

physics

*PART
TWO*

physics

PART TWO
THIRD EDITION

PROFESSOR OF PHYSICS
UNIVERSITY OF PITTSBURGH

DAVID HALLIDAY

PROFESSOR OF PHYSICS
RENSSALAER POLYTECHNIC INSTITUTE

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JOHN WILEY & SONS

NEW YORK SANTA BARBARA CHICHESTER BRISBANE TORONTO

SUPPLEMENTAL MATERIAL

Student Study Aid

Available for student use with *Physics* as well as with *Fundamentals of Physics* is the *Student Study Guide* by Williams, Brownstein, and Gray.
ISBN 94801-2

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Library of Congress Cataloging in Publication (Revised)

Resnick, Robert, 1923-

Physics.

Published in 1960 and 1962 under title: *Physics for students of science and engineering.*

Includes bibliographical references and index.

1. Physics. I. Halliday, David, joint author.

II. Title.

QC21.2.R47 1977 530 77-1295

ISBN 0-471-34529-6

preface to the third edition of part two

Physics is available in a single volume or in two separate parts; Part One includes mechanics, sound and heat, and Part Two includes electromagnetism, optics and quantum physics. The first edition was published in 1960 (*Physics for Students of Science and Engineering*) and the second in 1966 (*Physics*).

The text is intended for students studying calculus concurrently, such as students of science and engineering. The emphasis is on building a strong foundation in the principles of classical physics and on solving problems. Attention is given, however, to practical application, to the most modern theories, and to historical and philosophic issues throughout the book. This is accomplished by inclusion of special sections and thought questions, and by the entire manner of presentation of the material. There is a large set of worked-out examples, interspersed throughout the book, and an extensive collection of problems at the end of each chapter. Much care has been given to pedagogic devices that have proved effective for learning.

It has been eleven years since the publication of the second edition of *Physics*. During that time the book has continued to be well received throughout the world. We have had abundant correspondence with users over those years and concluded that a new edition is now appropriate.

In accordance with the increasing use of metric units in the United States and their general use throughout the world, we have greatly increased the emphasis on the metric system, using the *Système International* (SI) units and nomenclature throughout. Where it seems to be sensible, in this transition period for the United States, we retain some features of the British Engineering system.

The entire book was carefully reviewed for pedagogic improvement, based chiefly on the experiences of users — students and teachers, — and on the most recent scientific literature. As a result, we have rewritten selected areas significantly for improvements in presentation, accuracy, or physics. We have included new worked-out examples for topics or areas needing them. We have modernized all references, added new ones, and have improved many figures for greater clarity. The tables and the appendices have been expanded and updated to give newer data and more information than before. And we have added a supplementary topic on special relativity, in which the applications of this theory, scattered throughout Parts One and Two, are brought together as a cohesive whole.

Some subjects, not included in the second edition, are treated significantly in this third edition of Part Two. These include semiconductors, mutual inductance, earth magnetism, radio astronomy, virtual objects, and optical instruments. The long chapter on electromagnetic oscillations of the second edition has been divided into two chapters here, with extensive rewriting for greater clarity, and an entirely new chapter on alternating currents, for which there has been much demand, has been added.

As in Part One, we have made major improvements in the questions and the problems. In Parts One and Two combined, the number of questions has increased by 57%, from 778 in the second edition to 1219 in the present edition. For the problems the increase is 29%, from 1441 to 1864. Both problems and questions have been carefully edited, and most of the new ones have been classroom tested.

To assist students and teachers in organizing and evaluating the large number of problems, we have done several things. First, we have grouped problems within each chapter by section number, namely the first section needed to be covered in order to be able to work out the problem. Then, within each set of section problems, we have arranged the problems in the approximate order of increasing difficulty. Naturally, neither the assignment by section nor by difficulty is absolute, given different ways of solving some problems and different pedagogic values and tastes. We have coded the illustrations to the problems and have put the answers to the odd-numbered problems right at the end of these problems rather than at the end of the book. Finally, we have blended the supplementary problems, which appeared at the end of Part Two in the second edition, with the problems at the end of each chapter.

We have restyled the physical layout of the book to give it a less crowded appearance than formerly, making it easier now for the student to read the material, to make notations, and to differentiate between the various components of each chapter (text, figures, examples, tables, quotes, references, questions, problems, and so forth). We continue the practice of using somewhat reduced print for material which, in the context of a chapter, is of an advanced, specialized, or historical character.

We are grateful to John Wiley and Sons and to Donald Deneck, physics editor, for outstanding cooperation. We acknowledge the valuable assistance of Dr. Edward Derrinck with the problem sets and of Mrs. Carolyn Clemente with the wide range of secretarial services required. We are indebted to the many teachers and students who have sent us constructive criticisms of the 1966 edition and particularly to Robert P. Bauman, Kenneth Brownstein, Robert Karplus, and Brian A. McInnes, who have advised or assisted us in many ways. We hope that this third edition of *Physics* will contribute to the improvement of physics education.

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26 *charge and matter*

The science of electricity has its roots in the observation, known to Thales of Miletus in 600 B.C., that a rubbed piece of amber will attract bits of straw. The study of magnetism goes back to the observation that naturally occurring "stones" (that is, magnetite) will attract iron. These two sciences developed quite separately until 1820, when Hans Christian Oersted (1777–1851) observed a connection between them, namely, that an *electric* current in a wire can affect a *magnetic* compass needle (Section 33-1).

The new science of electromagnetism was developed further by many workers, of whom one of the most important was Michael Faraday (1791–1867). It fell to James Clerk Maxwell (1831–1879) to put the laws of electromagnetism in essentially the form in which we know them today. These laws, called *Maxwell's equations*, are displayed in Table 38-3, which you may want to examine at this time. These laws play the same role in electromagnetism that Newton's laws of motion and of gravitation do in mechanics.

Although Maxwell's synthesis of electromagnetism rests heavily on the work of his predecessors, his own contribution was central and vital. Maxwell deduced that light is electromagnetic in nature and that its speed can be found by making purely electric and magnetic measurements. Thus the science of optics was intimately connected with those of electricity and of magnetism. The scope of Maxwell's equations is remarkable, including as it does the fundamental principles of all large-scale electromagnetic and optical devices such as motors, radio, television, microwave radar, microscopes, and telescopes.

The development of classical electromagnetism did not end with Maxwell. The English physicist Oliver Heaviside (1850–1925) and es-

26-1 **ELECTROMAGNETISM— A PREVIEW**

pecially the Dutch physicist H. A. Lorentz (1853–1928) contributed substantially to the clarification of Maxwell's theory. Heinrich Hertz (1857–1894)* took a great step forward when, more than twenty years after Maxwell set up his theory, he produced in the laboratory electromagnetic "Maxwellian waves" of a kind that we would now call short radio waves. It remained for Marconi and others to exploit the practical application of the electromagnetic waves of Maxwell and Hertz.

Present interest in electromagnetism takes two forms. At the level of engineering applications Maxwell's equations are used constantly and universally in the solution of a wide variety of practical problems. At the level of the foundations of the theory there is a continuing effort to extend its scope in such a way that electromagnetism is revealed as a special case of a more general theory. Such a theory would also include (say) the theories of gravitation and of quantum physics. This grand synthesis has not yet been achieved.

The rest of this chapter deals with electric charge and its relationship to matter. We can show that there are *two kinds* of charge by rubbing a glass rod with silk and hanging it from a long thread as in Fig. 26-1. If a second rod is rubbed with silk and held near the rubbed end of the first rod, the rods will repel each other. On the other hand, a rod of plastic (Lucite, say) rubbed with fur will *attract* the glass rod. Two plastic rods rubbed with fur will repel each other. We explain these facts by saying that rubbing a rod gives it an *electric charge* and that the charges on the two rods exert forces on each other. Clearly the charges on the glass and on the plastic must be different in nature.

Benjamin Franklin (1706–1790), who, among his other achievements, was the first American physicist†, named the kind of electricity that appears on the glass *positive* and the kind that appears on the plastic (sealing wax or shell-lac in Franklin's day) *negative*; these names have remained to this day. We can sum up these experiments by saying that *like charges repel and unlike charges attract*.

Electric effects are not limited to glass rubbed with silk or to plastic rubbed with fur. Any substance rubbed with any other under suitable conditions will become charged to some extent; by comparing the unknown charge with a glass rod which had been rubbed with silk or a plastic rod which had been rubbed with fur, it can be labeled as either positive or negative.

The modern view of bulk matter is that, in its normal or neutral state, it contains equal amounts of positive and negative electricity. If two bodies like glass and silk are rubbed together, a small amount of charge is transferred from one to the other, upsetting the electric neutrality of each. In this case the glass would become positive, the silk negative.

* "Heinrich Hertz," by P. and E. Morrison, *Scientific American*, December 1957.

† To learn about practical applications of static electric charges, as in fly-ash precipitators, paint sprayers, electrostatic copying machines, etc., see "Modern Electrostatics" by A. W. Bright, *Physics Education*, 9, 381 [1974], and "Electrostatics" by A. D. Moore, *Scientific American*, March 1972.

‡ The science historian I. Bernard Cohen of Harvard University says of Franklin in his book *Franklin and Newton*: "To say . . . that had Franklin 'Not been famous as a publisher and a statesman, he might never have been heard of as a scientist,' is absolutely wrong. Just the opposite is more nearly the case; his international fame and public renown as a scientist was in no small measure responsible for his success in international statesmanship." See also "The Lightning Discharge" by Richard E. Orville, *The Physics Teacher*, January 1976 for a description of Franklin's famous kite experiment and a review of modern concepts about the nature of lightning.

26-2 ELECTRIC CHARGE†

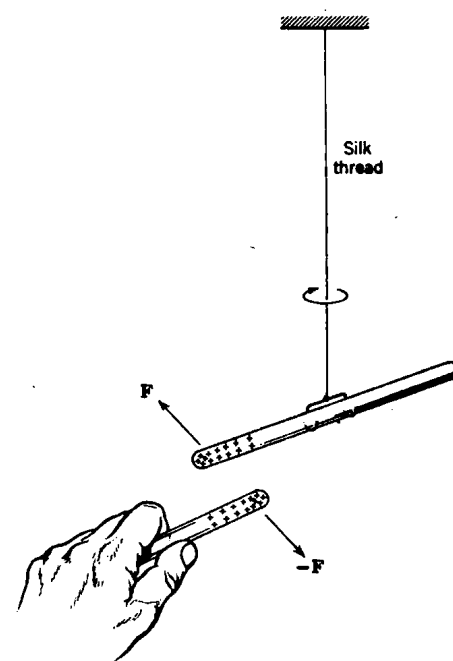


figure 26-1
Two positively charged glass rods repel each other.

A metal rod held in the hand and rubbed with fur will not seem to develop a charge. It is possible to charge such a rod, however, if it is furnished with a glass or plastic handle and if the metal is not touched with the hands while rubbing it. The explanation is that metals, the human body, and the earth are *conductors* of electricity and that glass, plastics, etc., are *insulators* (also called *dielectrics*).

In conductors electric charges are free to move through the material, whereas in insulators they are not. Although there are no perfect insulators, the insulating ability of fused quartz is about 10^{25} times as great as that of copper, so that for many practical purposes some materials behave as if they were perfect insulators.

In metals a fairly subtle experiment called the Hall effect (see Section 33-5) shows that only negative charge is free to move. Positive charge is as immobile as it is in glass or in any other dielectric. The actual charge carriers in metals are the *free electrons*. When isolated atoms are combined to form a metallic solid, the outer electrons of the atom do not remain attached to individual atoms but become free to move throughout the volume of the solid. For some conductors, such as electrolytes, both positive and negative charges can move.

A class of materials called *semiconductors* is intermediate between conductors and insulators in its ability to conduct electricity. Among the elements, silicon and germanium are well-known examples. Semiconductors have many practical applications, among which is their use in the construction of transistors. The way a semiconductor works cannot be described adequately without some understanding of the basic principles of quantum physics. Figure 26-2, however, suggests the principal features of the distinction between conductors, semiconductors, and insulators.

In solids, electrons have energies that are restricted to certain levels, the levels being confined to certain bands. The intervals between bands are forbidden, in the sense that electrons in the solid may not possess such energies. Electrons are assigned two to a level and they may not increase their energy (which means that they may not move freely through the solid) unless there are empty levels at higher energies into which they can readily move.

Figure 26-2a shows a conductor, such as copper. Band 1 is only partially filled so that electrons can easily move to the higher empty levels and thus travel through the solid. Figure 26-2b shows a (intrinsic) semiconductor such as silicon. Here band 1 is completely filled but band 2 is so close energetically that electrons can easily "jump" (absorbing energy from, say, thermal fluctuations) into the unfilled levels of that band. Figure 26-2c shows an insulator, such as sodium chloride. Here again band 1 is filled, but band 2 is too far above band 1 energetically to permit any appreciable number of the band-1 electrons to jump the energy gap.

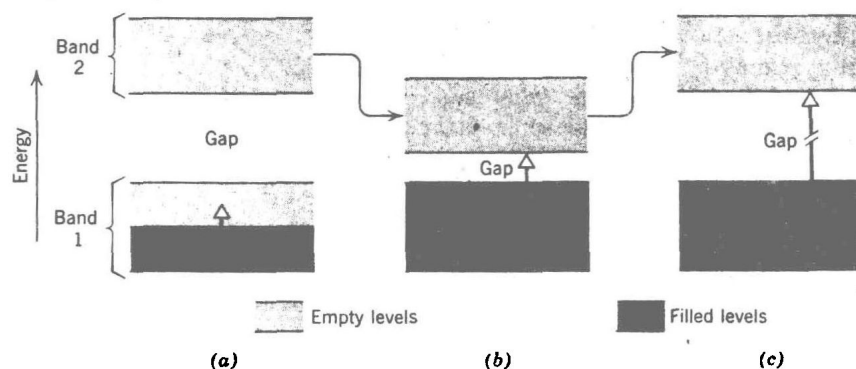


figure 26-2

Suggesting (a) a conductor, (b) an intrinsic semiconductor, and (c) an insulator. In (b) the gap is relatively small but in (c) it is relatively large. In intrinsic semiconductors the electrical conductivity can often be greatly increased by adding very small amounts of other elements such as arsenic or boron, a process called "doping."

Charles Augustin Coulomb (1736–1806) measured electrical attractions and repulsions quantitatively and deduced the law that governs them. His apparatus, shown in Fig. 26-3, resembles the hanging rod of Fig. 26-1, except that the charges in Fig. 26-3 are confined to small spheres a and b .

If a and b are charged, the electric force on a will tend to twist the suspension fiber. Coulomb canceled out this twisting effect by turning the suspension head through the angle θ needed to keep the two charges at the particular distance apart in which he was interested. The angle θ is then a relative measure of the electric force acting on charge a . The device of Fig. 26-3 is called a *torsion balance*; a similar arrangement was used later by Cavendish to measure gravitational attractions (Section 16-3).

Coulomb's first experimental results can be represented by

$$F \propto \frac{1}{r^2}.$$

Here F is the magnitude of the interaction force that acts on each of the two charges a and b ; r is their distance apart. These forces, as Newton's third law requires, act along the line joining the charges but point in opposite directions. Note that the magnitude of the force on each charge is the same, even though the charges may be different.

The force between charges depends also on the magnitude of the charges. Specifically, it is proportional to their product. Although Coulomb did not prove this rigorously, he implied it and thus we arrive at

$$F \propto \frac{q_1 q_2}{r^2}, \quad (26-1)$$

where q_1 and q_2 are relative measures of the charges on spheres a and b . Equation 26-1, which is called *Coulomb's law*, holds only for charged objects whose sizes are much smaller than the distance between them. We often say that it holds only for *point charges*.

Coulomb's law resembles the inverse square law of gravitation which was already more than 100 years old at the time of Coulomb's experiments; q plays the role of m in that law. In gravity, however, the forces are always attractive; this corresponds to the fact that there are two kinds of electricity but (apparently) only one kind of mass.

Our belief in Coulomb's law does not rest quantitatively on Coulomb's experiments. Torsion balance measurements are difficult to make to an accuracy of better than a few percent. Such measurements could not, for example, convince us that the exponent in Eq. 26-1 is exactly 2 and not, say, 2.01. In Section 28-7 we show that Coulomb's law can also be deduced from an indirect experiment (1971) which shows that the exponent in Eq. 26-1 lies between the limits $2 \pm 3 \times 10^{-16}$.

Although we have established the physical concept of electric charge, we have not yet defined a unit in which it may be measured. It is possible to do so operationally by putting equal charges q on the spheres of a torsion balance and by measuring the magnitude F of the force that acts on each when the charges are a measured distance r apart. One could then define q to have a unit value if a unit force acts on each charge when the charges are separated by a unit distance and one can give a name to the unit of charge so defined.*

* This scheme is the basis for the definition of the unit of charge called the *statcoulomb*. However, in this book we do not use this unit or the systems of units of which it is a part; see Appendix L, however.

26-4 COULOMB'S LAW

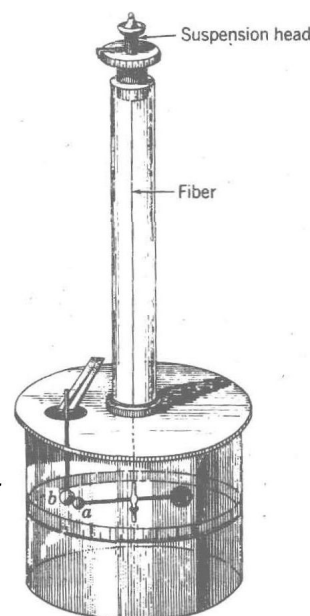


figure 26-3
Coulomb's torsion balance, from his 1785 memoir to the French Academy of Sciences.

For practical reasons having to do with the accuracy of measurements, the SI unit of charge is not defined using a torsion balance but is derived from the unit of electric current. If the ends of a long wire are connected to the terminals of a battery, it is common knowledge that an *electric current* i is set up in the wire. We visualize this current as a flow of charge. The SI unit of current is the *ampere* (abbr. A). In Section 34-4 we describe the operational procedures in terms of which the ampere is defined.

The SI unit of charge is the *coulomb* (abbr. C). A coulomb is defined as the amount of charge that flows through any cross section of a wire in 1 second if there is a steady current of 1 ampere in the wire. In symbols

$$q = it, \quad (26-2)$$

where q is in coulombs if i is in amperes and t is in seconds. Thus, if a wire is connected to an insulated metal sphere, a charge of 10^{-6} C can be put on the sphere if a current of 1.0 A exists in the wire for 10^{-6} s.

A copper penny has a mass of 3.1 g. Being electrically neutral, it contains equal amounts of positive and negative electricity. What is the magnitude q of these charges? A copper atom has a positive nuclear charge of 4.6×10^{-18} C and a negative electronic charge of equal magnitude.

The number N of copper atoms in a penny is found from the ratio

$$\frac{N}{N_0} = \frac{m}{M},$$

where N_0 is the Avogadro number, m the mass of the coin, and M the atomic weight of copper. This yields, solving for N ,

$$N = \frac{(6.0 \times 10^{23} \text{ atoms/mole})(3.1 \text{ g})}{64 \text{ g/mole}} = 2.9 \times 10^{22} \text{ atoms}.$$

The charge q is

$$q = (4.6 \times 10^{-18} \text{ C/atom})(2.9 \times 10^{22} \text{ atoms}) = 1.3 \times 10^5 \text{ C}.$$

In a 100-watt, 110-volt light bulb the current is 0.91 ampere. Verify that it would take 40 h for a charge of this amount to pass through this bulb.

EXAMPLE 1

Equation 26-1 can be written as an equality by inserting a constant of proportionality. Instead of writing this simply as, say, k , it is usually written in a more complex way as $1/4\pi\epsilon_0$ or

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}. \quad (26-3)$$

Certain equations that are derived from Eq. 26-3, but are used more often than it is, will be simpler in form if we do this.

In SI units we can measure q_1 , q_2 , r , and F in Eq. 26-3 in ways that do not depend on Coulomb's law. Numbers with units can be assigned to them. There is no choice about the so-called *permittivity constant* ϵ_0 ; it must have that value which makes the right-hand side of Eq. 26-3 equal to the left-hand side. This (measured) value turns out to be*

$$\epsilon_0 = 8.854187818 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2.$$

* For practical reasons this value is not actually measured by direct application of Eq. 26-3 but in an equivalent although more circuitous way.

In this book the value $8.9 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$ will be accurate enough for most problems. For direct application of Coulomb's law or in any problem in which the quantity $1/4\pi\epsilon_0$ occurs we may use, with sufficient accuracy for this book,

$$1/4\pi\epsilon_0 = 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2.$$

Let the total positive and the total negative charges in a copper penny be separated to a distance such that their force of attraction is 1.0 lb (= 4.5 N). How far apart must they be?

We have (Eq. 26-3)

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}.$$

Putting $q_1 q_2 = q^2$ (see Example 1) and solving for r yields

$$\begin{aligned} r &= q \sqrt{\frac{1/4\pi\epsilon_0}{F}} = 1.3 \times 10^5 \text{ C} \sqrt{\frac{9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{4.5 \text{ N}}} \\ &= 5.8 \times 10^9 \text{ m} = 3.6 \times 10^6 \text{ mi.} \end{aligned}$$

This is 910 earth radii and it suggests that it is not possible to upset the electrical neutrality of gross objects by any very large amount. What would be the force between the two charges if they were placed 1.0 m apart?

If more than two charges are present, Eq. 26-3 holds for every pair of charges. Let the charges be q_1, q_2, q_3 , etc.; we calculate the force exerted on any one (say q_1) by all the others from the vector equation

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_{14} + \cdots, \quad (26-4)$$

where \mathbf{F}_{12} , for example, is the force exerted on q_1 by q_2 .

Figure 26-4 shows three fixed charges q_1, q_2 , and q_3 . What force acts on q_1 ? Assume that $q_1 = -1.0 \times 10^{-6} \text{ C}$, $q_2 = +3.0 \times 10^{-6} \text{ C}$, $q_3 = -2.0 \times 10^{-6} \text{ C}$, $r_{12} = 15 \text{ cm}$, $r_{13} = 10 \text{ cm}$, and $\theta = 30^\circ$.

From Eq. 26-3, ignoring the signs of the charges, since we are interested only in the magnitudes of the forces,

$$\begin{aligned} F_{12} &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \\ &= \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.0 \times 10^{-6} \text{ C})(3.0 \times 10^{-6} \text{ C})}{(1.5 \times 10^{-1} \text{ m})^2} \\ &= 1.2 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{and } F_{13} &= \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.0 \times 10^{-6} \text{ C})(2.0 \times 10^{-6} \text{ C})}{(1.0 \times 10^{-1} \text{ m})^2} \\ &= 1.8 \text{ N.} \end{aligned}$$

The directions of \mathbf{F}_{12} and \mathbf{F}_{13} are as shown in the figure.

The components of the resultant force \mathbf{F}_1 acting on q_1 (see Eq. 26-4) are

$$\begin{aligned} F_{1x} &= F_{12x} + F_{13x} = F_{12} + F_{13} \sin \theta \\ &= 1.2 \text{ N} + (1.8 \text{ N})(\sin 30^\circ) = 2.1 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{and } F_{1y} &= F_{12y} + F_{13y} = 0 - F_{13} \cos \theta \\ &= -(1.8 \text{ N})(\cos 30^\circ) = -1.6 \text{ N.} \end{aligned}$$

Find the magnitude of \mathbf{F}_1 and the angle it makes with the x-axis.

EXAMPLE 2

EXAMPLE 3

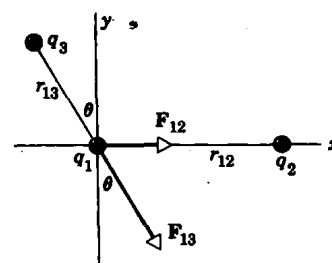


figure 26-4

Example 3. Showing the forces exerted on q_1 by q_2 and q_3 .

In Franklin's day electric charge was thought of as a continuous fluid, an idea that was useful for many purposes. The atomic theory of matter, however, has shown that fluids themselves, such as water and air, are not continuous but are made up of atoms. Experiment shows that the "electric fluid" is not continuous either but that it is made up of integral multiples of a certain minimum electric charge. This fundamental charge, to which we give the symbol e , has the magnitude $1.6021892 \times 10^{-19} \text{C}$. Any physically existing charge q , no matter what its origin, can be written as ne where n is a positive or a negative integer.

When a physical property such as charge exists in discrete "packets" rather than in continuous amounts, the property is said to be *quantized*. Quantization is basic to modern quantum physics. The existence of atoms and of particles such as the electron and the proton indicates that mass is quantized also. Later you will learn that several other properties prove to be quantized when suitably examined on the atomic scale; among them are energy and angular momentum.

The quantum of charge e is so small that the "graininess" of electricity does not show up in large-scale experiments, just as we do not realize that the air we breathe is made up of atoms. In an ordinary 110-volt, 100-watt light bulb, for example, 6×10^{18} elementary charges enter and leave the bulb every second.

There exists today no theory that predicts the quantization of charge (or the quantization of mass, that is, the existence of fundamental particles such as protons, electrons, muons, and so on). Even assuming quantization, the classical theory of electromagnetism and Newtonian mechanics are incomplete in that they do not correctly describe the behavior of charge and matter on the atomic scale. The classical theory of electromagnetism, for example, describes correctly what happens when a bar magnet is thrust through a closed copper loop; it fails, however, if we wish to explain the magnetism of the bar in terms of the atoms that make it up. The more detailed theories of quantum physics are needed for this and similar problems.

Matter as we ordinarily experience it can be regarded as composed of three kinds of particles, the proton, the neutron, and the electron. Table 26-1 shows their masses and charges. Note that the masses of the neutron and the proton are approximately equal but that the electron is less massive by a factor of about 1840.

26-5 CHARGE IS QUANTIZED

26-6 CHARGE AND MATTER

Table 26-1
Some properties of three particles

Particle	Symbol	Charge	Mass
Proton	p	$+e$	$1.6726485 \times 10^{-27} \text{ kg}$
Neutron	n	0	$1.6749543 \times 10^{-27} \text{ kg}$
Electron	e^-	$-e$	$9.109534 \times 10^{-31} \text{ kg}$

Atoms are made up of a dense, positively charged *nucleus*, surrounded by a cloud of electrons, see Fig. 26-5. The nucleus varies in radius from about $1 \times 10^{-15} \text{ m}$ for hydrogen to about $7 \times 10^{-15} \text{ m}$ for the heaviest atoms. The outer diameter of the electron cloud, that is, the diameter of the atom, lies in the range $1\text{--}3 \times 10^{-10} \text{ m}$, about 10^5 times larger than the nuclear diameter.