Ric Pimentel
Terry Wall

Cambridge checkpoint

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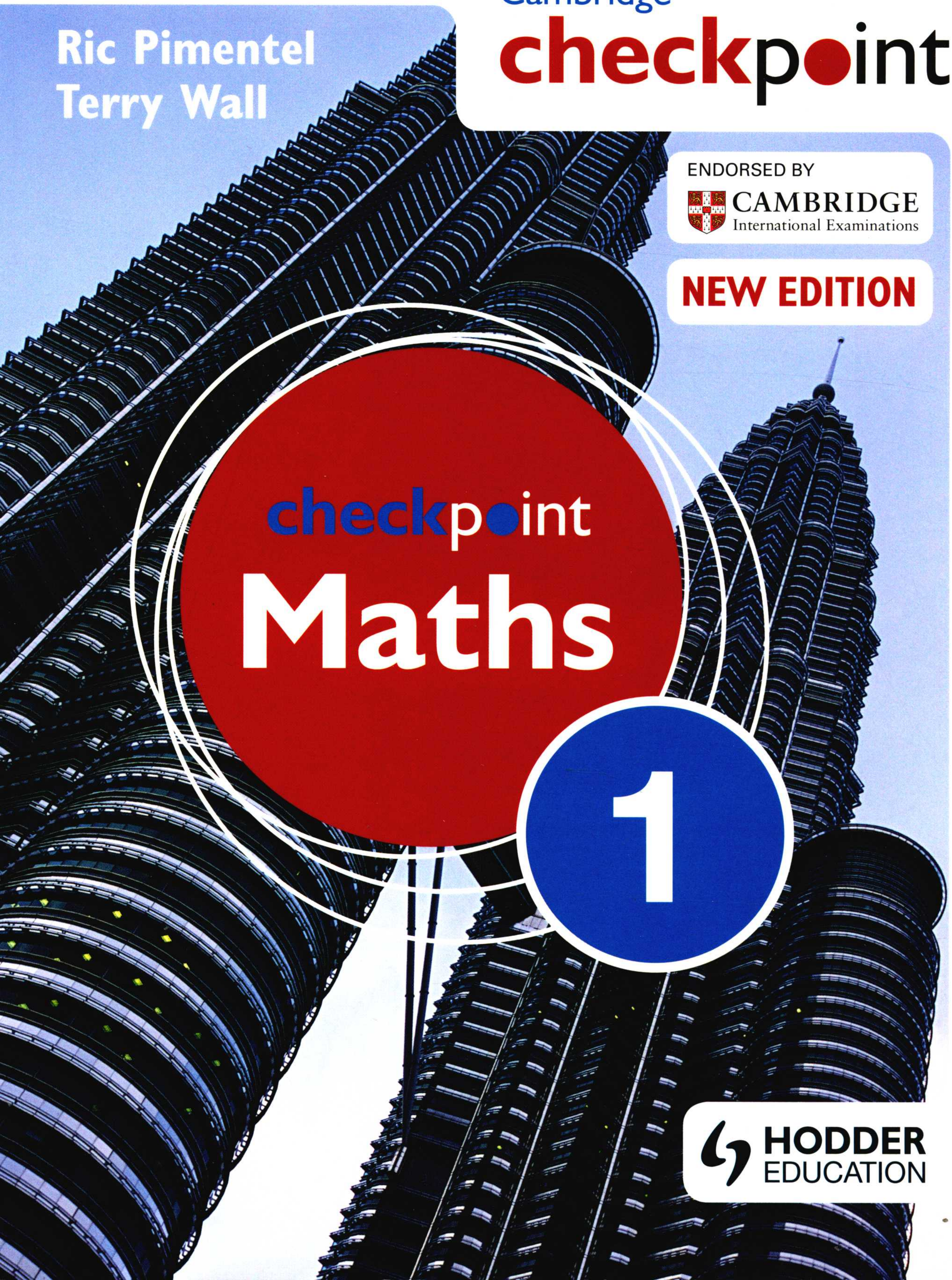

checkpoint Maths 1

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Introduction

This series of books follows the Cambridge Secondary 1 Mathematics Curriculum Framework drawn up by University of Cambridge International Examinations. It has been written by two experienced teachers who have lived or worked in schools in many countries, and worked with teachers from other countries, including England, Spain, Germany, France, Turkey, South Africa, Malaysia and the USA.

Students and teachers in these countries come from a variety of cultures and speak many different languages as well as English. Sometimes cultural and language differences make understanding difficult. However, mathematics is largely free from these problems. Even a maths book written in Japanese will include algebra equations with x and y.

We should also all be very aware that much of the mathematics you will learn in these books was first discovered, and then built upon, by mathematicians from all over the world, including China, India, Arabia, Greece and Western countries.

Most early mathematics was simply game play and problem solving. Later this maths was applied to building, engineering and sciences of all kinds. Mathematicians study maths because they enjoy it.

We hope that you will enjoy the work you do, and the maths you learn in this series of books. Sometimes the ideas will not be easy to understand at first. That should be part of the fun. Ask for help if you need it, but try hard first. Write down what you are thinking so that others can understand what you have done and help to correct your mistakes. Most students think that maths is about answers, and so it is, but it is also a way to exercise our brains, whether we find the solution or not. Some questions throughout this book are starred (✪). This means that these questions go slightly beyond the content of the curriculum at this level and will be an enjoyable challenge for those of you who try them.

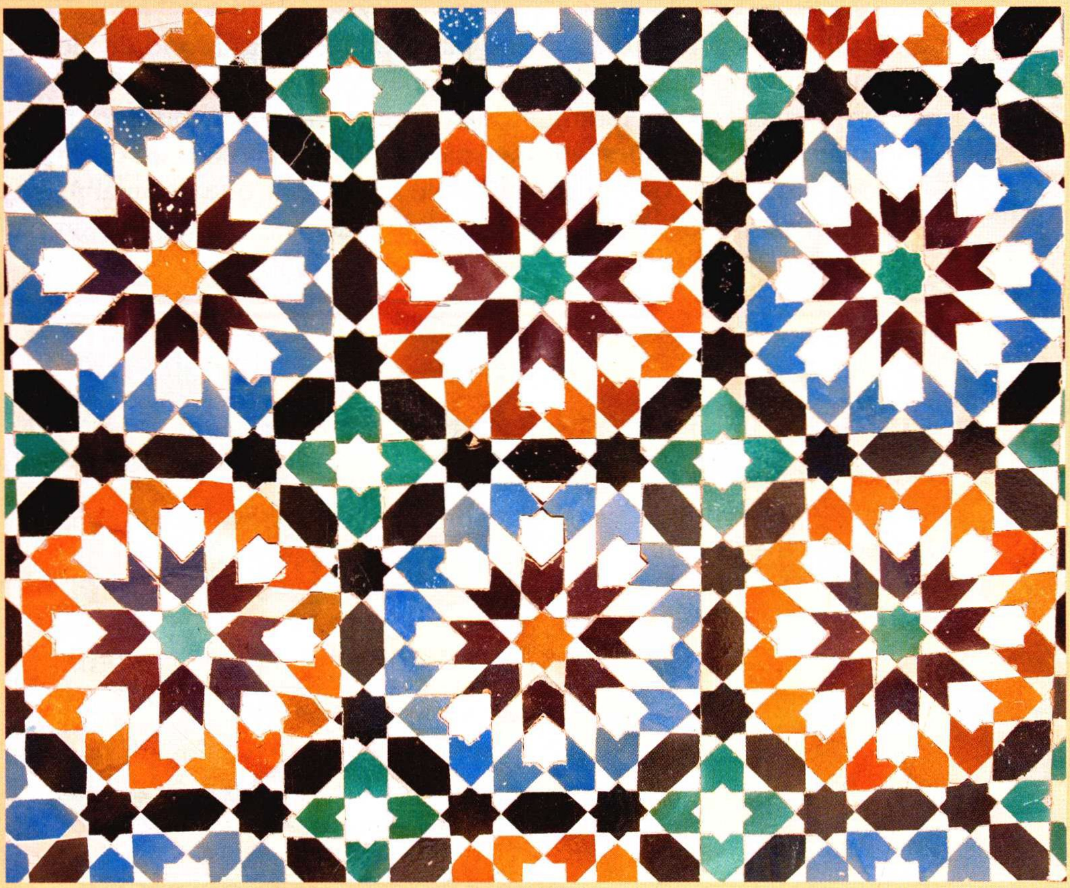
Ric Pimentel and Terry Wall

Contents

The chapters in this book have been arranged to match the Cambridge Secondary 1 Mathematics Curriculum Framework for stage 7 as follows:

- Number
- Algebra
- Geometry
- Measure
- Handling data
- Calculation and mental strategies
- Problem solving

SECTION 1

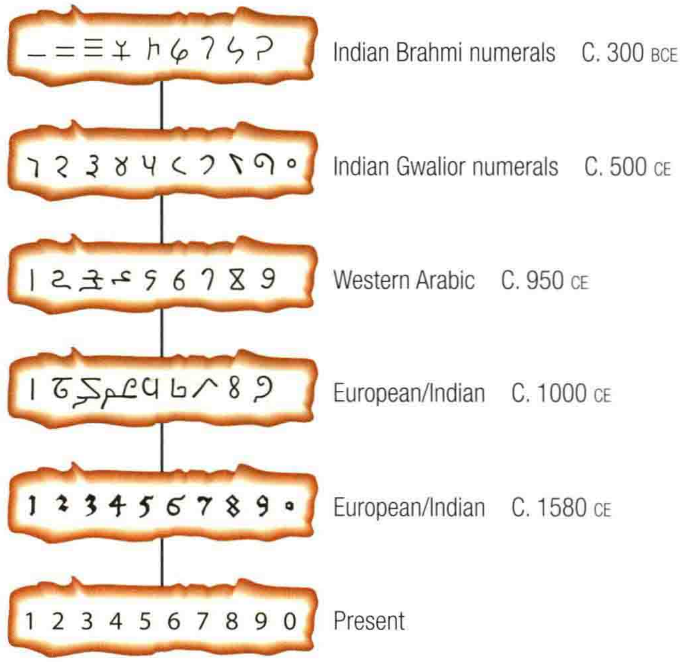


1 Place value, ordering and rounding

- Interpret decimal notation and place value; multiply and divide whole numbers and decimals by 10, 100 or 1000.
- Order decimals, including measurements, changing these to the same units.
- Round whole numbers to the nearest 10, 100 or 1000 and decimals, including measurements, to the nearest whole number or one decimal place.
- Use the order of operations, including brackets, to work out simple calculations.

Place value

Our present number system originated in India with the Brahmi numerals in about 300 BCE.



The position of a digit in our number system determines its value. For example, 6287.35 can be placed in a table like this:

Thousands	Hundreds	Tens	Units	•	Tenths	Hundredths
6	2	8	7	•	3	5

the decimal point

Looking at the table we can see that:

the 6 is worth	6000
the 2 is worth	200
the 8 is worth	80
the 7 is worth	7
the 3 is worth	0.3 or $\frac{3}{10}$
the 5 is worth	0.05 or $\frac{5}{100}$

EXERCISE 1.1A

1 What is the value of the 3 in the following numbers?
a) 37 **b)** 389 **c)** 2873 **d)** 6.13

2 What is the value of the 8 in the following numbers?
a) 0.8 **b)** 6.38 **c)** 0.048 **d)** 184 000

3 What is the value of the 5 in the following numbers?
a) 573 **b)** 8596 **c)** 0.54 **d)** 3.85

4 What is the value of the 1 in the following numbers?
a) 18 300 **b)** 0.71 **c)** 401 **d)** 0.041

5 Put the following sets of numbers in order, with the largest first.
a) 4.5, 7.2, 11, 0.9, 2.08, 0.07
b) 0.08, 0.008, 8, 80, 0.8
c) 4.6, 7.2, 6.7, 7.66, 4.07, 7.34
d) 0.1, 0.12, 0.09, 0.19, 0.92, 1.2
e) 1.12, 2.11, 3.1, 1.3, 2.13, 1.33, 3

6 Put the following sets of numbers in order, with the largest first.
a) 12.5, 7.67, 1, 3.59, 2.668, 0.097
b) 0.043, 0.009, 7.48, 8, 0.09, 20.8
c) 14.6, 25.2, 6.97, 7.0, 9.97, 7.34, 6.098
d) 0.31, 0.312, 0.309, 0.319, 0.392, 3.2
e) 0.512, 2.51, 0.1, 1.13, 1.113, 1.33, 1.433

7 Put the following sets of numbers in order, with the largest first.
a) 14.5, 372, 31, 10.9, 12.08, 10.07
b) 0.308, 0.3008, 3.8, 8, 0.88, 0.898
c) 74.6, 77.2, 76.7, 77.66, 74.07, 77.34
d) 0.91, 0.12, 0.909, 0.919, 0.992, 1.92
e) 31.12, 62.11, 33.81, 101.3, 52.13, 18.33, 3.98

8 Put the following sets of quantities in order, with the largest first.
a) 3 cm, 1.3 cm, 5.3 cm, 3.5 cm, 5.6 cm, 2.55 cm
b) 5 g, 6.4 g, 3.4 g, 8.75 g, 5.5 g, 0.9 g, 4.25 g
c) 30 cm, 0.4 m, 8.9 m, 2 m, 0.1 m, 250 cm
d) 400 g, 80 g, 2 kg, 1.3 kg, 2500 g
e) 2 litres, 0.6 litre, 400 m*l*, 0.5 litre, 550 m*l*

Multiplying and dividing by 10, 100 and 1000

Multiplying a number by 10, 100 or 1000 results in the digits moving one, two or three places to the left respectively. For example,

$28 \times 10 = 280$	$34.56 \times 10 = 345.6$
$28 \times 100 = 2800$	$34.56 \times 100 = 3456$
$28 \times 1000 = 28\,000$	$34.56 \times 1000 = 34\,560$

Similarly, dividing a number by 10, 100 or 1000 results in the digits moving one, two or three places to the right respectively. For example,

$28 \div 10 = 2.8$	$34.56 \div 10 = 3.456$
$28 \div 100 = 0.28$	$34.56 \div 100 = 0.3456$
$28 \div 1000 = 0.028$	$34.56 \div 1000 = 0.034\,56$

EXERCISE 1.1B

1 Multiply the following numbers by 10.
a) 63 **b)** 4.6 **c)** 0.84 **d)** 0.065 **e)** 1.07

2 Multiply the following numbers by 100.
a) 45 **b)** 7.2 **c)** 0.96 **d)** 0.0485 **e)** 6.033

3 Find the value of the following.
a) 46×1000
b) 6.8×1000
c) 3.8×1000
d) 0.0084×1000
e) 0.7×1000

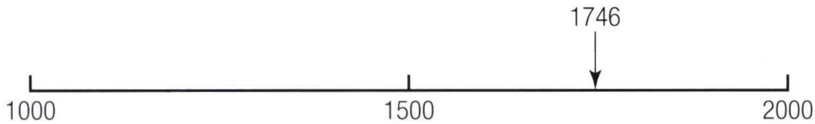
4 Divide the following numbers by 10.
a) 680 **b)** 72 **c)** 8.9 **d)** 0.64 **e)** 0.054

5 Divide the following numbers by 100.
a) 3500 **b)** 655 **c)** 5.62 **d)** 0.8 **e)** 0.034

6 Find the value of the following.
a) $6.4 \div 1000$
b) $46 \div 1000$
c) $950 \div 1000$
d) $0.0845 \div 1000$
e) $4 \div 1000$

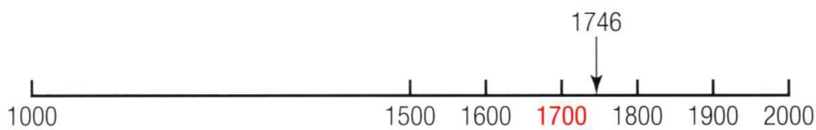
Rounding

A large school has 1746 students. This figure can be **rounded** (approximated) in several ways by showing its position on a number line.

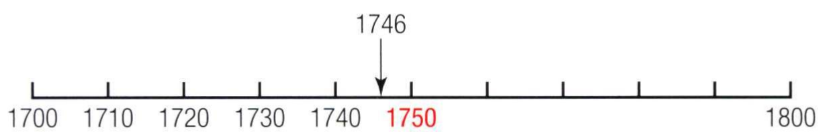


If 1746 is rounded to the nearest **thousand**, then it is written as 2000, because 1746 is closer to 2000 than it is to 1000.

If 1746 is rounded to the nearest **hundred**, then it is written as 1700, because 1746 is closer to 1700 than it is to 1800.



If 1746 is rounded to the nearest **ten**, then it is written as 1750, because 1746 is closer to 1750 than it is to 1740.



If a number is half way, then it is rounded up. For example, 1745 rounded to the nearest ten would be 1750 even though it is half way between 1740 and 1750.

EXERCISE 1.2

1 Draw number lines similar to those above and use them to round the following numbers **(i)** to the nearest thousand, **(ii)** to the nearest hundred and **(iii)** to the nearest ten.

a) 38 273 **b)** 21 793 **c)** 15 476 **d)** 58 437

2 107 638 people attended a football match in Madrid.
a) This was reported in the programme as 100 000 attendance.
b) The club estimate was 110 000 in the crowd.
c) A closer estimate in a newspaper was 108 000.

All of these estimates were acceptable. However, they are to different degrees of accuracy.
Write down the degree of accuracy for each one.

3 Round the following numbers to the nearest thousand.
a) 58 437 **b)** 9288 **c)** 68 400 **d)** 72 985

4 Round the following numbers to the nearest hundred.
a) 483 **b)** 1692 **c)** 93 **d)** 12 763

5 Round the following numbers to the nearest ten.
a) 63 **b)** 846 **c)** 5839 **d)** 8

Decimal places

Another way of *rounding* a number is to write it to a given number of **decimal places**. This refers to the number of digits written after the decimal point.

Worked example

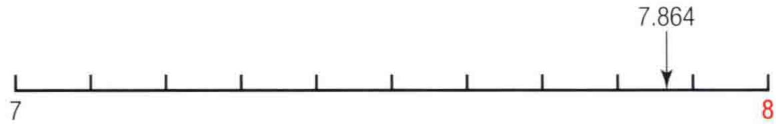
The length of this model car is 7.864 cm.
Write 7.864

a) to the nearest whole number
b) to one decimal place.

A number written to one decimal place has one digit after the decimal point.

Draw a number line to help you.

a)



7.864 is closer to 8 than it is to 7, so 7.864 written to the nearest whole number is 8.

b)

7.864 is closer to 7.9 than it is to 7.8, so 7.864 written to one decimal place is 7.9.

To round to a whole number or to a certain number of decimal places, look at the next digit after the one in question. If that digit is 5 or more, round up. If it is 4 or less, round down.

EXERCISE 1.3

1 Round the following numbers **(i)** to the nearest whole number and **(ii)** to one decimal place.

a) 6.37 **b)** 4.13 **c)** 0.85
d) 8.672 **e)** 1.093 **f)** 0.063

2 Round the following numbers **(i)** to the nearest whole number and **(ii)** to one decimal place.

a) 4.383 **b)** 5.719 **c)** 5.803
d) 1.477 **e)** 3.999 **f)** 6.273

3 Round the following numbers **(i)** to the nearest whole number and **(ii)** to one decimal place.

a) 0.5682 **b)** 3.4765 **c)** 8.8467 **d)** 3.6543 **e)** 3.4567

Estimating answers to calculations

Even though calculators are a quick and easy way of solving arithmetical problems, an **estimate** can be a useful check.

Worked examples

a) Estimate the answer to

$18 \times 71.$

To the nearest ten, 18 is 20 and 71 is 70.
So an easy estimate is

$20 \times 70 = 1400.$

b) Estimate the answer to

$3568 \div 28.$

To the nearest hundred, 3568 is 3600.
To the nearest ten, 28 is 30.

So a good estimate would be

$3600 \div 30 = 120.$

If 3568 was rounded to 4000 then 4000 ÷ 30 = 133 is still a reasonable estimate.

EXERCISE 1.4

1 Estimate the answers to the following calculations.
 a) 42×19 b) 63×27
 c) 198×39 d) 8.9×384
 e) 55×77

2 Estimate the answers to the following calculations.
 a) $3984 \div 41$ b) $5872 \div 32$
 c) $8.972 \div 2.8$ d) $0.414 \div 2.1$
 e) $0.414 \div 0.21$

3 Using estimation, write down which of these calculations are definitely wrong.
 a) $6357 \div 21 = 30.27$ b) $834 \times 7.9 = 6588$
 c) $189 \div 8.9 = 212$ d) $78.3 \times 11.2 = 8769$

Order of operations

The order in which mathematical calculations are done depends on the operations being used.

Look at this calculation:

$6 + 3 \times 2 - 1.$

Carrying out the calculation from left to right would give an answer of 17. However, if you do the calculation on a calculator, the answer it gives is 11.

This is because mathematical operations are carried out in a particular order:

- **B**rackets — Any operation in brackets is done first.
- **I**ndices — A number raised to a power (index) is done next.
- **D**ivision and/or **M**ultiplication — Multiplications and divisions are done next. Their order does not matter.
- **A**ddition and/or **S**ubtraction — Additions and subtractions are carried out last. Again, their order is not important.

A way of remembering this order is with the shorthand **BIDMAS**.

The correct answer to the calculation $6 + 3 \times 2 - 1$ is 11, because the 3×2 must be done first, followed by addition of the 6 and subtraction of the 1.

The 1 can be subtracted before the 6 is added; the answer is still 11.

Worked examples

a) Calculate

$7 + 4 \times 9 - 8.$

$7 + 4 \times 9 - 8$

$= 7 + 36 - 8$

$= 35$

The multiplication is done first.

b) Calculate

$25 - (2 + 3) \times 4.$

$25 - (2 + 3) \times 4$ — *The brackets are done first, (2 + 3) = 5.*

$= 25 - 5 \times 4$ — *This is multiplied by 4 next, giving 20.*

$= 25 - 20$ — *Lastly this is subtracted from 25.*

$= 5$

c) Calculate

$101 - (11 - 7) - 3 \times 8.$

$101 - (11 - 7) - 3 \times 8$

$= 101 - 4 - 3 \times 8$

$= 101 - 4 - 24$

$= 101 - 28$

$= 73$

EXERCISE 1.5

Work out the following.

1 $4 + 3 \times 2 - 1$

2 $6 \times 2 + 4 \times 3$

3 $3 \times 5 - 2 - 7$

4 $8 + 4 \times 8 - 40$

5 $6 - 2 \times 3 \times 4$

6 $7 \times (4 + 2) - 3$

7 $8 + (6 + 3) \div 3$

8 $16 \div (2 + 6) + 8$

9 $4 + 4 \times 4 - 4$

10 $(4 + 4) \div (8 - 4)$

2 Expressions

- Use letters to represent unknown numbers or variables; know the meanings of the words *term*, *expression* and *equation*.
- Know that algebraic operations follow the same order as arithmetic operations.
- Construct simple algebraic expressions by using letters to represent numbers.
- Simplify linear expressions, e.g. collect like terms; multiply a constant over a bracket.

'The yield of two sheaves of superior grain, three sheaves of medium grain and four sheaves of inferior grain is each less than one tou. But if one sheaf of medium grain is added to the superior grain, or if one sheaf of inferior grain is added to the medium, or if one sheaf of superior grain is added to the inferior, then in each case the yield is exactly one tou.

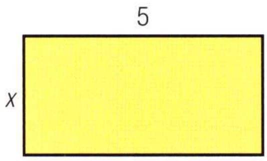
What is the yield of one sheaf of each grade of grain?'

The problem above comes from the most important book of ancient Chinese mathematics. The book was called *Nine Chapters on the Mathematician's Art*. It was written approximately 2000 years ago. As its name suggests, it consists of nine chapters. Each chapter presents a series of mathematical problems related to life in China at the time.

Expressions

An **expression** is used to represent a value in algebraic form. For example,

The length of the line is given by the expression $x + 3$.



The perimeter of the rectangle is given by the expression $x + 5 + x + 5$. This can be simplified to $2x + 10$.

The area of the rectangle is given by the expression $5x$.

In the examples above, x, $2x$ and $5x$ are called **terms** in the expressions.

In the expression $2a + 3b + 4c - 5$, each of $2a$, $3b$, $4c$ and -5 is a term in the expression.

An expression is different from an **equation**. An equation contains an equals sign (=), which shows that the expressions either side of it are equal to each other. For example, the equation

$$x + 1 = y - 2$$

tells us that the expressions $x + 1$ and $y - 2$ are equal to each other.

Order of operations when simplifying expressions

In Chapter 1 you saw that calculations need to be carried out in a particular order. This order is not necessarily from left to right.

For example, the calculation $2 + 3 \times 4$ has the answer 14 (rather than 20) because the multiplication is done before the addition.

The order in which operations are carried out is as follows:

Brackets
Indices
Division/**M**ultiplication
Addition/**S**ubtraction

A useful way of remembering the order is with the shorthand **BIDMAS**.
The same order of operations applies when working with algebraic expressions.