cindy Anfinson, Palomar College
Elaine Barber, Germanna
Community College
Anna Baumgartner, Carthage
College

James E. Beamer, Northeastern
State University
Elliot Benjamin, Unity College
Jaime Bestard, Barry University
Joyce Blair, Belmont University
Gus Brar, Delaware County
Community College
Roger L. Brown, Davenport
College

John Busovicki, Indiana University of Pennsylvania Ann Cascarelle, St. Petersburg Junior College Kenneth Chapman, St. Petersburg Junior College Gordon M. Clarke, University of the Incarnate Word M. Marsha Cupitt. Durham Technical Community College James Curry, American River College Ken Davis, Mesa State College Nancy Davis, Brunswick Community College George DeRise, Thomas Nelson Community College Catherine Dermott, Hudson Valley Community College Diana C. Dwan, Yavapai College Laura Dyer, Belleville Area College Jan Eardley, Barat College Joe Eitel, Folsom College Azin Enshai, American River College Gayle Farmer, Northeastern State University Michael Farndale, Waldorf College Gordon Feathers, Passaic County Community College Thomas Flohr, New River Community College Bill Fulton, Black Hawk College -East Anne Gardner, Wenatchee Valley College Donald Goral, Northern Virginia Community College Glen Granzow, Idaho State University Larry Green, Lake Tahoe Community College Arthur D. Grissinger, Lock Haven University

Don Hancock, Pepperdine University Denis Hanson, University of Regina Marilyn Hasty, Southern Illinois University Shelby L. Hawthorne, Thomas Nelson Community College Jeff Heiking, St. Petersburg Junior College Emanuel Jinich, Endicott College Karla Karstens, University of Vermont Hilary Kight, Wesleyan College Barbara J. Kniepkamp, Southern Illinois University at Edwardsville Suda Kunyosying, Shepherd College Pam Lamb, J. Sargeant Reynolds Community College John W. Legge, Pikeville College Leo Lusk, Gulf Coast Community College Sherrie Lutsch, Northwest Indian College Rhonda Macleod, Florida State University Andrew Markoe, Rider University Darlene Marnich, Point Park College Victoria Martinez, Okaloosa Walton Community College Mark Maxwell, Maryville University Carol McCarron, Harrisburg Area Community College Delois McCormick, Germanna Community College Daisy McCoy, Lyndon State College Vena McGrath, Davenport College Robert Moyer, Fort Valley State University Shai Neumann, Brevard Community College

Barbara Nienstedt, Gloucester County College Chaitanya Nigam, Gateway Community-Technical College Jean Okumura, Windward Community College Bob Phillips, Mesabi Range Community College Kathy Pinchback, University of Memphis Priscilla Putman, New Jersey City University Scott C. Radtke, Davenport College Beth Reynolds, Mater Dei College John Reily, Montclair State University Shirley I. Robertson, High Point University Andrew M. Rockett, CW Post Campus of Long Island University D. Schraeder, McLennan Community College Wilfred Schulte, Cosumnes College Melinda Schulteis, Concordia University Gary D. Shaffer, Allegany Community College of Maryland Jane Sinibaldi, York College of

Pennsylvania

Larry Smith, Peninsula College Marguerite Smith, Merced College Charlene D. Snow. Lower Columbia College H. Jeannette Stephens, Whatcom Community College Suzanne J. Stock, Oakton Community College Dian Thom, McKendree College Claude C. Thompson, Hollins University Mark Tom, College of the Sequoias Ida Umphers, University of Arkansas at Little Rock Karen Villarreal, University of New Orleans Wayne Wanamaker, Central Florida Community College David Wasilewski, Luzerne County Community College William Watkins, California State University, Northridge Susan Williford, Columbia State Community College Tom Witten, Southwest Virginia Community College

Fred Worth, Henderson State

University

A project of this magnitude cannot be conceived and completed without the help of many other dedicated individuals. It was our good fortune to be reunited with Anne Kelly, who served as project manager for this edition. Gina Linko of Elm Street Publishing Services provided excellent production supervision. Ed Moura, Christine O'Brien, Greg Tobin, Karen Guardino, Becky Malone, Barbara Atkinson, Carter Fenton, Jennifer Kerber, and Bill Poole of Addison-Wesley gave us their unwavering support. In addition, thanks go to Dr. Margaret L. Morrow of Plattsburgh State University of New York and Dr. Jill Van Newenhizen of Lake Forest College, who wrote the material on graph theory and voting/apportionment, respectively, for the Expanded Edition. Kitty Pellissier did her usual outstanding job of answerchecking, Paul van Erden prepared a thorough and accurate index, and Becky Troutman contributed the comprehensive Index of Applications. Tim Mogill and Frank Purcell also provided accuracy checks in the last stages of production. Finally, we thank our loyal users. Over the past thirty-three years, you have made this book one of the most successful in its market.

Vern E. Heeren John Hornsby

#### Contents

#### Preface xi

with						
C H	A	P	T	E	R	

#### The Art of Problem Solving

- 1.1 Solving Problems by Inductive Reasoning 2
- 1.2 An Application of Inductive Reasoning: Number Patterns 11
- 1.3 Strategies for Problem Solving 21
- 1.4 Calculating, Estimating, and Reading Graphs 33

Extension: Using Writing to Learn About Mathematics 42

Collaborative Investigation: Discovering Mathematics in Pascal's

Triangle 49 Chapter 1 Test 50



#### The Basic Concepts of Set Theory 52

- 2
- 2.1 Symbols and Terminology 53
- 2.2 Venn Diagrams and Subsets 60
- 2.3 Set Operations and Cartesian Products 68
- 2.4 Cardinal Numbers and Surveys 82
- 2.5 Infinite Sets and Their Cardinalities 89

Collaborative Investigation: A Survey of Your Class 96 Chapter 2 Test 97



#### Introduction to Logic 100

- 3
- 3.1 Statements and Quantifiers 101
- 3.2 Truth Tables and Equivalent Statements 108
- 3.3 The Conditional and Circuits 120
- 3.4 More on the Conditional 130
- 3.5 Analyzing Arguments with Euler Diagrams 136

Extension: Logic Puzzles 141

3.6	Analyzing	g Arguments	with Truth Tables	146	
Colla	borative In	vestigation:	Logic Puzzles Rev	isited	157
Chap	ter 3 Test	158			



#### Numeration and Mathematical Systems 159



- 4.1 Historical Numeration Systems 160
- 4.2 Arithmetic in the Hindu-Arabic System 170
- 4.3 Conversion Between Number Bases 179
- 4.4 Finite Mathematical Systems 190
- 4.5 Groups 198

Collaborative Investigation: A Perpetual Calendar Algorithm 205 Chapter 4 Test 206



#### Number Theory 208

5

- 5.1 Prime and Composite Numbers 209
- 5.2 Selected Topics from Number Theory 220
- 5.3 Greatest Common Factor and Least Common Multiple 227
- 5.4 Modular Systems 234
- 5.5 The Fibonacci Sequence and the Golden Ratio 244

Extension: Magic Squares 252

Collaborative Investigation: Investigating an Interesting Property of Number Squares 257

Chapter 5 Test 258



## The Real Numbers and Their Representations 259

6

- 6.1 Real Numbers, Order, and Absolute Value 260
- 6.2 Operations, Properties, and Applications of Real Numbers 270
- 6.3 Rational Numbers and Decimal Representation 283
- 6.4 Irrational Numbers and Decimal Representation 299
- 6.5 Applications of Decimals and Percents 309

Extension: Complex Numbers 322

Collaborative Investigation: Budgeting to Buy a Car 326

Chapter 6 Test 327



#### The Basic Concepts of Algebra 331

7

- 7.1 Linear Equations 332
- 7.2 Applications of Linear Equations 342
- 7.3 Ratio, Proportion, and Variation 356

				~=
7.4	Inear	Inequal	11100	370
1.4	Lilleai	Incquai	ILLO	2/0

- 7.5 Properties of Exponents and Scientific Notation 379
- 7.6 Polynomials and Factoring 393
- 7.7 Quadratic Equations and Applications 402

Collaborative Investigation: How Does Your Walking Rate Compare to That of Olympic Race-Walkers? 412

Chapter 7 Test 413



## Graphs, Functions, and Systems of Equations and Inequalities 415

- 8.1 The Rectangular Coordinate System and Circles 416
- 8.2 Lines and Their Slopes 426
- 8.3 Equations of Lines and Linear Models 437
- 8.4 An Introduction to Functions: Linear Functions, Applications, and Models 447
- 8.5 Quadratic Functions, Applications, and Models 461
- 8.6 Exponential and Logarithmic Functions, Applications, and Models 471
- 8.7 Systems of Equations and Applications 483

Extension: Using Matrix Row Operations to Solve Systems 504

8.8 Linear Inequalities, Systems, and Linear Programming 509 Collaborative Investigation: Olympic Track and Field Results 517 Chapter 8 Test 518



#### Geometry 520

9

- 9.1 Points, Lines, Planes, and Angles 522
- 9.2 Curves, Polygons, and Circles 532
- 9.3 Perimeter, Area, and Circumference 542
- 9.4 The Geometry of Triangles: Congruence, Similarity, and the Pythagorean Theorem 556

Extension: Right Triangle Trigonometry 570

- 9.5 Space Figures, Volume, and Surface Area 575
- 9.6 Non-Euclidean Geometry, Topology, and Networks 585
- 9.7 Chaos and Fractal Geometry 597

Collaborative Investigation: Generalizing the Angle Sum Concept 604 Chapter 9 Test 604



#### Counting Methods 608



- 10.1 Counting by Systematic Listing 609
- 10.2 Using the Fundamental Counting Principle 619
- 10.3 Using Permutations and Combinations 628

10.4 Using Pascal's Triangle and the Binomial Theorem 640 10.5 Counting Problems Involving "Not" and "Or" 648 Collaborative Investigation: Approximating Factorials Using Stirling's Formula 655 Chapter 10 Test 656



#### Probability 657



- 11.1 Basic Concepts 658
- 11.2 Events Involving "Not" and "Or" 669
- 11.3 Events Involving "And" 677
- 11.4 Binomial Probability
- 11.5 Expected Value 694
- 11.6 Estimating Probabilities by Simulation 703

Collaborative Investigation: Finding Empirical Values of  $\pi$  707 Chapter 11 Test 708



#### **Statistics** 710

- 12.1 Frequency Distributions and Graphs
- 12.2 Measures of Central Tendency 724
- 12.3 Measures of Dispersion 739
- 12.4 Measures of Position 748
- 12.5 The Normal Distribution 756

Extension: How to Lie with Statistics 766

12.6 Regression and Correlation 773

Collaborative Investigation: Combining Sets of Data 782

Chapter 12 Test 783



#### **Consumer Mathematics**

13.1 Interest and Inflation 787

Extension: Annuities 801

- 13.2 Consumer Credit 805
- 13.3 Truth in Lending 813
- 13.4 Buying a House 822
- 13.5 Investing in the Stock Market 834

Collaborative Investigation: To Buy or to Rent? 844

Chapter 13 Test 845



#### Graph Theory 846



- 14.1 Basic Concepts 84714.2 Euler Circuits 863
- Hamilton Circuits 881

14.4 Trees and Minimum Spanning Trees 897
 Collaborative Investigation: Finding the Number of Edges in a Complete Graph 911
 Chapter 14 Test 912



#### Voting and Apportionment 917

- 15.1 The Possibilities of Voting 918
- 15.2 The Impossibilities of Voting 939
- 15.3 The Possibilities of Apportionment 960
- 15.4 The Impossibilities of Apportionment 977

Collaborative Investigation: Class Favorites, An Election Exploration 990

Chapter 15 Test 992

Appendix The Metric System A-1

Answers to Selected Exercises A-9

Credits A-59

Index 1-1

Index of Applications 1-18

CHAPTER

## The Art of Problem Solvine

Modern supercomputers can perform billions of calculations in a single second. With such computing power available, why do scientists need even faster computers to solve some problems?

Consider the question of how a steel beam supporting a bridge might fracture under extreme stress. To approach this problem, a new science called **molecular dynamics** is being used to model the motion of individual atoms in the metal. Modeling a rectangular slab of metal only 1000 atoms wide, 2000 atoms long, and 19 atoms thick requires over 50 hours for a supercomputer. In order to simulate the motion of 100 million atoms in a steel beam, 7.6 billion numbers must be stored in computer memory. Because an atom's movement is so rapid, scientists must calculate its velocity and location every 1 millionth of 1 billionth of a second. Each one of these steps in time takes 90 billion arithmetic calculations. Scientists have so far succeeded in modeling 600 million atoms at one time. But, a single speck of dust can contain more than a billion atoms, so only extremely small pieces of metal can currently be modeled. Modeling an entire steel beam as it fractures under stress is not within reach of today's supercomputers simply because they are too slow.

Real applications requiring high-speed computation are becoming increasingly important. From modeling the heart over long periods of time to predicting weather 24 hours in advance, real-world problems are indeed very complex. Understanding them will require not only faster computers, capable of performing trillions rather than billions of calculations per second, but also new problem-solving strategies.

Source: SIAM News, Vol. 28, No. 5, 1995.

- I.I Solving Problems by Inductive Reasoning
- I.2 An Application of Inductive Reasoning: Number Patterns
- 1.3 Strategies for Problem Solving
- 1.4 Calculating, Estimating, and Reading Graphs

#### Extension

Using Writing to Learn
About Mathematics

#### Collaborative

Investigation:

Discovering Mathematics in Pascal's Triangle

Chapter 1 Test