Second Edition

Thinking Mathematically

Robert Blitzer

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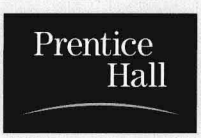
Robert Blitzer
Miami-Dade Community College



PRENTICE HALL,
Upper Saddle River, New Jersey 07458

Library of Congress Cataloging-in-Publication Data
Blitzer, Robert.
Thinking mathematically / Robert Blitzer.-- 2nd ed.
p. cm.
Includes index.
ISBN 0-13-065601-1
1. Mathematics. I. Title.

Qa39.3 .B59 2003
510—dc21 2002059825

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Printed in the United States of America
10 9 8 7 6 5 4 3 2 1

ISBN 0-13-065601-1

Pearson Education LTD., *London*
Pearson Education Australia PTY., *Sydney*
Pearson Education Singapore, Pte. Ltd.
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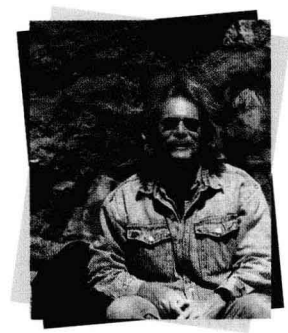

Bob Blitzer is a native of Manhattan and received a Bachelor of Arts degree with dual majors in mathematics and psychology (minor: English literature) from the City College of New York. His unusual combination of academic interests led him toward a Master of Arts in mathematics from the University of Miami and a doctorate in behavioral sciences from Nova University. Bob is most energized by teaching mathematics and has taught a variety of mathematics courses at Miami-Dade Community College for nearly 30 years. He has received numerous teaching awards, including Innovator of the Year from the League for Innovations in the Community College, and was among the first group of recipients at Miami-Dade for an endowed chair based on excellence in the classroom. In addition to *Thinking Mathematically,* Bob has written *Introductory Algebra for College Students, Intermediate Algebra for College Students, Introductory* and *Intermediate Algebra for College Students, Algebra for College Students, College Algebra, Algebra and Trigonometry,* and *Precalculus*, all published by Prentice Hall.

Preface

Thinking Mathematically, Second Edition, provides a general survey of mathematical topics that are useful in our contemporary world. My primary purpose in writing the book was to show students how mathematics can be applied to their lives in interesting, enjoyable, and meaningful ways. The book's variety of topics and flexibility of sequence make it appropriate for a one- or two-term course in liberal arts mathematics, finite mathematics, mathematics for education majors, as well as for courses specifically designed to meet state-mandated requirements in mathematics.

I wrote the book with three major goals: first, to help students acquire knowledge of fundamental mathematics; second, to show students how mathematics can solve authentic problems that apply to their lives; and third, to enable students to develop problem-solving skills, fostering critical thinking, within a varied, interesting, and contemporary setting.

New and Expanded Features

I have added the following new features in the Second Edition:

- **Section-Opening Scenarios.** Every section now opens with a compelling image that supports a scenario presenting an application of mathematics in students' lives outside the classroom. Each scenario is revisited later in the section or chapter.

- **Check Point Examples.** Each worked example is followed by a similar matched problem for the student to work while reading the material. This actively involves students in the learning process and gives them the opportunity to work with a concept as soon as they have learned it. Answers to all Check Points are given in the answer section.

- **Updated Real-World Data.** Real-world data are used to bring relevance to examples, discussions, and applications. Data from the first edition have been replaced to include data and applications that extend as far up to the present as possible. For example, Section 8.5 (Investing in Stocks, Bonds, and Mutual Funds) includes a discussion of the economic downturn prior to September 11, 2001, and the ramifications of that horrific day for the long-range investor.

- **Mathematical Modeling.** Mathematical modeling is now introduced in the algebra chapters—Chapters 6 and 7. Using mathematical models, the Second Edition presents dozens of new algebraic applications in the examples and exercises. Mathematical modeling is also used in the new chapter on graph theory, Chapter 15, in the book's expanded edition.

- **Expanded Exercise Sets.** New exercises have been added to many of the exercise sets. Most of the new exercises can be found in the Application Exercises category, and involve problem solving and current real-world data.

- **New Enrichment Essays.** Enrichment essays provide historical, interdisciplinary, and otherwise interesting connections. A variety of new essays, ranging from vampire numbers (Chapter 4) to modeling a thinning Miss America with a regression line (Chapter 12), appear throughout the book.

- **Voting and Apportionment.** A new chapter on voting and apportionment, Chapter 14, has been added to the expanded second edition.

- **Graph Theory.** A new chapter on graph theory, Chapter 15, has been added to the expanded second edition.

I am avidly interested in psychology, philosophy, and literature, as well as mathematics. I have worked as a musician, psychotherapist, and even attempted book, music, and lyrics to an original musical. (Compared to writing for musical theater, authoring mathematics textbooks is a breeze.) I hope that my love for learning, as well as my respect for the diversity of students I have taught and learned from over the years, is apparent in the hundreds of applications that appear throughout the book. By connecting mathematics to the whole spectrum of learning, it is my intent to show students that their world is profoundly mathematical and , indeed, π is in the sky.

Robert Blitzer

To The Student

I have written this book to give you control over the part of your life that involves numbers and mathematical ideas. Gaining an understanding and appreciation of mathematics will help you participate fully in the twenty-first century. In some ways, you cannot get along in life without the mathematics in this book. For example, if you do not understand the basic ideas of investment, you may find it impossible to achieve your financial goals. If you do not have at least a rudimentary understanding of set theory, you may not be able to meaningfully interpret the vast amount of survey data we are presented with almost daily.

This book has been written so that you can learn about the power of mathematics directly from its pages. All concepts are carefully explained, important definitions and procedures are set off in boxes, and worked-out examples that present solutions in a step-by-step manner appear in every section. Each example is followed by a similar matched problem, called a Check Point, for you to try so that you can actively participate in the learning process as you read the book. (Answers to all Check Points appear in the back of the book.) Study Tip boxes offer hints and suggestions and often point out common errors to avoid. A great deal of attention has been given to applying algebra to your life to make your learning experience both interesting and relevant.

As you begin your studies, I would like to offer some specific suggestions for using this book and for being successful in this course:

1. **Attend all lectures.** No book is intended to be a substitute for valuable insights and interactions that occur in the classroom. In addition to arriving for lecture on time and being prepared, you will find it useful to read the section before it is covered in lecture. This will give you a clear idea of the new material that will be discussed.

2. **Read the book.** Read each section with pen (or pencil) in hand. Move through the illustrative examples with great care. These worked-out examples provide a model for doing exercises in the exercise sets. As you proceed through the reading, do not give up if you do not understand every single word. Things will become clearer as you read on and see how various procedures are applied to specific worked-out examples.

3. **Work problems every day and check your answers.** The way to learn mathematics is by doing mathematics, which means working the Check Points and assigned exercises in the exercise sets. The more exercises you work, the better you will understand the material.

4. **Prepare for chapter exams.** After completing a chapter, study the summary chart, work the exercises in the Chapter Review, and work the exercises in the Chapter Test. Answers to all these exercises are given in the back of the book.

5. **Use the supplements available with this book.** A solutions manual containing worked-out solutions to the book's odd-numbered exercises, all review exercises, and all Check Points, a dynamic web page, and videotapes created for every section of the book are among the supplements created to help you tap into the power of mathematics. Ask your instructor or bookstore what supplements are available and where you can find them.

I wrote this book in Point Reyes National Seashore, 40 miles north of San Francisco. The park consists of 75,000 acres with miles of pristine surf-washed beaches, forested ridges, and bays bordered by white cliffs. It was my hope to convey the beauty and excitement of mathematics using nature's unspoiled beauty as a source of inspiration and creativity. Enjoy the pages that follow as you empower yourself with the mathematics needed to succeed in college, your career, and in your life.

Regards,

Bob

Robert Blitzer

Acknowledgments

I would like to express my appreciation to all the reviewers for their helpful criticisms and suggestions, frequently transmitted with wit, humor and intelligence. In particular, I would like to thank:

Reviewers of Second Edition:
Jaime Langille, *University of Nevada Las Vegas*
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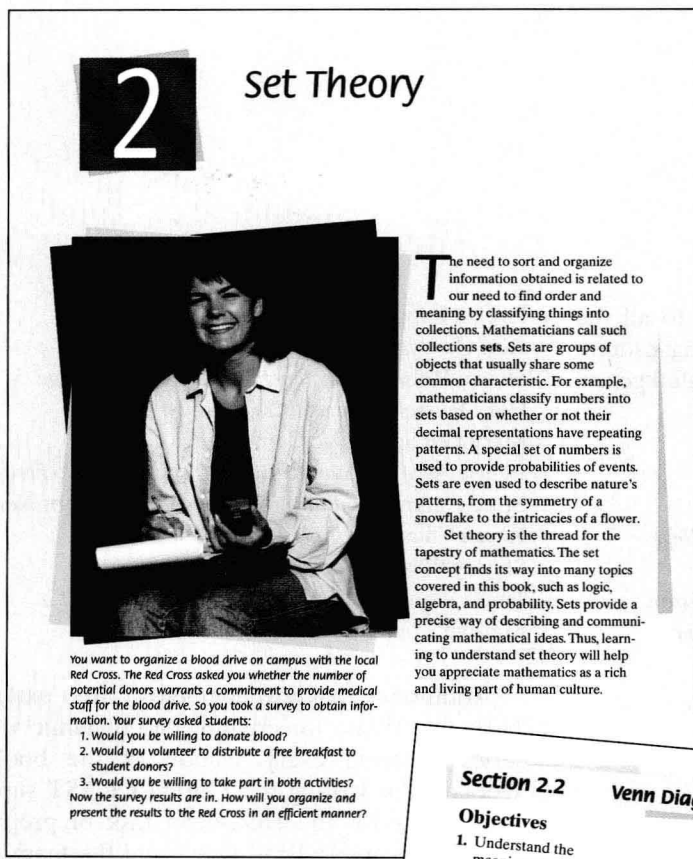
Mike Tomme, *Community College of Southern Nevada*
Karen Villareal, *University of New Orleans*
Shirley Wilson, *North Central College*
James Wooland, *Florida State University*
Marilyn Zopp, *North Illinois University*

Additional acknowledgments are extended to Jacquelyn White for creating the dynamic video tape series covering every section of the book; James Wooland for writing the Florida CLAST supplement; Dan Miller for the Herculean task of preparing the solutions manuals; Brad Davis and the team at Laurel Technical Services for preparing the answer section and also serving as accuracy checker; Melinda Alexander, photo researcher, for obtaining the book's photographs; Brian Morris and the graphic artists at Scientific Illustrators, whose superb illustrations and graphs provide visual support to the verbal portions of the text; the Prentice Hall formatting team for the countless hours they spent paging the book; and Bayani Mendoza de Leon, whose talents as production editor contributed to the book's wonderful look.

I would like to thank my editors at Prentice Hall, Sally Yagan and Dawn Murrin, who guided and coordinated the many details of this project. Thank you Patrice Lumumba Jones, Senior Marketing Manager, for your innovative marketing efforts, as well as the Prentice Hall sales force for your confidence and enthusiasm over the book.

A Guide to Using This Text



2 Set Theory

You want to organize a blood drive on campus with the local Red Cross. The Red Cross asked you whether the number of potential donors warrants a commitment to provide medical staff for the blood drive. So you took a survey to obtain information. Your survey asked students:

1. *Would you be willing to donate blood?*
2. *Would you volunteer to distribute a free breakfast to student donors?*
3. *Would you be willing to take part in both activities?*

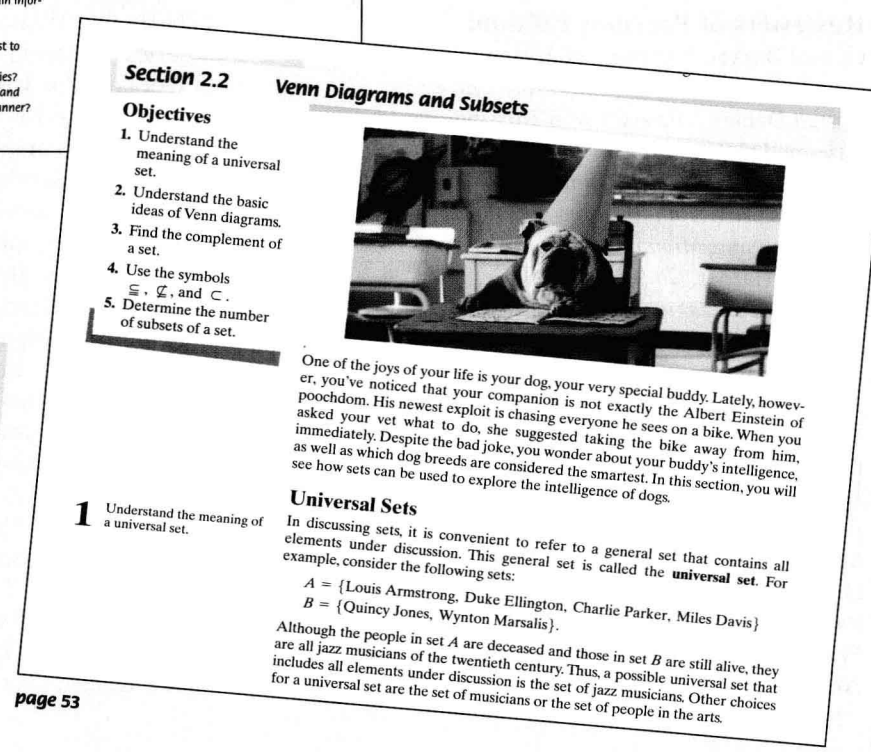
Now the survey results are in. How will you organize and present the results to the Red Cross in an efficient manner?

The need to sort and organize information obtained is related to our need to find order and meaning by classifying things into collections. Mathematicians call such collections **sets**. Sets are groups of objects that usually share some common characteristic. For example, mathematicians classify numbers into sets based on whether or not their decimal representations have repeating patterns. A special set of numbers is used to provide probabilities of events. Sets are even used to describe nature's patterns, from the symmetry of a snowflake to the intricacies of a flower.

Set theory is the thread for the tapestry of mathematics. The set concept finds its way into many topics covered in this book, such as logic, algebra, and probability. Sets provide a precise way of describing and communicating mathematical ideas. Thus, learning to understand set theory will help you appreciate mathematics as a rich and living part of human culture.

Relevant Chapter Openers

Every chapter highlights a scenario from everyday life and how the mathematics relates to it. These scenarios are revisited later in the chapter.



Section 2.2 **Venn Diagrams and Subsets**

Objectives

1. Understand the meaning of a universal set.
2. Understand the basic ideas of Venn diagrams.
3. Find the complement of a set.
4. Use the symbols $\subseteq$, $\not\subseteq$, and $\subset$.
5. Determine the number of subsets of a set.

One of the joys of your life is your dog, your very special buddy. Lately, however, you've noticed that your companion is not exactly the Albert Einstein of poochdom. His newest exploit is chasing everyone he sees on a bike. When you asked your vet what to do, she suggested taking the bike away from him, immediately. Despite the bad joke, you wonder about your buddy's intelligence, as well as which dog breeds are considered the smartest. In this section, you will see how sets can be used to explore the intelligence of dogs.

1 Understand the meaning of a universal set.

Universal Sets

In discussing sets, it is convenient to refer to a general set that contains all elements under discussion. This general set is called the **universal set**. For example, consider the following sets:

A = {Louis Armstrong, Duke Ellington, Charlie Parker, Miles Davis}
B = {Quincy Jones, Wynton Marsalis}.

Although the people in set A are deceased and those in set B are still alive, they are all jazz musicians of the twentieth century. Thus, a possible universal set that includes all elements under discussion is the set of jazz musicians. Other choices for a universal set are the set of musicians or the set of people in the arts.

Section-Opening Scenarios

Each and every section opens with a unique application of math in students' lives outside the classroom. These scenarios are revisited later in the section.

Learning Objectives at the beginning of each section give students a clear idea of the concepts they will cover. Objectives reappear in the margin at their point of use.

Section 4.3 Computation in Positional Systems

Objectives

1. Add in bases other than ten.
2. Subtract in bases other than ten.
3. Multiply in bases other than ten.
4. Divide in bases other than ten.

People have always looked for ways to make calculations faster and easier. The Hindu-Arabic system of numeration made computation simpler and less mysterious. More people were able to perform computation with ease, leading to the widespread use of the system.

All computations in bases other than ten are performed exactly like those in base ten. However, when a computation is equal to or exceeds the given base, use the mental conversions discussed in the previous section to convert from the base ten numeral to a numeral in the desired base.

1 Add in bases other than ten.

Addition

EXAMPLE 1 Addition in Base Four

Add:

$$\begin{array}{r} 33_{four} \\ +\ 13_{four}. \end{array}$$

Solution We will begin by adding the numbers in the right-hand column. In base four, the digit symbols are 0, 1, 2, and 3. If a sum in this, or any, column exceeds 3, we will have to convert this base ten number to base four. We begin by adding the numbers in the right-hand, or ones', column:

The 4's, or fours' column

The ones' column

$$\begin{array}{r} 33_{four} \\ +\ 13_{four} \end{array}$$

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$3_{four} = 6.$

ever, we can express 6 as one group of four

$\times 4) + (2 \times 1) = 12_{four}.$

nd column, 12_{four}:

Study Tip

With increased practice in solving linear equations, try working some steps mentally. For example, consider

$7x - 8 = -22,$

the simplified equation, on the right. Add 8 to both sides without listing all the steps:

$7x = -14.$

Now, divide both sides by 7:

$x = -2.$

EXAMPLE 6 Solving a Linear Equation by First Simplifying

Solve the equation: $2(x - 4) + 5x = -22$.

Solution

$2(x - 4) + 5x = -22$	This is the given equation.
$2x - 8 + 5x = -22$	Apply the distributive property on the left.
$7x - 8 = -22$	Combine like terms on the left.
$7x - 8 + 8 = -22 + 8$	Add 8 to both sides.
$7x = -14$	Combine like terms.
$\frac{7x}{7} = \frac{-14}{7}$	Divide both sides by 7.
$x = -2$	Simplify.

Check

Check by substituting the proposed solution, -2, into the original equation.

$2(x - 4) + 5x = -22$	This is the original equation.
$2(-2 - 4) + 5(-2) \stackrel{?}{=} -22$	Substitute -2 for x.
$2(-6) + 5(-2) \stackrel{?}{=} -22$	Simplify inside parentheses.
$-12 + (-10) \stackrel{?}{=} -22$	Multiply.
$-22 = -22$	This true statement indicates that -2 is the solution.

The solution set is $\{-2\}$.

Check Point 6 Solve the equation: $6(x - 3) + 7x = -57$.

Study Tip

You can solve

In Example 7, we solve a linear equation by subtracting the same algebraic expression from both sides.

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Well-Constructed Examples are clearly written and provide students with step-by-step solutions containing both numerical and verbal explanations. Well-presented examples are essential to students mastering procedural concepts.

Essays provide historical, interdisciplinary, and interesting connections throughout the text.

Einstein's Famous Formula: $E = mc^2$

One of the most famous formulas in the world is $E = mc^2$, formulated by Albert Einstein. Einstein showed that any form of energy, has mass and that mass itself is a form of energy. In this formula, E represents energy, in ergs, m represents mass, in grams, and c represents the speed of light. Because light travels at 30 billion centimeters per second, the formula indicates that 1 gram of mass will produce 900 billion ergs of energy.

Einstein's formula implies that the mass of a golf ball could provide the daily energy needs of the metropolitan Boston area. Mass and energy are equivalent, and the transformation of even a tiny amount of mass releases an enormous amount of energy. If this energy is released suddenly, a destructive force is unleashed, as in an atom bomb. When the release is gradual and controlled, the energy can be used to generate power.

The theoretical results implied by Einstein's formula $E = mc^2$ have not been realized because scientists have not yet developed a way of converting a mass completely to energy.

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The text uses **Compelling Images** to help students make the connection between their world and the mathematics that permeates it.

Section 7.5 Systems of Linear Equations

Objectives

1. Decide whether an ordered pair is a solution of a linear system.
2. Solve linear systems by graphing.
3. Solve linear systems by substitution.
4. Solve linear systems by addition.
5. Identify systems that do not have exactly one ordered-pair solution.
6. Solve problems using systems of linear equations.

Key West residents Brian Goss (left), George Wallace, and Michael Mooney (right) hold on to each other as they battle 90-mph winds along Houseboat Row in Key West, Fla., on Friday, Sept. 25, 1998. The three had sought shelter behind a Key West

Solving real-world problems often involves thousands of equations, sometimes a million variables. Problems ranging from scheduling airline flights to controlling traffic flow to routing phone calls over the nation's communication network often require solutions in a matter of moments. AT&T's domestic long-distance network involves 800,000 var

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Check Points offer students the opportunity to test their understanding of the example by working a similar exercise while they are reading the material. The answers to all the Check Points are given in the answer section.

Check Point 1 Consider the equation $y = x^2 - 6x + 8$.

a. Is the graph a parabola that opens upward or downward?

b. Use point plotting to graph the parabola. Select integers for x, starting with 0 and ending with 6.

A number of points are important when graphing a quadratic equation. These points, labeled in Figure 7.22, are the x-intercepts (although not every parabola has two x-intercepts), the y-intercept, and the vertex. Let's see how we can locate these points.

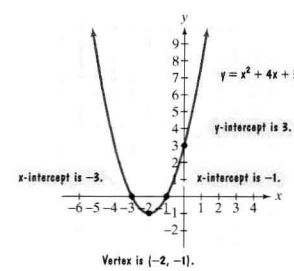

Figure 7.22 Useful points in graphing a parabola

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Technology

You can use a calculator to evaluate exponential expressions. For example, to evaluate 5^3, press the following keys:

Scientific Calculator

5 [y^x] 3 [=]

Graphing Calculator

5 [^] 3 [ENTER]

Although calculators have special keys to evaluate powers of ten and squaring bases, you can always use one of the sequences shown above.

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Technology Boxes suggest hints for using both computers and graphing calculators. Technology is integrated into both the chapter material and section exercises. Students using technology will find that the text appropriately supports their mode of study.

Study Tip

A familiar algebraic statement should help you remember the logical equivalency

$$\sim(\sim p) \equiv p.$$

If a represents a number, then

$$-(-a) = a.$$

For example, $-(-4) = 4$.

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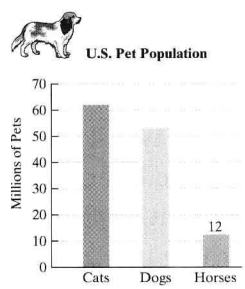
Study Tips offer suggestions for problem solving, point out common student errors, and provide informal tips and suggestions throughout the text.

EXAMPLE 3 Pet Population

Americans love their pets. The number of cats in the United States exceeds the number of dogs by 7.5 million. The number of cats and dogs combined is 114.7 million. Determine the number of dogs and cats in the United States.



Source: American Veterinary Medical Association

Americans spend more than $21 billion a year on their pets, 31.4% of households have cats and 34.3% have dogs.

Solution

Step 1 **Let x represent one of the quantities.** We know something about the number of cats: the cat population exceeds the dog population by 7.5 million. This means that there are 7.5 million more cats than dogs. We will let

x = the number (in millions) of dogs in the United States.

Step 2 **Represent other quantities in terms of x.** The other unknown quantity is the number of cats. Because there are 7.5 million more cats than dogs, let

$x + 7.5$ = the number (in millions) of cats in the United States.

Step 3 **Write an equation in x that describes the conditions.** The number of cats and dogs combined is 114.7 million

The number (in millions) of dogs in the U.S.	plus	the number (in millions) of cats in the U.S.	equals	114.7 million.
x	+	$x + 7.5$	=	114.7

Step 4 **Solve the equation and answer the question.**

$$x + x + 7.5 = 114.7 \quad \text{This is the equation specified by the conditions of the problem.}$$

$$2x + 7.5 = 114.7 \quad \text{Combine like terms on the left side.}$$

$$2x + 7.5 - 7.5 = 114.7 - 7.5 \quad \text{Subtract 7.5 from both sides.}$$

$$2x = 107.2 \quad \text{Simplify.}$$

$$\frac{2x}{2} = \frac{107.2}{2} \quad \text{Divide both sides by 2.}$$

$$x = 53.6 \quad \text{Simplify.}$$

Because x represents the number (in millions) of dogs, there are 53.6 million dogs in the United States. Because $x + 7.5$ represents the number (in millions) of cats, there are53.6 + 7.5, or 61.1 million cats in the United States.

Step 5 **Check the proposed solution in the original wording of the problem.** The problem states that the number of cats and dogs combined is 114.7 million. By adding 53.6 million, the dog population, and 61.1 million, the cat population, we do, indeed, obtain a sum of 114.7 million.

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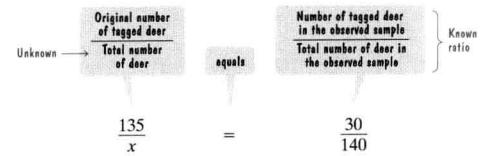
Real-World Data brings relevance to examples, discussions, and applications.

Step-by-Step Procedures provide a structured resource for learning.

Voice Balloons allow for more specific annotations in examples, further clarifying procedures and concepts for students.

Step 2 Set up a proportion.



$$\frac{135}{x} = \frac{30}{140}$$

Steps 3 and 4 Apply the cross products principle, solve, and answer the question.

$$\frac{135}{x} = \frac{30}{140}$$

$(135)(140) = 30x$ Apply the cross products principle.

$18{,}900 = 30x$ Multiply.

$\frac{18{,}900}{30} = \frac{30x}{30}$ Divide both sides by 30.

$630 = x$ Simplify.

There are approximately 630 deer in the refuge.

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Exercise Set 4.2

Practice Exercises

In Exercises 1–18, convert the numeral to a numeral in base ten.

1. 43_{five}
2. 34_{five}
3. 52_{eight}
4. 67_{eight}
5. 132_{four}
6. 321_{four}
7. 1011_{two}
8. 1101_{two}
9. 2035_{six}
10. 2073_{nine}
11. 70355_{eight}
12. 41502_{six}
13. $2096_{sixteen}$
14. $3104_{fifteen}$
15. 110101_{two}
16. 101101_{two}
17. $ACE5_{sixteen}$
18. $EDF7_{sixteen}$

In Exercises 19–28, mentally convert each base ten numeral to a numeral in the given base.

19. 7 to base five
20. 9 to base five
21. 11 to base seven
22. 12 to base seven
23. 2 to base two
24. 3 to base two
25. 13 to base four
26. 19 to base four
27. 37 to base six
28. 25 to base six

In Exercises 29–40, use divisions to convert each base ten numeral to a numeral in the given base.

29. 87 to base five
30. 85 to base seven
31. 108 to base four
32. 199 to base four
33. 19 to base two
34. 23 to base two
35. 138 to base three
36. 129 to base three
37. 386 to base six
38. 428 to base nine
39. 1599 to base seven
40. 1346 to base eight

Application Exercises

Use a procedure similar to the one used in Exercises 29–40 to solve Exercises 41–43.

41. Change 153 days to weeks and days.
42. Change 273 hours to days and hours.
43. Change \$8.79 to quarters, nickels, and pennies.

Writing in Mathematics

44. Explain how to determine the place values for a four-digit numeral in base six.
45. Describe how to change a numeral in a base other than ten to a base ten numeral.
46. Describe how to change a base ten numeral to a numeral in another base.

Critical Thinking Exercises

In Exercises 47–48, write in the indicated base the counting numbers that precede and follow the number expressed by the given numeral.

47. 888_{nine}
48. $EC5_{sixteen}$
49. Arrange from smallest to largest: 11111011_{two}, $3A6_{twelve}$, 673_{eight}.

Technology Exercises

In Exercises 50–54, use the scientific calculator in Windows (or any scientific calculator that handles different base conversions) to convert each numeral to a numeral in the indicated base. Note: If you are unsure how to convert to a different base on your calculator, consult the owner's manual.

50. 45 to octal
51. 100101_{two} to decimal
52. 100101_{two} to hexadecimal
53. 567 to binary
54. 333_{eight} to hexadecimal

Group Exercises

The following topics are appropriate for either individual or group research projects. A report should be given to the class on the researched topic. Useful references include history of mathematics books, books whose purpose is to excite the reader about mathematics, encyclopedias, and the World Wide Web.

55. Societies That Use Numeration Systems with Bases Other Than Ten
56. The Use of Fingers to Represent Numbers
57. Applications of Bases Other Than Ten
58. Binary, Octal, Hexadecimal Bases and Computers
59. Negative Bases (See "Numeration Systems with Unusual Bases," by David Ballow in *The Mathematics Teacher*, May 1974, pp. 413–414.)
60. Babylonian and Mayan Civilizations and Their Contributions

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Exercises are graded by level within six category types:

- Practice Exercises
- Application Exercises
- Writing in Mathematics
- Critical Thinking Exercises
- Technology Exercises
- Group Exercises

This format makes it easy to create well-rounded homework assignments.

Chapter Summaries and Review Exercises are organized around each section within the chapter. The review exercises conclude with a group exercise based on the chapter opener. *More review is available on the Website at:* ***www.prenhall.com/blitzer.***

CHAPTER SUMMARY, REVIEW, AND TEST

SUMMARY

4.1 Our Hindu-Arabic System and Early Positional Systems

a. In a positional-value, or place-value, numeration system, the value of each symbol, called a digit, varies according to the position it occupies in the number.

b. The Hindu-Arabic numeration system is a base ten system with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The place values in the system are

$$\ldots, 10^5, 10^4, 10^3, 10^2, 10^1, 1.$$

c. The Babylonian numeration system is a base 60 system, with place values given by

...,	60^3,	60^2,	60^1,	1.
	or	or	or	
	216,000	3600	60	

Babylonian numerals are given in Table 4.1 on page 174.

d. The Mayan numeration system has place values given by

...,	18×20^3,	18×20^2,	18×20,	20, 1.
	or	or	or	
	144,000	7200	360	

Mayan numerals are given in Table 4.2 on page 175.

4.2 Number Bases in Positional Systems

a. The base of a positional numeration system refers to the number of individual digit symbols used in the system as well as to the way in which grouping is to occur. In base b, there are b digit symbols (from 0 through $b - 1$ inclusive) with place values given by

b. To change a numeral in a base other than ten to a base ten numeral,
1. Multiply each digit in the numeral by its respective place value.
2. Find the sum of the products in step 1.

c. To change a base ten numeral to a base b numeral, use mental conversions or repeated divisions by powers of b to find how many groups of each place value are contained in the base ten numeral.

4.3 Computation in Positional Systems

a. Computations in bases other than ten are performed using the same procedures as in ordinary base ten arithmetic. When a computation is equal to or exceeds the given base, use mental conversions to convert from the base ten numeral to a numeral in the desired base.

b. To divide in bases other than ten, it is convenient to use a multiplication table for products in the required base.

4.4 Looking Back at Early Numeration Systems

a. A successful numeration system expresses numbers with relatively few symbols and makes computation with these numbers fairly easy.

b. By the standard in (a), the Egyptian system (Table 4.6 on page 191), the Roman system (Table 4.7 on page 193), the Chinese system (Table 4.8 on page 195), and the Greek system (Table 4.9 on page 196) are all unsuccessful. Unlike our Hindu-Arabic, system, these systems are ot positional and contain no symbol for zero.

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CHAPTER 1 TEST

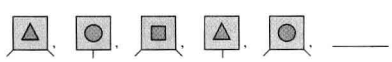
1. Find a counterexample to show that the following statement is false:
If a two-digit number is multiplied by a one-digit number, the answer is a two-digit number.

In Exercises 2–3, identify a pattern in each list of numbers. Then use this pattern to find the next number.

2. 0, 5, 10, 15, _

3. $\frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \frac{1}{48}$, _

4. Identify a pattern in the following sequence of figures. Then use the pattern to find the next figure in the sequence.



5. Consider the following procedure:
Select a number. Multiply the number by 4. Add 8 to the product. Divide the sum by 2. Subtract 4 from the quotient.

a. Repeat this procedure for three numbers of your choice. Write a conjecture that relates the result of the process to the original number selected.

b. Represent the original number by ■ or n, and use deductive reasoning to prove the conjecture in part (a).

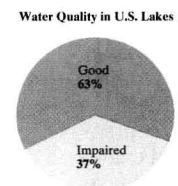
Work Exercises 6–9 without a calculator. Different rounding results in different estimates, so there is not a single correct answer to these exercises. Use rounding to make the resulting calculations simple.

6. For a spring break vacation, a student needs to spend $47.00 for gas, $311.00 for food, and $405.00 for a hotel room. If the student takes $681.79 from savings, estimate how much more money is needed for the vacation.

7. The cost for opening a restaurant is $485,000. If 19 people decide to share equally in the business, estimate the amount each must contribute.

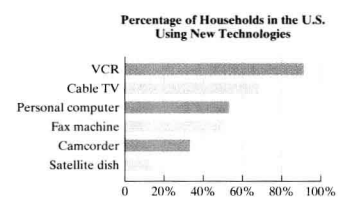
8. Find an estimate of 0.48992×120.

9. The circle graph shown is based on a survey of 17.1 million acres of lakes in the United States. Using this survey by the Environmental Protection Agency, estimate the number of acres of impaired lakes.



Source: U.S. Environmental Protection Agency

The bar graph shows the percentage of households in the United States using various technologies. Use the graph to solve Exercises 10–11.



Source: U.S. Consumer Electronics Industry

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Chapter Tests enable students to test their understanding of the contents from the chapter.

Resources

For the Instructor

Instructor's Solutions Manual

Daniel Miller, Niagara County Community College (NY)
Includes fully worked out solutions to the text exercises, both odds and evens.
ISBN: 0-13-009250-9

Test Item File

Tamarack Software, Inc.
Contains four different test forms for each chapter paralleling the chapter tests found in the text, plus an additional 125 to 150 test items for each chapter. It also contains teaching tips and suggested course outlines.
ISBN: 0-13-009261-4

TestGen-EQ

Tamarack Software, Inc.
An algorithmic test generator allows for multiple tests to be created from the same section objectives in the text.
Fully editable.
ISBN: 0-13-009299-1

PowerPoint Presentations

Jacquelyn White, St. Leo University
Designed to enhance an instructor's classroom presentation or create a framework on-line materials, these mini-presentations correspond to each chapter in the text. These materials can be edited by the user and, if desired, printed and used as overheads. The PowerPoint Presentations can be found on the Website.
www.prenhall.com/blitzer

Mathpak 5.0 Instructor Version

Tutorial, Homework and Assessment Anytime, Anywhere.
MathPak 5.0 features Mathpro 5.0, Prentice Halls revolutionary technology solution. Mathpro 5 is online, customizable tutorial and assessment software integrated with the text at the learning objective level. The easy-to-use course management system enables instructors to track and evaluate student performance on tutorial work, quizzes and tests. The MathPak CD launches to a premium website that features the SSM, Graphing Calculator Help, Clast Manual, as well as a wealth of tests and quizzes.
ISBN: 0-13-009302-5

For the Student

Student Solutions Manual

Daniel Miller, Niagara County Community College (NY)
Fully worked out solutions to the odd numbered text exercises.
ISBN: 0-13-044971-7

Lecture Videotapes

Jacquelyn White, St. Leo University
This comprehensive set of videos provides a short lecture and worked examples for almost every section in the text. These videos provide excellent support for students who missed class or need to study the material at their own pace. They are also an invaluable resource for distance learning or self-paced programs.
ISBN: 0-13-009304-1

Website

The Website provides additional student resources, including:

- On-line graphing calculator manual, including steps, screen grabs and keystrokes for the TI 81,83,85, 86, and 92 calculators as well as Casio and Sharp models.
- Additional exercises
- Chapter tests and quizzes
- Syllabus builder
- On-Line CLAST manual
- PowerPoint Presentations
- Electronic Study Guide
  www.prenhall.com/blitzer

CLAST Guide

James Wooland, Florida State University
This study guide helps students to prepare for the Florida College Level Academic Skills Test. Available in print format as well as electronically on the MathPak Website.
ISBN: 0-13-009290-8

MathPak 5.0 Student Version

Tutorial, Homework and Assessment Anytime, Anywhere.
MathPak 5.0 features Mathpro 5.0, Prentice Halls revolutionary technology solution. Mathpro 5 is online, customizable tutorial and assessment software integrated with the text at the learning objective level. The easy-to-use course management system enables instructors to track and evaluate student performance on tutorial work, quizzes and tests. The MathPak CD launches to a premium website that features the SSM, Graphing Calculator Help, Clast Manual, as well as a wealth of tests and quizzes.
ISBN: 0-13-009305-X

CD Lecture Series

Jacquelyn White, St. Leo University
This CD-based, comprehensive set of videos provides a short lecture and worked examples for almost every section in the text. These videos provide excellent support for students who missed class or need to study the material at their own pace. They are also an invaluable resource for distance learning or self-paced programs.
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Applications Index