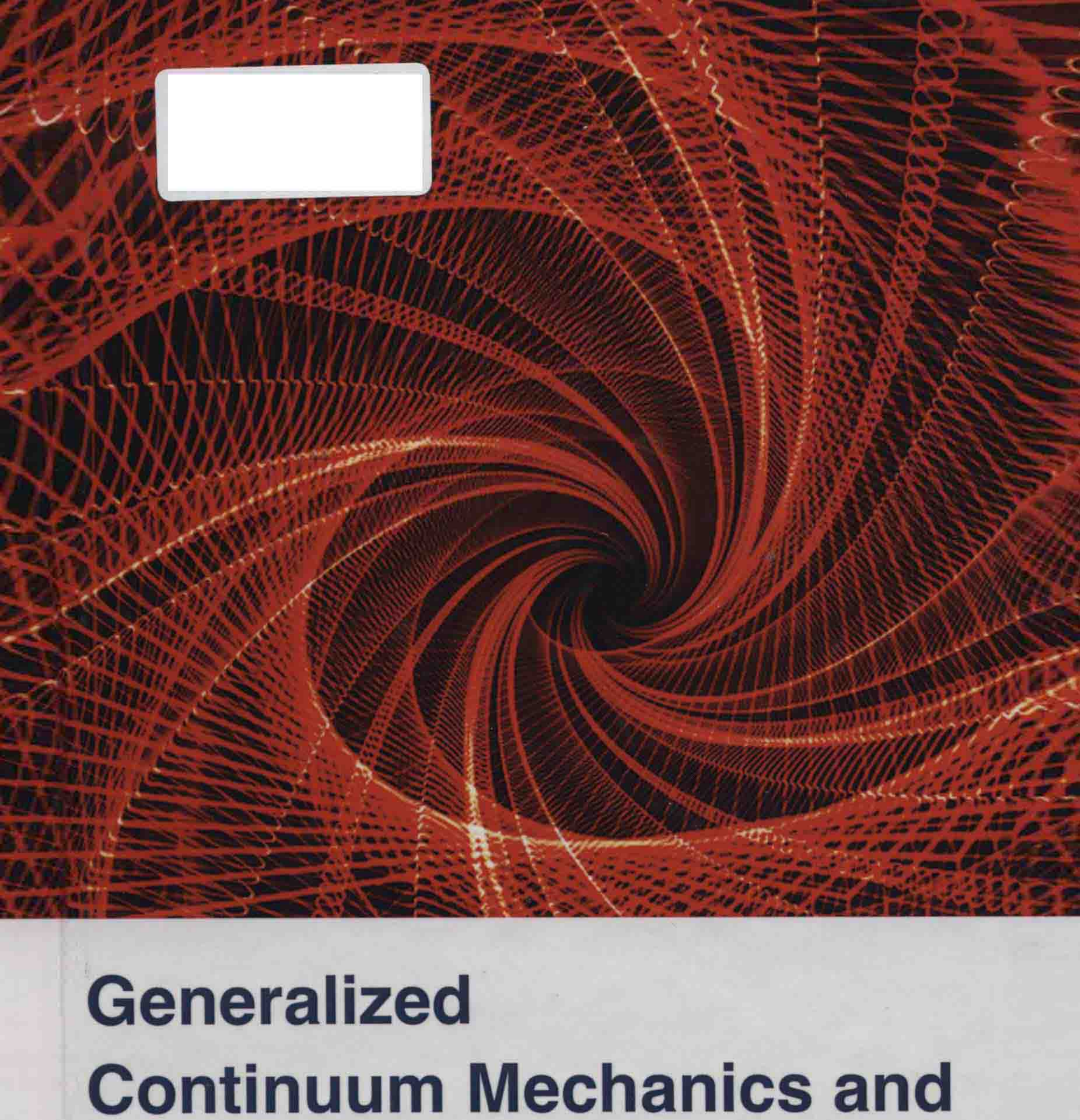

Generalized Continuum Mechanics and Engineering Applications

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iSTE PRESS

ELSEVIER

Series Editor
Yves Rémond

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First published 2015 in Great Britain and the United States by ISTE Press Ltd and Elsevier Ltd

ISTE Press Ltd
27-37 St George's Road
London SW19 4EU
UK

www.iste.co.uk

Elsevier Ltd
The Boulevard, Langford Lane
Kidlington, Oxford, OX5 1GB
UK

www.elsevier.com

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British Library Cataloguing-in-Publication Data
A CIP record for this book is available from the British Library
Library of Congress Cataloging in Publication Data
A catalog record for this book is available from the Library of Congress
ISBN 978-1-78548- 032-4

Transferred to Digital Printing, 2015

Printed in the United States of America

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Acknowledgments

The idea behind this book came from the writing of my Habilitation Thesis that I defended in Lyon in December 2014.

As a matter of fact, this event was for me the trigger of multiple reflections concerning both my personal and professional life. Indeed, when we are faced with the realization of an achievement of the type mentioned above, very naturally and, let me say, abruptly, we are also brought to make a point on what was before and on who were the people contributing to make you the person that you actually are. I can count such persons on one hand, but despite the scareceness of such important encounters, what they left with me is, to my own scale of values, priceless.

The first person that I have to thank is, for sure, Francesco dell'Isola, my PhD supervisor and mentor for several years. I am grateful to him for a long series of reasons that would be unfair to summarize in a few lines. He initiated me into the scientific method and allowed me to see the problems of life with new awareness with respect to what I could do before. The clarity of thought, the way of approaching and solving difficult problems, the pleasure of understanding, the aptitude of looking at life as it is are all things that I feel I possess today and that I owe him to a big extent. It is thus more than natural for me of think of Francesco dell'Isola when a section of acknowledgments is being written, since I look at him as the predominant contributor to my current way of being and I feel that it is again time to thank him for that. And since when an achievement is realized we are brought to simultaneously think of the past and the future, I finally mention here my hope that future times will be as productive and enriching as the past ones have undoubtedly been.

Before starting my PhD, I spent a period of time at Virginia Tech for an exchange program which allowed me to obtain an American Master of Science degree. I feel the need to mention the important role that my advisor Norman Dowling played for the successful conclusion of such an experience. He and his wife helped me by letting me feel at home though I was so far from home that the only fact of quantifying the distance could have scared a 22 year-old girl. He mentored my studies in mechanical behavior of materials giving me the instruments for appreciating the power of a sane application of theoretical tools. I thank him for the fundamental contribution that he gave to my growth in that delicate period of my life.

In the last year of my PhD studies, I had the chance to meet Pierre Seppecher, who greatly contributed to the successful completion of my PhD Thesis. From him, I could learn how creativeness and originality can still be accompanied by humbleness with respect to science. The time that he spent explaining to me how to approach a complex problem in the clearest possible way is worth its weight in gold. I thank him for having taught me that “one needs to come back to his work one thousand times” before it can be considered to be fully metabolized and hence intelligible for others. Such lessons of science and life will accompany me for the future and my determination is now to become able to transform them into something concrete.

Arriving at INSA-Lyon as Associate Professor, I had hence the fortune to meet Philippe Boisse who was working on a class of materials which will be deeply discussed in one chapter of this book: that of fibrous composite reinforcements. It rapidly turned out while working with him that such microstructured materials could have taken advantage of a generalized continuum modeling by second gradient theories. I then had the intellectual satisfaction of finding a first application of the models that I had developed up to that point, actually becoming aware of their true potentialities and also of their limits. I thank Philippe for having helped me in pursuing my inclination toward science notwithstanding the external difficulties that often arise in everyday academic life. I will be forever grateful to him for such invaluable help in the search of a stable academic direction.

Among the people that I have to thank, a special place is reserved to those who are friends before covering any other specific role. Once again, they are very rare, but the evidence of their disinterested presence and support gives me the strength of looking at life with the awareness that we are somehow not alone in confronting it.

Thank you to my current and former PhD students Gabriele Barbagallo, Marco Valerio d’Agostino and Manuel Ferretti for their commitment to science and for their constant work also concerning the typesetting of figures and text in this book.

Last, but not least, I thank my father and mother, for having brought me up, loved me and accepted me as I am. Today that I am wise enough to understand the difficulties that they faced, I feel even more grateful to them and I do not need any other specific reason for doing so.

Contents

1

General Introductory Aspects

1.1. Introduction

The microstructure of materials is an essential feature for the design of engineering structures with improved performances. In these last decades, a huge effort has been made in the direction of conceiving new materials with specific microstructures for the sake of producing exotic mechanical behaviors both in the static and the dynamic regime. Such man-made artifacts, usually called metamaterials, indeed show peculiar material properties that cannot be found in natural materials and that can have multiple engineering applications [ENG 06, ZOU 09, ZHO 09, MAN 13, VAS 98, VAS 01, BOU 13].

It is conceivable, at the present stage of knowledge and technology, to direct a consistent scientific effort toward the conception of microstructured materials showing unusual behaviors which may be beneficial for the functioning of engineering structures and for their optimization. In fact, engineering structures designed using microstructured materials may show very interesting mechanical properties such as light weight, improved stiffness, easy forming processes and so on. Moreover, such materials could also be used for innovative applications in the field of vibration control and stealth technology. In fact, metamaterials are good candidates for the conception of wave screens and wave absorbers since they may show particular properties with respect to elastic and electromagnetic wave propagation.

It is thus understandable that the new concept of metamaterial is nowadays increasingly attracting the interest of physicists and mechanicians and that different microstructures are being conceived in order to obtain the desired macroscopic properties. Usually, metamaterials are obtained by suitably assembling multiple individual elements but arranged in periodic or quasi-periodic substructures in order to show exotic global mechanical behaviors. The particular shape, geometry, size, orientation and arrangement of their constituting elements can affect, for instance, the propagation of waves of light or sound in a manner not observed in natural materials, creating material properties which may give rise to unexpected engineering applications. Particularly promising in the design and description of metamaterials are those microstructures which present high contrasts in their mechanical properties: these microstructures, once homogenized, may produce generalized continuum media (see, for example, [PID 97, ALI 03, FOR 98, FOR 99a, FOR 02, KRU 98]).

Another way to conceive and produce metamaterials is that of optimizing their microstructures by means of statistical approaches (see, for example, [MAN 13] and references there cited). In this way, the obtained microstructures are not periodic anymore, but nevertheless they possess a statistical "hidden order" which allows the macroscopic material to exhibit very particular characteristics, especially for what concerns their behavior with respect to wave propagation. Such materials have been called hyperuniform and have the very interesting property of being isotropic at sufficiently large scales: their response to wave propagation does not depend on the direction of propagation of the considered wave. More particularly, the width of the band gaps which are observed experimentally does not depend on such a direction of propagation. This fact opens very interesting perspectives to the continuum modeling of these metamaterials. Indeed, an isotropic relaxed micromorphic model of the type presented in [GHI 13, MAD 13, MAD 14b, NEF 13] could be used for the macroscopic description of the onset of band gaps by introducing very few elastic parameters which could subsequently be fitted on the available experimental evidence.

1.1.1. *Mechanical models for metamaterials*

The main theoretical challenge related to the modeling of the mechanical behavior of metamaterials is the choice of the model which one wants to use. In fact, there are several possible approaches to the complex problem of considering the effect of microstructures on the overall mechanical behavior of real materials which basically belong to two philosophically distinct categories:

– start from the detailed description of the microscale to arrive to the description of the macroscale;

– start directly from the description of the macroscale somehow accounting for the presence of microscales.

We refrain here from a deep analysis of these two "philosophies", limiting ourselves to briefly discussing some of their advantages and disadvantages. Indeed, a remarkable literature exists based on the adoption of the first viewpoint: start from the microscopic properties of complex materials to arrive to the homogenized ones (bottom-up approaches, see, for example, [FRA 86, FOR 98, KRU 98, FOR 02, FOR 99a, PID 97, GRÜ 88, SEP 11]). From this respect, we can cite so-called homogenization models, multi-scale methods, upscaling procedures and so on. The common idea to all such approaches is to establish "*a priori*" the characteristics of the microstructure (e.g. topology, mechanical stiffnesses, distribution of different phases, etc.) and develop suitable tools to arrive at the global mechanical properties at higher scales. The main advantage of these methods is that they allow us to directly know how the macroscopic parameters are related to the microscopic ones. It is clear that such information is a really useful tool since it suffices to observe the characteristics of a given microstructure to arrive to the homogenized descriptors which can henceforth be used to describe the material behavior at higher scales. Nevertheless, some drawbacks can also be reported about such methods which are substantially related to the fact that a certain number of simplifying assumptions concerning the characteristics of the microstructure are usually needed and often become too restrictive to be able to give rise to a homogenized behavior which is fully representative of the real material behavior at higher scales. For example, some standard homogenization techniques intrinsically need the imposition of boundary conditions between representative cells and it is difficult to establish whether one type of boundary condition is more realistic than another. As a result, we can summarize by saying that it is true that the homogenized system keeps in its memory some peculiar informations about the microscopic characteristics of the system itself, but often the simplifying hypotheses which have been made at the level of the microstructure are too restrictive to assure that the obtained homogenized system is fully able to describe the real macroscopic material behavior.

The second possible type of approach is to start directly from the description of the macroscopic scale by developing models which are able to describe the average mechanical behavior of the considered microstructured materials by means of a relatively small set of macroscopic descriptors (top-down approach). The main advantage of this kind of approach is that real material behaviors can be described by means of few constitutive parameters at those macroscopic scales which are interesting from an engineering point of view. Moreover, the efficacy of the adopted macroscopic theory can be easily compared with experiments which can be conceived and reproduced on specimens having reasonable sizes to be handled without problems related, for example, to the smallness of the samples themselves. Finally, the real material behavior being described by a limited number of parameters, it is conceivable to design structures which have rather sophisticated shapes and large dimensions just relying on a few equations describing the global mechanical behavior of the considered structure. However, the drawbacks of such a type of procedure are twofold:

– one must know that, even if in a simplified macroscopic framework, the global theory must be complemented with some additional macroscopic descriptors if we want to model some macroscopic manifestations of the microstructure;

– it is often hard to accomplish the inverse task of relating the proposed macroscopic descriptors to precise characteristics of the microstructure.

Hence, we can conclude by saying that, if such macroscopic models are able to be more easily handled at scales which are particular to engineering design, some difficulties arise when one needs to precisely relate the used macroscopic descriptors to detailed microscopical properties.

In summary, at the current state of knowledge, there is no common agreement on which would be the correct approach to be used to model at best the mechanical behavior of metamaterials. Would a bottom-up approach be more consistent than a top-down? In other words, is it better to start from the characteristics of the single components of the microstructure and to obtain the homogenized properties, or conversely to try to get a simplified model with relatively few parameters which is somehow able to account for the macroscopic manifestation of the underlying presence of a microstructure inside the material? To our feeling, the answer is: it depends. If the scope is to control in detail how the microstructures affect the macroscopic behavior of the system, then a bottom-up approach seems to be mandatory. However, if with an averaged model we are able to describe the phenomena we are interested in, then there is no reason for not doing so. In the optic of dealing with big pieces of metamaterials in view of engineering design, it is not reasonable to propose the use of a model accounting for the single presence of all the constituents of the considered microstructures. A continuum model would possibly be the desirable choice.

In the framework of continuum theories, the systematic use of Cauchy theories may sometimes represent a too drastic simplification of reality, especially when dealing with metamaterials, since some essential characteristics related to the heterogeneity of microstructures are implicitly neglected in such models. Every material is actually heterogeneous if we consider sufficiently small scales: it suffices to go down to the molecular or atomic level to be aware of such heterogeneity. Nevertheless, very often, the effect of microstructure cannot be detected at the engineering scale. In such cases, continuum Cauchy theory is a suitable choice for modeling the mechanical behavior of considered materials in the simplest and more effective way. However, there are some cases in which the considered materials are heterogeneous even at relatively large scales and, as a result, the effect of microstructure on the overall mechanical behavior of the medium cannot be neglected. In such situations, Cauchy continuum theory may not be sufficient to fully describe the mechanical behavior of considered materials especially when considering particular loading and/or boundary conditions. It is in fact well known that such continuum theory is not able to catch significant phenomena related to concentrations of stress and strain or to specific deformation patterns in which high gradients of deformation occur and which are, in turn, connected to particular phenomena which take place at lower scales. Moreover, Cauchy models are not able to catch in an appropriate way the dynamical response of some microstructured materials showing dispersive behaviors or even frequency band gaps. Generalized continuum theories may be good candidates to model such microstructured materials in a more appropriate way (both in the static and dynamic regime) since they are able to account for the description of some macroscopic manifestations of the presence of microstructure in a rather simplified way.

We have to explicitly say that the heterogeneity of the microstructures alone is not sufficient to unveil the need of using generalized continuum theories against classical continuum ones. Indeed, anisotropic constitutive laws can bring a lot of information concerning the microstructures of considered materials even when remaining in the framework of classical continuum theories. For example, orthotropic constitutive laws can be useful for considering the fact that there are two preferred directions inside the material as a consequence of the fact that the components of the microstructure are oriented in some privileged patterns (as is the case, for example, for woven fibrous composite reinforcements). Nevertheless, such orthotropic constitutive laws are sometimes insufficient to account for some complex microstructure-related deformation patterns in which high gradients of deformation occur. If, as an example, we consider the case of woven fibrous reinforcements, we can easily convince ourselves that the local bending of the yarns is a microscopic deformation mechanism which has a concrete impact on the macroscopic behavior of the piece. Such local bending can be associated with a rapid variation of the shear angle between initially mutually orthogonal yarns which can be interpreted as a concentration of high gradients of shear deformation in thin transition layers. In order to describe such particular patterns in the framework of a continuum theory, second gradient or micromorphic theories must be used instead of classical first gradient ones. Hence, we can summarize by saying that the presence of microstructures can lead to different modeling needs at the level of macroscopic theories:

– the need for considering particular anisotropic constitutive laws which account for the fact that the underlying microstructure has a macroscopic effect on the material behavior for the simple fact of giving rise, for example, to privileged material directions or completely anisotropic behaviors. Such anisotropic constitutive laws can classically be integrated in standard Cauchy continuum models and are sensible to account for a wealthy of microstructure-related effects;

– the need for accounting for some specific behaviors which are usually associated with the description of microstructure-driven concentration of stress and strain inside the material. In such cases, generalized continuum theories may be of use for an improved modeling of the mechanical behavior of microstructured materials.

In the remainder of this chapter, we will present different specific problems in which generalized continuum theories actually bring important complementary information which is essential for a precise modeling of the behavior of the considered mechanical systems.

1.2. Generalized continuum theories and some possible applications

Generalized continuum theories naturally belong to the second of the categories mentioned at the beginning of section 1.1.1 (top-down approaches) and, in this chapter, we will try to analyze whether their possible use can provide some advantages when dealing with real engineering problems. We are of course aware that the first category previously discussed (bottom-up approaches) is as legitimate as the second one for approaching a wealthy of problems, but its study will not be the subject of the present book. Instead, we will focus on a discussion about the use of generalized continuum theories to model materials with microstructure: we regard such theories as a reasonable "engineering" compromise between the complexity of the model which we want to use and the detail at which microstructures can be described.

1.2.1. *Some basic aspects concerning generalized continuum theories*

In this section, we recall some very basic aspects concerning generalized continuum theories in order to let them also be accessible to the non-specialist readers. More precisely, we will make a point about some of the different existing generalized continuum models and we will try to point out which model can be useful to describe specific phenomena of engineering interest. Indeed, a vast literature exists concerning the development of "second gradient", "couple stress", "Cosserat", "micropolar", "micromorphic" models and so on which dates back to the works of the Cosserat brothers, Mindlin, Toupin, Germain, Eringen, Bleustein, etc. (see, for example, [MIN 64, MIN 68, MIN 65, BLE 67, COS 09, GER 73a, GER 73b, TOU 62, TOU 64, ERI 99, DEL 14c, DEL 14b]). Such generalized continuum theories are today experiencing a vehement revival since it is becoming more evident which are their potentialities concerning the macroscopic mechanical description of microstructured materials (see among many others [BOU 13, MAD 12, MAD 14b, DEL 09a, DEL 14a, FER 14, FOR 10, ASK 11, PLA 13, SCI 08, EXA 01, NEF 07, NEF 13, FOR 99b, FOR 01, LAK 82, NEF 06, YAN 82, YAN 81, PLA 14, RIN 14, ALT 13, AUF 13, PIE 09a, PIE 09b, ALT 10, ERE 14, ROS 13, YAN 10]). In this section we present and compare a class of such generalized theories and we highlight some of their possible applications which may be worth further study in view of technological innovation.

In order to review, in a concise way, the different possible types of generalized continuum theories which are usually encountered in the literature, we need to clarify that there are two main ways of generalizing classical Cauchy continuum theories, namely:

– keep the same kinematics as Cauchy theory (only the displacement field), but envisage more complicated expressions of the strain energy density letting it depend on higher gradients of such a displacement field;