Finite Mathematics
Practical Applications

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David B. Johnson
Diablo Valley College

Thomas A. Mowry
Diablo Valley College

Brooks/Cole Publishing Company
I(T)P® *An International Thomson Publishing Company*

Pacific Grove • Albany • Belmont • Bonn • Boston • Cincinnati • Detroit • Johannesburg
London • Madrid • Melbourne • Mexico City • New York • Paris • Singapore
Tokyo • Toronto • Washington

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Typesetting: *Carlisle Communications, Ltd.*
Printing and Binding: *R. R. Donnelley and Sons, Inc.*

For more information, contact:

BROOKS/COLE PUBLISHING COMPANY
511 Forest Lodge Road
Pacific Grove, CA 93950
USA

International Thomson Publishing Europe
Berkshire House 168-173
High Holborn
London WC1V 7AA
England

Thomas Nelson Australia
102 Dodds Street
South Melbourne, 3205
Victoria, Australia

Nelson Canada
1120 Birchmount Road
Scarborough, Ontario
Canada M1K 5G4

International Thomson Editores
Seneca 53
Col. Polanco
11560 México, D. F., México

International Thomson Publishing GmbH
Königswinterer Strasse 418
53227 Bonn
Germany

International Thomson Publishing Asia
60 Albert Street
#15-10 Albert Complex
Singapore 189969

International Thomson Publishing Japan
Hirakawacho Kyowa Building, 3F
2-2-1 Hirakawacho
Chiyoda-ku, Tokyo 102
Japan

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

Library of Congress Cataloging-in-Publication Data

Johnson, David B. (David Bruce), [date]
Finite mathematics : practical applications / David B. Johnson, Thomas A. Mowry.
p. cm.
Includes bibliographical references (p. -) and index.
ISBN 0-534-94782-4 (alk. paper)
1. Mathematics. I. Mowry, Thomas A. II. Title.
QA39.2.J59 1999
510—dc21 98-24798
CIP

Preface

Typically, a student takes a finite mathematics course to satisfy either a major or graduation requirement. Thus, the course can be populated with students ranging from business majors to biology majors to liberal arts majors. The goal of *Finite Mathematics: Practical Applications* is to familiarize students with the mathematics used in their major fields of study and to expose liberal arts students to topics in mathematics that are usable and relevant to any educated person. It is our hope that each student will encounter several topics that will prove useful over the course of his or her life. In addition, we hope that students will see that mathematics is relevant to their education and that there is a human aspect to mathematics.

Tour of the Book

Ease of Use

This book is user-friendly. The examples don't skip steps; key points are boxed for emphasis; step-by-step procedures are given; and there is an abundance of explanation. The Instructor's Manual includes a "prerequisite map" so that the instructor can easily tell which earlier topics must be covered.

Course Prerequisite

Finite Mathematics: Practical Applications is written for the student who has successfully completed a course in algebra. A background in beginning algebra may be sufficient; however, the authors have found that the student who has a background in intermediate algebra is significantly better prepared for the course.

Algebra Review

Where appropriate, algebraic topics are reviewed, but in a very selective and focused manner. Only those topics that are used in the book are covered. There is no "Review of Algebra" chapter; instead, the reviews are placed as close as possible to the topics that utilize them, usually in a Section 0 at the beginning of the chapter. These sections are direct and to the point. They do not attempt to provide a thorough treatment of the algebra in question but rather focus on the algebra that will be used in the sections that follow. Typically, they do not cover applications of the algebra, which are covered in the following sections.

Algebra courses vary significantly from school to school. Among the Section 0 topics are some that students may not have seen before, such as matrix arithmetic and the elimination method. In these cases, the reviews are more detailed and assume less prior knowledge.

Technology

Calculators and computers are useful and powerful tools that have become an integral part of the classroom and workplace. However, many students are unable to use their calculators effectively and have no mathematical experience using computers. Therefore, instructions for graphing calculator, scientific calculator, and computer use are included.

Detailed instructions for both scientific and graphing calculator use are given in calculator boxes throughout the text:

Scientific calculator instructions are identified with this scientific calculator icon.

Graphing calculator instructions are identified with this graphing calculator icon.

Furthermore, a number of optional technology subsections address some of the more advanced capabilities of graphing calculators and computers. These subsections allow instructors to incorporate technology into their classes if and when they desire, but they are entirely optional, and the text is in no way technology dependent. The subsections, identified with italics in the table of contents, are clearly identified in the text with an icon at the beginning of the subsection, and with a colored bar at the edge of the page as in this portion of the preface. The subsections are always preceded by technology-free discussion and exercises.

The technology subsections that focus on graphing calculators were specifically written for Texas Instruments models TI-82, TI-83, TI-85, and TI-86; however, they frequently apply to other brands as well. They are identified by the graphing calculator icon. See the table of contents for a complete listing.

The text also features Amortrix, a computer software supplement written specifically to accompany this text. It is available for Macintosh and Windows-based computers; it is also accessible on the World Wide Web (see Ancillaries section, below). The software shows students the value of using a computer in computationally intensive areas, without relieving the student of decision-making responsibilities. Amortrix has two capabilities:

- It will execute specific matrix row operations. After inputting a matrix, the student can instruct the computer to multiply row 2 by 3, and add the result to row 1. However, Amortrix will not "take over" and do a problem for the student—the student must decide where and how to pivot, and the computer will perform only the calculations.
- It will create an amortization schedule. However, Amortrix will not compute the last line correctly; instead, it uses the same algorithm on *all* lines of the schedule, forcing the student to correct the last line so that there is a zero balance.

The use of Amortrix is addressed in optional technology subsections in Chapter 2 (Systems of Linear Equations and Inequalities), Chapter 3 (Linear Programming: The Simplex Method), and Chapter 10 (Finance). The software is *not* an integral part of this book; the topics can be covered quite reasonably without any computer use.

Finally, an optional technology subsection gives instructions on the use of computerized spreadsheets (such as Microsoft Excel and Lotus 1-2-3) in creating amortization schedules.

Subsections that make use of the Amortrix software or spreadsheets are identified by a computer icon. Some technology subsections provide support for both graphing calculators and software, and are identified by both icons.

History

The history of the subject matter is interwoven throughout most chapters. In addition, Historical Notes give in-depth biographies of the prominent people involved. It is our hope that students will see that there is a human aspect to mathematics. After all, mathematics was invented by real people for real purposes and is a part of our culture. Interesting research topics are given, and writing assignments are suggested. Short-answer historical questions are also included; they are intended to focus and reinforce the students' understanding of the historical material. They also serve to warn students that history questions may appear on exams.

Exercises

The exercises vary in difficulty. Some are exactly like the examples, and others expect more of the students. The exercises are not explicitly graded into A, B, and C categories, nor are any marked "optional" (students in this audience tend to react negatively if asked to do anything labeled in this manner). The more difficult exercises are indicated in the Instructor's Resource Manual.

Applications are stressed, and the student is usually given real or realistic data. Furthermore, the student is usually given information at a realistic level. For example, in Chapter 1 on Linear Equations, the student is not given cost and revenue functions, since it is not realistic to assume that this level of information would be available. Instead, he or she is given data and is asked to compute the cost and revenue functions, as well as the break-even point.

Critical thinking is also stressed. For example, the student is frequently asked to interpret a quantitative answer, give advice based on a quantitative answer, discuss assumptions, or make a prediction. Writing exercises are common, as are exercises that could be used in a group situation. Essay questions are also common; they can be used as an integral part of the students' grades, as a background for classroom discussion, or as extra credit work. Many are research topics and are kept as open-ended as possible.

Throughout the text, there is emphasis on the importance of checking one's answers. Thus, students learn to evaluate the reasonableness of their answers rather than accepting them at face value.

Answers

Answers to the odd-numbered exercises are given in the back of the book, with two exceptions:

- Answers to historical questions, interpretive questions, essay questions, and other open-ended questions are not given.
- Answers are not given when the exercises instruct the students to check the answers themselves.

The Students' Solutions Manual contains solutions to every other odd exercise. Thus, the instructor has access to four different types of exercises:

- Exercises that have an answer in the back of the book and a solution in the Student Solutions Manual.

- Exercises that have an answer in the back of the book but no solution in the Student Solutions Manual.
- Exercises that have neither an answer in the back of the book nor a solution in the Student Solutions Manual.
- Exercises that require the student to check his or her answer.

ANCILLARIES

Instructor's Resource Manual (ISBN 0-534-93407-2), by David Johnson, Thomas Mowry, and Michael Rosenborg, contains answers to all even-numbered exercises, chapter summaries, and suggestions for teaching from the text.

Student Solutions Manual (ISBN 0-534-93408-0) provides the solutions to every other odd-numbered text exercise.

Student Tutorial CD (ISBN 0-534-36425-X) provides the student with tutorial items and practice problems.

Amortrix (Macintosh ISBN 0-534-35597-8; Windows 95/NT ISBN 0-534-35699-0; Windows 3.x ISBN 0-534-35695-8; Java http://www.brookscole.com/math/amortrix/) This software accompanies Chapter 2 (Systems of Linear Equations and Inequalities), Chapter 3 (Linear Programming: The Simplex Method), and Chapter 4 (Matrix Equations). The software executes matrix row operations and creates amortization schedules. It shows students the value of using a computer in computationally intensive areas. It will run on a network, independent computer, or over the Web, and is free to adopters of the text.

Printed Test Items (ISBN 0-534-35849-7) contains printed test forms, with answers, for instructors.

Thomson World Class Testing Tools (Macintosh ISBN 0-534-35859-4 and 0-534-36284-2; Windows ISBN 0-534-35848-9 and 0-534-36285-0) This fully-integrated suite of test creation, delivery, and classroom management tools includes World Class Test, Test Online, and World Class Management software. World Class Testing Tools allows professors to deliver tests via print, floppy, hard drive, LAN, or Internet. With these tools, professors can create cross-platform exam files from publisher files or existing WESTest 3.2 test banks, create and edit questions, and provide their own feedback to objective test questions—enabling the system to work as a tutorial or an examination. In addition, professors can generate questions algorithmically, creating tests that include multiple-choice, true/false, and matching questions. Professors can also track the progress of an entire class or an individual student. Testing and tutorial results can be integrated into the class management tool, which offers scoring, gradebook, and reporting capabilities.

Acknowledgments

Special thanks go to the reviewers who evaluated the manuscript:

Alfred Bachman
Cal Poly State University

Gary G. Bard
Taylor University

Ronald Barnes
University of Houston, Downtown

Carol A. Barnett
St. Louis Community College at Meramec

Barbara Beaudin
University of Hartford

Barbara A. Becker
St. Xavier University

W. E. Bonnice
University of New Hampshire

Chris Braun
Coconino County Community College

Eleanor Canter
Wentworth Institute of Technology

Frederick J. Carter
St. Mary's University

Jerry Davis
Johnson State College

Joseph W. Fidler
Triton College

Bennette Harris
University of Wisconsin–Whitewater

Marian Harty
Edgewood College

Yvette Hester
Texas A&M University

Vern H. Klotz
Concordia University

Cen-Tsong Lin
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Lewis Lum
University of Portland

Thomas Ralley
Portland State University

Jane M. Rood
Eastern Illinois University

Arthur Rosenthal
Salem State College

Janet S. Schachtner
San Jacinto College North

Dan Schapiro
Yakima Valley Community College

Linda Schultz
McHenry County College

Ghulam M. Shah
University of Wisconsin–Waukesha

Mary Jane Sterling
Bradley University

We wish to thank our wives for their patience and support. We are also grateful to Margot Hanis, Bob Pirtle, Melissa Duge, Kirk Bomont, Roy Neuhaus, and all the wonderful people at Brooks/Cole who worked on this project, as well as Deann Christianson, Sarah Merz, Mike Rosenborg, and Amy Wasserman.

DAVID JOHNSON
THOMAS MOWRY

CONTENTS

Linear Equations 1

You have studied three different branches of mathematics: arithmetic, algebra, and geometry. Algebra involves equations and inequalities, while geometry involves shapes and graphs. **Analytic geometry** is a bridge between algebra and geometry. It emphasizes a correspondence between equations and graphs; every equation has a graph, and most graphs have equations. This correspondence is a fruitful one; it allows algebra problems to be attacked with the tools of geometry in addition to the tools of algebra.

Business analysts and economists use a great deal of analytic geometry in their fields. In this chapter we will investigate some of the mathematics used in business and economics. In particular, we will investigate the Central State University's Business Club and its attempt to make money selling T-shirts at football games. This enterprise involves the careful determination of the quantity of shirts to order as well as of the shirts' sales price. Ordering too many shirts or charging too much would result in the club's buying shirts they can't sell, and ordering too few or charging too little would result in not having enough to sell; in either event, the club would lose revenue.

1.0 LINES AND THEIR EQUATIONS

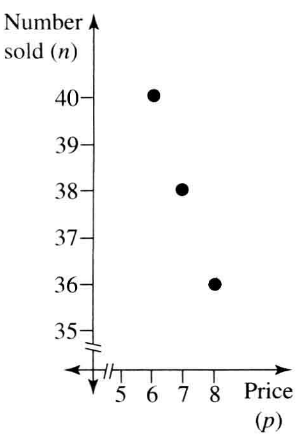
Cartesian Coordinates

Stuart sells sunglasses from a stand at Venice Beach. After experimenting with prices, he discovered (not surprisingly) that the more he charges, the less he sells. For several days Stuart charged \$6 per pair. He kept records on the number of pairs sold and found that he sold an average of 40 pairs per day at that price. This and similar data are given in Figure 1.1.

FIGURE 1.1

Price	Number sold
\$6	40
\$7	38
\$8	36

FIGURE 1.2



Stuart's data can be illustrated graphically by drawing two perpendicular number lines, where the horizontal number line represents price and the vertical number line represents number sold, as shown in Figure 1.2. Notice that both number lines have breaks; the price is always above 5, and the number sold is always above 35.

The upper-left point in Figure 1.2 is directly above 6 on the horizontal number line, so it corresponds to a price of \$6. It is also directly across from 40 on the vertical number line, so it corresponds to 40 pairs sold. If we let p refer to price and n refer to number sold, the upper-left point could be labeled $p = 6$, $n = 40$. A more traditional way of labeling this point is to write $(p, n) = (6, 40)$. This is called an **ordered pair,** because it is a pair of numbers written in a certain order. The order is important; if we write (40, 6), we get the incorrect statement that at a price of \$40, 6 pairs are sold.

The number 6 is called the ***p*-coordinate** of the ordered pair, and the number 40 is called the ***n*-coordinate.** The system of graphing is called **Cartesian coordinates,** in honor of René Descartes, a mathematician and philosopher. (Oddly, while Descartes explored the relationship between algebra and geometry, he neither invented nor utilized the system that bears his name.)

There are two different traditions of the use of letters in this type of situation:

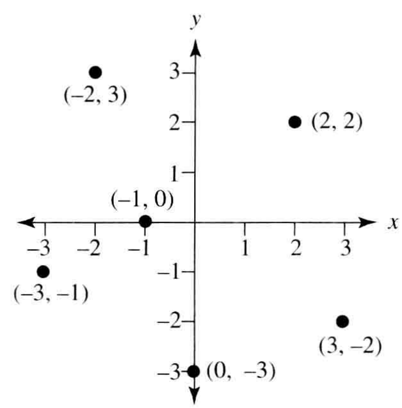
- Use x and y
- Use letters that refer to the quantity being measured, as p refers to price and n refers to number sold above

Descartes started the former tradition; he used letters from the end of the alphabet to represent variables. This tradition is usually adhered to in algebra classes. However, in an application like this one, the latter tradition can serve as a valuable memory aid.

When the "x and y" tradition is followed, we have ***x*-coordinates** and ***y*-coordinates** rather than p-coordinates and n-coordinates. The horizontal axis is called the ***x*-axis,** and the vertical axis is called the ***y*-axis.** In the above discussion, we used p and n rather than x and y, respectively, so the horizontal axis is the p-axis, and the vertical axis is the n-axis. The two axes meet where both p and n are 0; this point, (0, 0), is called the **origin.**

In Figure 1.3, we show x- and y-axes that include both positive and negative values (the negative values are on the left end of the x-axis and on the lower end of the y-axis). The upper-left point corresponds to the ordered pair $(x, y) = (-2, 3)$, since it is above -2 on the x-axis and across from 3 on the y-axis.

FIGURE 1.3



Slope

A line's steepness is measured by its slope. **Slope** (usually denoted by the letter m) is the ratio of the **rise** (the change in y) to the **run** (the change in x):

$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

EXAMPLE 1

a. Calculate the slope of the line between the ordered pairs $(p, n) = (6, 40)$ and $(p, n) = (7, 38)$ from Stuart's sales data.

b. Calculate the slope of the line between the ordered pairs $(p, n) = (7, 38)$ and $(p, n) = (8, 36)$ from Stuart's sales data.

c. Determine what these slopes measure in the context of the problem.

d. Use the slope to predict the number of sunglasses that will sell at \$9. What is this prediction based on?

Solution Here our ordered pairs are (p, n), rather than (x, y). Thus,

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } n}{\text{change in } p}$$

a. In moving from the point $(p, n) = (6, 40)$ to the point $(p, n) = (7, 38)$, p increases from 6 to 7, so p changes by $7 - 6 = 1$. Similarly, n decreases from 40 to 38, so n changes by $38 - 40 = -2$. Thus, the slope is

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } n}{\text{change in } p} = \frac{-2}{1} = -2$$

b. In moving from the point $(p, n) = (7, 38)$ to the point $(p, n) = (8, 36)$, p increases from 7 to 8, so p changes by 1. Similarly, n decreases from 38 to 36, so n changes by -2. Thus, the slope is

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } n}{\text{change in } p} = \frac{-2}{1} = -2$$

c. In the context of the problem, the 1 in the denominator represents a price increase of \$1, and the -2 in the numerator indicates that the number sold decreased by 2. The fact that the slopes are the same in parts (a) and (b) indicates that a price increase of \$1 *consistently* corresponds to a sales decrease of 2 pairs of sunglasses. In the context of the graph, equal slopes means that the steepness doesn't change; that is, the 3 points lie on a line.

d. A charge of \$9 per pair of sunglasses is a \$1 increase above an \$8 price. Each \$1 increase consistently corresponded to a sales decrease of 2 pairs, so sales should decrease to 34 pairs per day, if future sales are consistent with past sales. •

Since a change in y is calculated by subtracting y-values, and a change in x is calculated by subtracting x-values, we have the following formula.

Slope Formula
The **slope** m of the line passing through the points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$

In the preceding formula, each of the symbols is meant to be a memory aid. For example, y_2 means the y-coordinate of the second point, and x_1 means the x-coordinate of the first point.

EXAMPLE 2 Find and interpret the slope of the line passing through the points $(-1, -3)$ and $(4, 7)$. Graph the points and the line, and show the rise and the run.

Solution Select $(-1, -3)$ as the first point [so $(x_1, y_1) = (-1, -3)$] and $(4, 7)$ as the second point [so $(x_2, y_2) = (4, 7)$], and substitute into the slope definition.