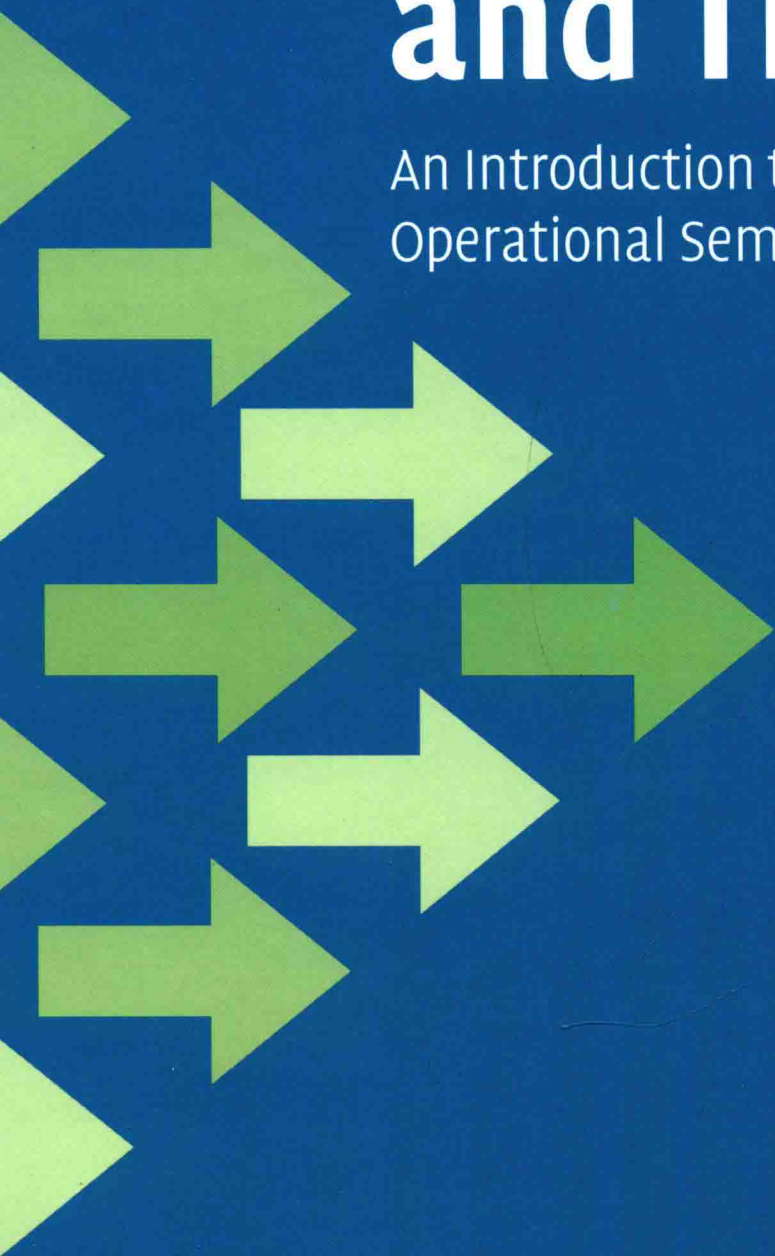


Hans Hüttel

Transitions and Trees

An Introduction to Structural
Operational Semantics



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An Introduction to Structural Operational Semantics

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Transitions and Trees

An Introduction to Structural Operational Semantics

Structural operational semantics is a simple, yet powerful mathematical theory for describing the behaviour of programs in an implementation-independent manner. This book provides a self-contained introduction to structural operational semantics, featuring semantic definitions using big-step and small-step semantics of many standard programming language constructs, including control structures, structured declarations and objects, parameter mechanisms and procedural abstraction, concurrency, non-determinism and the features of functional programming languages. Along the way, the text introduces and applies the relevant proof techniques, including forms of induction and notions of semantic equivalence (including bisimilarity).

Thoroughly class-tested, this book has evolved from lecture notes used by the author over a 10-year period at Aalborg University to teach undergraduate and graduate students. The result is a thorough introduction that makes the subject clear to students and computing professionals without sacrificing its rigour. No experience with any specific programming language is required.

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Preface

About this book

This is a book about structural operational semantics; more precisely it is a book that describes how this approach to semantics can be used to describe common programming language constructs and to reason about the behaviour of programs.

The text grew out of the lecture notes that I have used over a period of more than 10 years in the course *Syntax and semantics* which is taught to all students following the various degree programmes in computer science at Aalborg University. What began as a 10-page set of notes in Danish is now a textbook in English.

The book also includes chapters on related material, namely short introductions to type systems, denotational semantics and the mathematics necessary to understand recursive definitions.

Related work

This work was inspired by lecture notes by Plotkin (1981) (also written in Denmark), where this approach to programming language semantics was first presented.

The topic of structural operational semantics also appears in later books, three of which I will mention here.

Reynolds' book (Reynolds, 1999) is an excellent text that covers some of the same topics as this book but uses denotational and axiomatic semantics as well as structural operational semantics.

The book by Winskel (1993) is another very good textbook that covers many of the same topics as Reynolds' book.

Finally, I should mention Nielson and Nielson (2007), which introduces

and relates denotational, axiomatic and structural operational semantics and then gives an introduction to how these can be used in connection with static program analysis.

The book that you are now reading differs from the ones just mentioned in three important ways. First, the main topic is exclusively that of structural operational semantics. Second, both Reynolds and Winskel introduce domain theory early on; this book aims at developing the theory of structural operational semantics and making use of it with mathematical prerequisites of a more modest nature. Third, unlike the book by the Nielsons, the focus here is not that of program analysis. Instead, it is on how operational semantics can be used to describe common features of programming languages.

What you need to know in advance

This text is *not* intended as an introduction to programming; if you are a reader expecting this to be the case, you will probably be disappointed! The ideal reader should already have some experience with programming in a high-level imperative programming language such as C, Java or Pascal.

Programming language semantics is a mathematical theory. Therefore, the reader should also have some mathematical maturity. In particular, you should be familiar with basic notions of discrete mathematics – sets, functions and graphs and the proof techniques of proof by induction and proof by contradiction. Chapter 2 gives a short overview of some of this material. There are several good textbooks that you can consult as a supplement; one that I would recommend in particular is Velleman's book (Velleman, 2006).

Ways through the book

The book falls into four parts. The first two parts must be covered in any course for which this book is the main text, since the contents of these first four chapters are necessary to understand the material in the rest of the book. After that, there are the following dependences:

- Chapter 7 and Chapter 9 are independent of each other but both extend the language introduced in Chapter 6,
- Chapter 8 assumes knowledge of the parallel operator introduced in Chapter 5,
- Chapter 10 on small-step semantics for procedures and blocks also assumes knowledge of Chapter 6,
- Chapter 11 assumes knowledge of Chapters 8 and 10 and finally

- Chapter 15 assumes knowledge of the contents of Chapter 14.

Problems and thoughts

To learn a mathematical subject, one should of course read the text carefully but also learn to apply the content. For this reason, there are quite a few problems scattered throughout the text. As a rule of thumb, a problem will appear at the point in the text where it becomes relevant. I have chosen this approach since I would like you, the reader, to focus on the connection between the problem and the context in which it appears. You can read most of the text without solving the problems but I encourage you to solve as many of them as possible. In some places, I have put problems that are important for understanding the text and they are then marked as such.

I have also introduced mini-problems which I call **A moment's thought**. Here, the idea is to make you think carefully about what you have just read. Do *not* read the text without finding the answers to these mini-problems.

Related resources

The book has its own website, <http://www.operationalsemantics.net>, which has more information, including hints to the mini-problems. The website also holds information about the Danish-language version of the book, *Pilen ved træets rod*, including how to obtain a copy of it.

Acknowledgements

This book grew out of the years I have spent teaching, so, first, I would like to thank the students who have lived with the various incarnations of this text over the years and have made many useful comments that have helped improve and shape its content and form.

Second, I want to thank the people who have inspired me to reflect on the task of teaching mathematical subjects and teaching in general over the years: Jens Friis Jørgensen, Steffen Lauritzen, Finn Verner Jensen, Anette Kolmos, Helle Alrø and Ole Skovsmose.

Third, I would like to thank all those who helped me make this book a reality. My thanks go to the people and organizations who have kindly allowed me to use the pictures in Chapter 1 and to David Tranah from Cambridge University Press for his encouragement.

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On an entirely personal level, there are others who also deserve thanks. Over the years, I have come to know many inspiring people through my extracurricular activities in human rights activism and music and, most recently, through the extended family of sisters and brothers that I now have. I am very grateful for knowing you all.

Finally, and most importantly, I want to thank my wife Maria and our daughter Nadia for being in my life. This book is dedicated to you.

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Contents

<i>Preface</i>	page ix
<i>About the illustrations</i>	xiii
<i>List of illustrations</i>	xiv
<i>List of tables</i>	xv

PART I BACKGROUND 1

1 A question of semantics	3
1.1 Semantics is the study of meaning	3
1.2 Examples from the history of programming languages	4
1.3 Different approaches to program semantics	6
1.4 Applications of program semantics	10
2 Mathematical preliminaries	16
2.1 Mathematical induction	16
2.2 Logical notation	17
2.3 Sets	19
2.4 Operations on sets	20
2.5 Relations	21
2.6 Functions	22

PART II FIRST EXAMPLES 25

3 The basic principles	27
3.1 Abstract syntax	27
3.2 Transition systems	30
3.3 Big-step vs. small-step semantics	31
3.4 Operational semantics of arithmetic expressions	31

3.5	Proving properties	38
3.6	A semantics of Boolean expressions	39
3.7	The elements of an operational semantics	40
4	Basic imperative statements	43
4.1	Program states	43
4.2	A big-step semantics of statements	45
4.3	A small-step semantics of statements in Bims	53
4.4	Equivalence of the two semantics	55
4.5	Two important proof techniques	60
	 PART III LANGUAGE CONSTRUCTS	 63
5	Control structures	65
5.1	Some general assumptions	65
5.2	Loop constructs	66
5.3	Semantic equivalence	70
5.4	Abnormal termination	72
5.5	Nondeterminism	73
5.6	Concurrency	76
6	Blocks and procedures (1)	79
6.1	Abstract syntax of Bip	79
6.2	The environment-store model	80
6.3	Arithmetic and Boolean expressions	83
6.4	Declarations	84
6.5	Statements	85
6.6	Scope rules	86
7	Parameters	94
7.1	The language Bump	94
7.2	Call-by-reference	96
7.3	On recursive and non-recursive procedure calls	97
7.4	Call-by-value	99
7.5	Call-by-name	100
7.6	A comparison of parameter mechanisms	110
8	Concurrent communicating processes	113
8.1	Channel-based communication – Cab	113
8.2	Global and local behaviour	114
8.3	Synchronous communication in Cab	115
8.4	Other communication models	119

8.5	Bisimulation equivalence	122
8.6	Channels as data – the π -calculus	123
9	Structured declarations	134
9.1	Records	134
9.2	The language Bur	135
9.3	The class-based language Coat	142
10	Blocks and procedures (2)	154
10.1	Run-time stacks	154
10.2	Declarations	155
10.3	Statements	155
11	Concurrent object-oriented languages	161
11.1	The language Cola	161
11.2	A small-step semantics of concurrent behaviour	163
11.3	Transition systems	163
12	Functional programming languages	171
12.1	What is a functional programming language?	171
12.2	Historical background	173
12.3	The λ -calculus	174
12.4	Flan – a simple functional language	177
12.5	Further reading	181
	 PART IV RELATED TOPICS	 183
13	Typed programming languages	185
13.1	Type systems	185
13.2	Typed Bump	187
13.3	Typed Flan	198
13.4	Type polymorphism and type inference	209
14	An introduction to denotational semantics	211
14.1	Background	211
14.2	λ -Notation	212
14.3	Basic ideas	214
14.4	Denotational semantics of statements	216
14.5	Further reading	220
15	Recursive definitions	222
15.1	A first example	222
15.2	A recursive definition specifies a fixed-point	224
15.3	The fixed-point theorem	225

15.4	How to apply the fixed-point theorem	231
15.5	Examples of cpos	232
15.6	Examples of continuous functions	236
15.7	Examples of computations of fixed-points	240
15.8	An equivalence result	241
15.9	Other applications	246
15.10	Further reading	248
<i>Appendix A</i> A big-step semantics of Bip		249
<i>Appendix B</i> Implementing semantic definitions in SML		257
<i>References</i>		264
<i>Index</i>		269

Illustrations

1.1	Alfred Tarski	4
1.2	An ALGOL 60 procedure. What is its intended behaviour?	5
1.3	Dana Scott (left) and Christopher Strachey (right)	7
1.4	Gordon Plotkin	8
1.5	Robin Milner	9
1.6	Tony Hoare	9
1.7	Joseph Goguen	10
1.8	The start and end of the maiden voyage of the Ariane 5	12
1.9	The Mars Climate Orbiter	14
3.1	A very small transition system	31
3.2	Derivation tree for a big-step transition for $(\underline{2}+\underline{3}) * (\underline{4}+\underline{9}) \rightarrow 65$	35
3.3	Comparison between the derivation trees for the individual steps of a small-step transition sequence and that of a big-step transition	41
6.1	Example of a variable environment and a store	82
6.2	An example Bip statement whose behaviour is dependent on the choice of scope rules	88
7.1	A Bump statement with recursive calls	98
7.2	Exploiting call-by-name for computing the sum $\sum_{i=1}^{10} i^2$	104
7.3	A Bump statement showing where name clashes could occur as a result of an incorrectly defined substitution	108
9.1	A program with nested record declarations	135
9.2	A Coat program example	144
13.1	A type system is an overapproximation of safety	198
15.1	Part of the Hasse diagram for (\mathbb{N}, \leq)	226
15.2	A Hasse diagram for $(\mathcal{P}(\{1, 2, 3\}), \subseteq)$	233

Tables

3.1	Abstract syntax of Bims	29
3.2	Big-step transition rules for Aexp	33
3.3	Small-step transition rules for Aexp	37
3.4	Big-step transition rules for Bexp	40
4.1	Big-step transition rules for Aexp	45
4.2	Big-step transition rules for Bexp	46
4.3	Big-step transition rules for Stm	47
4.4	Small-step transition rules for Stm	53
5.1	Big-step transition rules for repeat-loops	67
5.2	Big-step transition rules for for-loops	72
5.3	Big-step transition rules for the or -statement	74
5.4	Small-step transition rules for the or -statement	75
5.5	Small-step semantics of the par -statement	77
5.6	An attempt at big-step transition rules for the par -statement	77
6.1	Big-step operational semantics of Aexp using the environment-store model	83
6.2	Big-step semantics of variable declarations	84
6.3	Big-step transition rules for Bip statements (except procedure calls)	87
6.4	Transition rules for procedure declarations (assuming fully dynamic scope rules)	89
6.5	Transition rule for procedure calls (assuming fully dynamic scope rules)	89
6.6	Transition rules for procedure declarations assuming mixed scope rules (dynamic for variables, static for procedures)	90
6.7	Transition rules for procedure calls assuming mixed scope rules (dynamic for variables, static for procedures)	91
6.8	Transition rules for procedure declarations assuming fully static scope rules	92

6.9	Transition rules for procedure calls assuming fully static scope rules	93
7.1	Rules for declaring procedures with a single parameter assuming static scope rules	96
7.2	Transition rule for calling a call-by-reference procedure	97
7.3	Revised transition rule for procedure calls that allow recursive calls	99
7.4	Transition rules for procedure calls using call-by-value	101
7.5	Transition rules for declaration of call-by-name procedures assuming fully static scope rules	105
7.6	Transition rules for procedure calls using call-by-name	105
7.7	Substitution in statements	111
8.1	Transition rules defining local transitions	116
8.2	Rules defining the capability semantics	117
8.3	Synchronous communication: transition rules for the global level	118
8.4	Asynchronous communication: communication capabilities	120
8.5	Asynchronous communication: the global level	121
8.6	The structural congruence rules	128
8.7	Rules of the reduction semantics of the π -calculus	130
8.8	Rules of the labelled semantics of the π -calculus	132
9.1	Transition rules for generalized variables	137
9.2	Transition rules for arithmetic expressions	138
9.3	Transition rules for variable declarations	139
9.4	Transition rules for procedure declarations	139
9.5	Transition rules for generalized procedure names	140
9.6	Transition rules for record declarations	140
9.7	Transition rules for statements in Bur	141
9.8	The semantics of class declarations	146
9.9	The semantics of variable declarations	147
9.10	The semantics of method declarations	147
9.11	The semantics of object declarations	148
9.12	The semantics of object sequences	149
9.13	The semantics of object expressions	150
9.14	Evaluating extended variables (in the semantics of arithmetic expressions)	150
9.15	Important transition rules for statements	151
9.16	Transition rule for programs	152
10.1	Transition rules for statements other than procedure calls	157
10.2	Transition rules for procedure calls assuming static scope rules	159
10.3	Transition rule for procedure calls assuming dynamic scope rules	160
11.1	Transition rules for the local transition system	165
11.2	Transition rules of the labelled transition systems	166
11.3	Transition rules for the global level (1) – initialization	167

11.4	Transition rules for the global level (2) – the connection between global and local behaviour	168
11.5	Transition rules for the global level (3) – rendezvous	169
12.1	Big-step semantics for Flan	179
12.2	Some of the small-step semantics of Flan	180
13.1	Big-step operational semantics of Exp (arithmetic part)	188
13.2	Big-step transition rules for Bump statements (except procedure calls)	189
13.3	Type rules for Bump expressions	191
13.4	Type rules for variable and procedure declarations in Bump	191
13.5	Type rules for Bump statements	192
13.6	The error predicate for variable declarations	194
13.7	The error predicate for statements	195
13.8	Type rules for Flan	200
13.9	The small-step semantics of Flan	201
13.10	The type rule for letrec	209
A.1	Big-step operational semantics of Aexp	252
A.2	Big-step transition rules for \rightarrow_b	253
A.3	Big-step semantics of variable declarations	254
A.4	Transition rules for procedure declarations assuming fully static scope rules	254
A.5	Big-step transition rules for Bip statements	256