PERSPECTIVES IN NONLINEAR DYNAMICS

Editors:
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W Zachary

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28 — 30 May 1985
Naval Surface Weapons Center

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INTRODUCTION

Dynamics, the oldest discipline of classical physics had its initial grand success with Newton's derivation of Kepler's laws. The clarity and beauty of classical dynamics went as far as to inspire a mechanistic clockwork view of the universe, which incidentally was not shared by Newton. This world view was over-turned with the advent of quantum mechanics with a God who plays dice with the universe. A joke making the rounds is "nonlinear dynamics is what physicists would have done 60 years ago if the computer had been invented and quantum mechanics had not".

Until recently classical mechanics was relegated to a first course in the graduate physics curriculum stressing its Hamiltonian formulation as a precursor to quantum mechanics. Mechanics was certainly not a popular field of active research. However, all of this has now changed and we are in the midst of a remarkable revolution which has vaulted dynamics into one of the most exciting and promising fields of science with consequences that cut across many disciplines. Nonlinear dynamics has been a unifying factor in science, helping to reverse the trend of specialization.

How, almost 300 years after the publication of Newton's *Principia* (1687) did dynamics once again reach the forefront of science? The answer lies in the shift from linear to nonlinear equations of motion. When compared to linear systems, nonlinear systems exhibit ergodic behavior or competition between nonlinear modes, they exhibit mode selection instead of mode superposition, effective reduction of degrees of freedom instead of completeness, relevant inherent geometries can be fractal instead of Euclidean, one can obtain solitons instead of waves, chaos instead of diffusion, and intermittency achieves a natural existence instead of being arbitrarily introduced as arising from noisy external sources.

A second part of the answer to the question lies in a historic reformulation of the harder fundamental problem of Newton's calculus: integration. Poincaré, Lie and Birkhoff fathered the global approach from which so many of the most important modern results have come, in which the problem of solving differential equations by effecting quadratures was replaced by the problem of asking for typical behaviors of the solution trajectories. Thus, two centuries after the publication of the *Principia* dynamics was geometrized. It was to take nearly another century before the computer, with assistance from Kolmogorov, Arnol'd and Moser, and from Smale, would make the study of complicated behavior accessible and exciting. The development of nonlinear dynamics has begun to take off now that experiment has joined this novel interplay between technology and theory. Nonlinear dynamics is truly a new calculus!

The emphasis has been placed on finding typical or universal properties of nonlinear systems. The progress in nonlinear dynamics has been so rapid that any Rip Van Winkle wandering into a modern dynamics conference could be startled not to find many landmarks. He would encounter with puzzlement a new vivid terminology such as chaos, strange attractors, fractal dimensions, fractal basic boundaries, solitons, crises, metamorphoses, KAM tori, horseshoes, devil's staircases, etc.

The Department of the Navy has recognized the importance of the new developments in dynamics in several ways. Research scientists at the Naval Research Laboratory and the Naval Surface Weapons Center formed in 1983 the Navy dynamics institute under partial sponsorship of the Office of Naval Research. The institute is involved with numerous research efforts, conducts weekly seminars, and organizes conferences. Under the sponsorship of the above three organizations, a workshop entitled "Perspectives in Nonlinear Dynamics," was held 28-30 May 1985 at the Naval Surface Weapons Center. In part, the lecturers were asked to provide the ONR Physics Division (which funds basic research in dynamics) with their views on what are the outstanding issues in nonlinear dynamics, their relative importance, and what types of research can resolve these issues. Their responses form the content of this book.

Broad areas covered include fluids, electrical systems, plasma, optics, mechanical systems, proteins, and mathematical and statistical approaches to investigating nonlinear dynamical systems.

We take this opportunity to thank all the lecturers and session chairmen for openly revealing their research plans in a forum attended by their keenest competitors. Their effort has been most helpful in helping the Navy chart new research directions in nonlinear dynamics.

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28-30 May, 1985

Naval Surface Weapons Center
White Oak, Silver Spring, Maryland

Morning Session Chairman:
Dr. Michael Shlesinger, Office of Naval Research, Arlington, Virginia

Welcoming address:
Dr. Lemmuel Hill, Technical Director, Naval Surface Weapons Center, Dahlgren, Virginia

Introductory remarks:
Dr. David Nagel, Naval Research Laboratory, Washington, D.C.

Introductory remarks:
Dr. William Condell, Office of Naval Research, Arlington, Virginia

"Chaos in semi-conductors"
Prof. Robert Westervelt, Department of Physics, Harvard University, Cambridge, Massachusetts.

"Experiments on nonlinear dynamics of solid state systems"
Prof. Carson Jeffries, Department of Physics, University of California, Berkeley, California

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*Organized and sponsored by Office of Naval Research and the Navy Dynamics Institute Program of the Naval Research Laboratory and the Naval Surface Weapons Center.

"Fractals concepts in experiments on chaotic mechanical systems"
Prof. Francis Moon, Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, New York.

"Studies of nonlinear dynamics in lasers and other nonlinear optical systems"
Prof. Neal Abraham, Department of Physics, Bryn Mawr College, Bryn Mawr, Pennsylvania.

Moderated discussion (45 minutes)

Afternoon Session Chairman:
Dr. A. W. Saenz, Naval Research Laboratory, Washington, D.C.

"Atmospheric and oceanic models as dynamical systems"
Prof. Edward Lorenz, Department of Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, Massachusetts.

"Attractors for infinite dimensional systems"
Prof Jack Hale, Lefshetz Center for Dynamical Systems, Division of Applied Mathematics, Brown University, Providence, Rhode Island.

"Nonlinear dynamical problems in channeling in crystals"
Prof. James Ellison, Department of Mathematics, University of New Mexico, Albuquerque, New Mexico.

"Stability and prechaotic motion in Hamiltonian and nearly Hamiltonian systems"
Prof. Jerrold Marsden, Department of Mathematics, University of California, Berkeley, California.

Moderated discussion (1 hour)

Wednesday, 29 May

Morning Session Chairman
Dr. William Caswell, Naval Surface Weapons Center, Silver Spring, MD

"Chaos and confusion in two-dimensional hydrodynamics"
Prof. Leo Kadanoff, James Franck Institute, University of Chicago, Chicago Illinois.

"Interdisciplinary chaos"
Prof. Henry Abarbanel, Scripps Institution of Oceanography, University of California, San Diego, California.

"Coherent structures and chaos in parital differential equations"
Prof. Alan Newell, Program in Applied Mathematics, University of Arizona, Tucson, Arizona.

"Bifurcation and the integration of nonlinear differential systems"
Prof. Melvyn Berger, Department of Applied Mathematics, University of Massachusetts, Amherst, Massachusetts.

Moderated discussion (1 hour)

Afternoon Session Chairman:
Dr. Woodford Zachary, Naval Research Laboratory, Washington, D.C.

"Computation theory, randomness and cellular automata"
Prof. Stephen Wolfram, School of Natural Sciences, Institute for Advanced Study, Princeton, New Jersey.

"Computing/quantum chaos"
Dr. Bernardo Huberman, Xerox Corporation, Palo Alto, California.

"(Chaotic) ionization of atoms by microwave fields"
Prof. Peter Koch, Department of Physics, State University of New York, Stony Brook, New York.

"Aspects of strange attractors in physical systems"
Prof. John Guckenheimer, Department of Mathematics, University of California, Santa Cruz, California.

Moderated discussion (1 hour)

Banquet Lecture

"Zoe: superhealth for body and brain"
Arnold Mandell, M.D. Professor of Psychiatry, Laboratory of Biological Dynamics and Theoretical Medicine, University of California, San Diego, California

Abstract: The application will be demonstrated of some of the concepts in mathematics and nonlinear dynamics to protein function, cardiovascular physiology, polypeptide-endocrine regulation and brain behavior. An attempt will be made to demonstrate the possibility that an ideal health exists, can be characterized, facilitated, and its incipient loss predicted.

Thursday, 30 May

Morning Session Chairman:
Dr. Robert Cawley, Naval Surface Weapons Center, Silver Spring, MD

"Quasi-periodicity and chaos"
Prof. Albert Libchaber, The James Franck and Enrico Fermi Institutes, University of Chicago, Chicago, Illinois.

"Nonlinear Dynamics and chaos in oscillatory Rayleigh-Benard convection"
Dr. Robert Ecke, Los Alamos National Laboratory, Los Alamos, New Mexico.

"Low dimensional dynamics in high dimensional systems"
Prof. Harry Swinney, Department of Physics, University of Texas, Austin, Texas.

"Nonlinear pattern formation from instabilities"
Prof. Jerry Gollub, Department of Physics, Haverford College, Haverford, Pennsylvania.

Moderated discussion (1 hour)

Afternoon (Closing) Session Chairman:
Prof. James Yorke, Institute for Physical Science and Technology and Department of Mathematics, University of Maryland, College Park, Maryland.

"Effects of diffusion on islands in phase space in many dimensions"
Prof. Alan Lichtenberg, Electronics Research Laboratory, University of California, Berkeley, California.

"Nonlinear dynamics beyond chaos"
Dr. Celso Grebogi, Center for Plasma Theory and Fusion Energy Research, Department of Physics and Astronomy, University of Maryland, College Park, Maryland.

"Nonlinear nights"
Prof. Robert H. G. Helleman, Theoretical Physics Center, Twente University of Technology, Enschede, The Netherlands, and La Jolla Institute for Nonlinear Problems, La Jolla, California.

Round Table Discussion, with additional moderators:

Prof Paul Linsay, Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Dr. Gottfried Mayer-Kress, Los Alamos National Laboratory, Los Alamos, New Mexico.

Prof. Sheldon Newhouse, Department of Mathematics, University of North Carolina, Chapel Hill, North Carolina.

PERSPECTIVES IN NONLINEAR DYNAMICS

CONTENTS

ATMOSPHERIC MODELS AS DYNAMICAL SYSTEMS

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Massachusetts Institute of Technology
Cambridge, MA 02139, U.S.A.

ABSTRACT

We describe various types of approximation which have been introduced into the atmospheric equations to convert them into models. These models may be treated as dynamical systems. We examine one model in detail, and we enumerate some atmospheric problems where a nonlinear-dynamical approach might yield beneficial results.

1. Introduction--models

The laws which govern the atmosphere may be expressed as a system of nonlinear equations. Deducing the typical behavior of the atmosphere from these equations constitutes a challenging problem in nonlinear dynamics. In attacking this problem one might expect to be guided by some of the recent studies in dynamical-systems theory, and one's first reaction to our title might be, "Why models of the atmosphere? Why not the real thing?" To understand our preference for models one needs to know what constitutes a dynamical system. One must also take a close look at the real atmosphere, and at the nature of the systems which comprise most atmospheric models.

We sometimes define a dynamical system as a finite system of coupled deterministically formulated ordinary differential equations in as many dependent variables [1]. Sometimes we relax the requirements to allow a countably infinite number of equations. Sometimes our systems consist of difference equations rather than differential equations. Whatever modifications we may permit, our interest is mainly in the long-term properties of typical solutions of the equations, rather than in methods of finding the solutions. We expect to encounter some special solutions, perhaps steady or periodic, whose properties differ considerably from those of most other solutions, but we expect that in some meaningful sense the special solutions will form a set of measure zero, so that their properties will not contribute to the overall average behavior.

What about the system of equations representing the laws which govern the atmosphere? Among these laws are the fundamental laws of hydrodynamics and thermodynamics, and we ordinarily take the attitude that they are known. A few details still elude us; for example, we do not know what determines just when a cloud, consisting of suspended water droplets or ice crystals, will release its water in the form of larger rain drops or snowflakes. Nevertheless, we are reasonably confident that a system obeying the atmospheric equations, as we have formulated them, will closely resemble the real atmosphere in its gross features and in many of its details.

What are typical solutions of these equations like? The equations are highly nonlinear, the most prominent nonlinear terms representing the quadratically nonlinear process of advection--the transport of momentum, heat, or moisture by the atmospheric motion. Any time-dependent solutions which we may be skillful or fortunate enough to discover by analytic procedures are likely to represent highly specialized behavior. In principle we can obtain typical solutions to any desired degree of approximation by numerical integration, although the actual task may be impractical. However, if our assumption regarding the exactness of the equations is correct, we can determine the nature of the typical solutions by observing the behavior of the atmosphere itself.

An outstanding characteristic of the atmosphere is the simultaneous presence of features of many spatial and temporal scales, and, in particular, many horizontal scales. There are globe-encircling westerly-wind currents, culminating in the jet streams. There are migratory vortices of subcontinental size, whose progression is responsible for many of the day-to-day weather changes in middle latitudes. There are tropical hurricanes, otherwise known as typhoons or tropical cyclones, which are less extensive but equally vigorous. There are intricately structured thunderstorms, comparable in size to large mountains. There are fair-weather cumulus clouds, often no larger than small hills. There are individual wind gusts, sometimes only broad enough to sway a single tree at one moment. Our list is but a sampling.

The above are not simply features which may be present in a correct solution of the equations; they are features which must be present in almost all time-dependent solutions. Any solution which describes only the meanderings of a westerly current, or only the progression of a chain of cyclonic and anticyclonic vortices, is a special solution, belonging to the set of measure zero whose existence we have noted.

It is evident that we lack the means for representing, even at a single instant, global fields of wind, temperature, and moisture which contain several thousand thunderstorms and hundreds of millions of gusts. In short, we are limited by the speed and capacity of today's most powerful computers, or of our brains, from determining typical solutions of the most realistic atmospheric equations which we can formulate. As a dynamical system the real atmosphere does not lend itself to convenient investigation.

In view of these limitations, how is it possible for dynamic meteorology, which was actually a well-established discipline long before the advent of computers, to accomplish anything? Several lines of pursuit are available.

We may use the equations, without actually solving them, to study various atmospheric phenomena and processes. For example, we may derive from the exact equations an expression for the time derivative of the total energy of the vortices, and we may identify the various terms in the expression with particular physical processes. If adequate observational data are available, we may then evaluate the long-term averages of the various terms, and learn which physical processes play leading roles.

Alternatively, we may introduce various approximations. A common procedure consists of linearizing the equations. The great advantage of linear systems, aside from relative ease of solution, is superposability of solutions. Thus, we may find solutions in which all