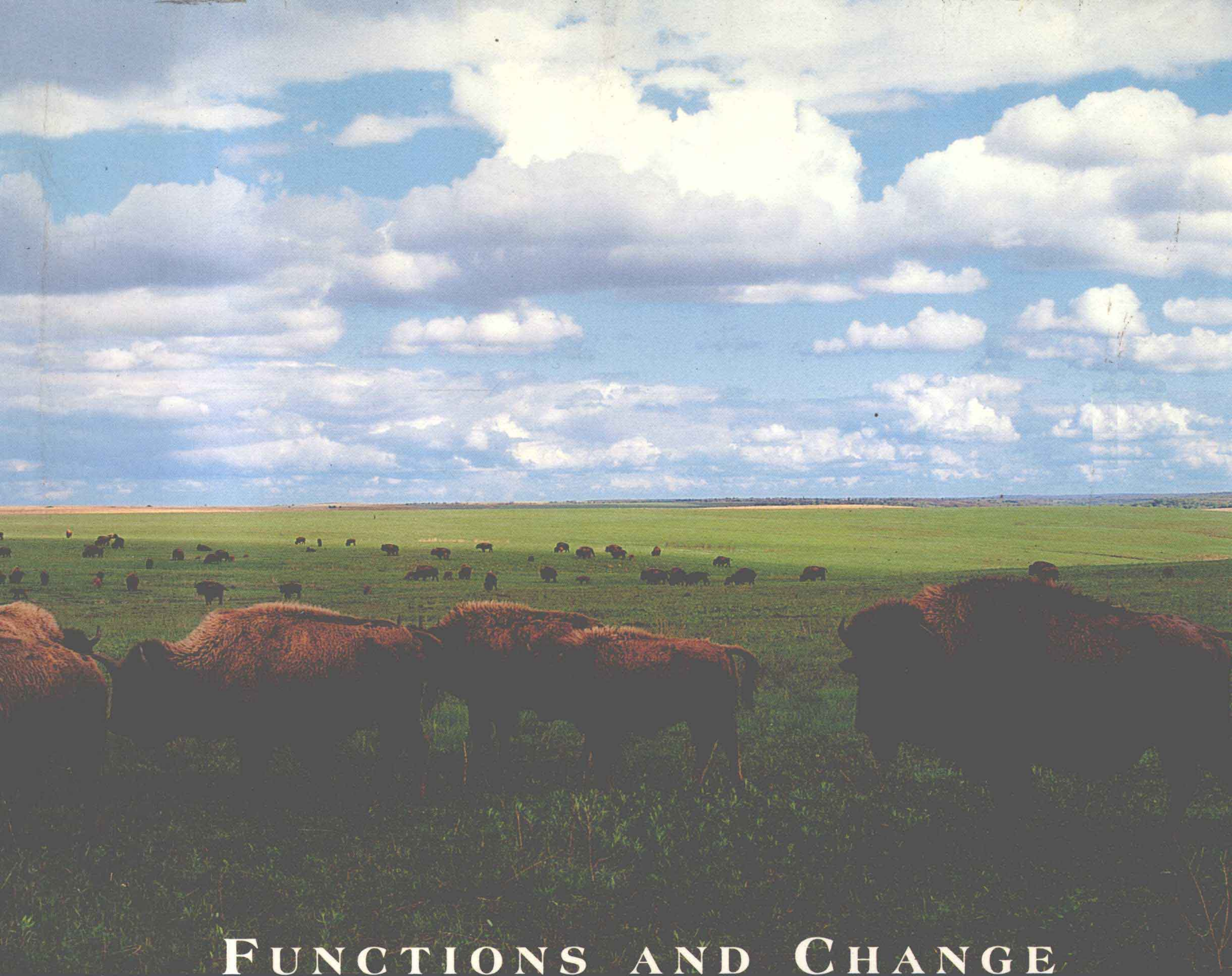

Functions and Change

A Modeling Alternative to College Algebra

Preliminary Edition

Crauder / Evans / Noell

FUNCTIONS AND CHANGE

A Modeling Alternative to College Algebra

Preliminary Edition

Bruce Crauder / *Benny Evans* / *Alan Noell*

Oklahoma State University

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by the
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Opinions expressed are those of the authors
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Gratitude and Thanks

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PREFACE

To Instructors

Mathematics is among the purest and most powerful of sciences, an art form of unsurpassed beauty, and a descriptive language that codifies ideas from many areas, including business, engineering, and the natural, physical, and social sciences, alway showing that major concepts drawn from many different sources are in their essence the same. Mathematics pervades modern society and, to the knowledgeable eye, is everywhere evident in nature.

Practicing engineers, mathematicians, and scientists require a deep understanding of mathematics as well as a level of exactitude and facility with symbol manipulation that is sometimes difficult for entering students to master. For many, frustration with these aspects of mathematics obscures its power, beauty, and utility. But modern technology in the form of graphing calculators or computers can supplant much of the drudgery of mathematics, move the focus toward important concepts and ideas, and make mathematics more accessible.

> The goal of this text is to use technology and informal descriptions to empower entering students with mathematics as a descriptive problem-solving tool, and to reveal mathematics as an integral part of nature, science, and society.

Mathematics in context: style, pedagogy, and topics

This text differs from traditional textbooks in many ways. A quick glance through the book shows that there are more words and fewer formulas than usual. Also evident are the extensive examples and problems. The choices of examples and exercises are part of an important theme in the text: mathematics is easily learned from carefully chosen, realistic examples. In general, mathematical principles are developed through examples. Only after the examples give good intuition and understanding are more general and abstract notions made explicit. In practice, this style has worked very well at Oklahoma State University, as well as at the other schools class testing earlier drafts. Students are able to read the text and understand the examples. Students have opinions and bring their own independent understanding of the topics in the examples and exercises, so classroom discussions are more lively.

This text easily accommodates pedagogies other than traditional lecture. Because so much of the text focuses on examples, spending class time in discussions or working in groups is easy. The examples are realistic, which promotes bringing outside materials into the classroom and having students find examples from their other classes.

This course arose through an effort to provide students with the mathematical tools they will need in courses that traditionally require college algebra. The topics were selected after lengthy consultations with our colleagues from departments across the campus. The skills our colleagues wanted students to learn from an entry-level mathematics course included facility with graphs, tables of values, linear algebraic manipulations, and, most importantly, some level of confidence in relating sentences to formulas, tables, and graphs. Perhaps surprisingly, our colleagues also wanted a qualitative understanding of rates of change, leading us to make this one of the most inportant themes in the text. Rates of change is a unifying concept, following from the observation that knowledge of the initial value and how a function changes are sufficient to understand the function. This concept is fully developed and exploited for linear and exponential functions in Chapters 3 and 4 and is carried through the rest of the text.

Our consultations also indicated that these goals are met only marginally, if at all, by a traditional college algebra course. The most common perception we got from our colleagues in other areas is that even when entering students have some facility with elementary mathematics, they may be afraid to apply it and indeed may see no relation at all between mathematics that they have been exposed to and applications of mathematics in other areas. As a result, compared to traditional college algebra texts, our treatment of topics is more geared toward applications in other disciplines. For example, we often use data tables to display functions, and students become proficient in recovering the formula for the function from data.

One of the most important goals of this text is to provide students with the opportunity to succeed at sophisticated mathematics. Our experience with the material in its fourth year of use is that the appropriately used calculator gives students the power needed to perform significant mathematical analyses. Many of our students report that success in mathematics is a new and finally pleasing experience. We also wanted students to see mathematics as part of their life experience, and we have anecdotal stories to indicate some success in that area as well.

This text was not designed as a prerequisite for calculus, but, as it develops, the text provides an excellent preparation for some of the newer reformed applied calculus texts. We are using it as such at Oklahoma State University and are pleased with the results.

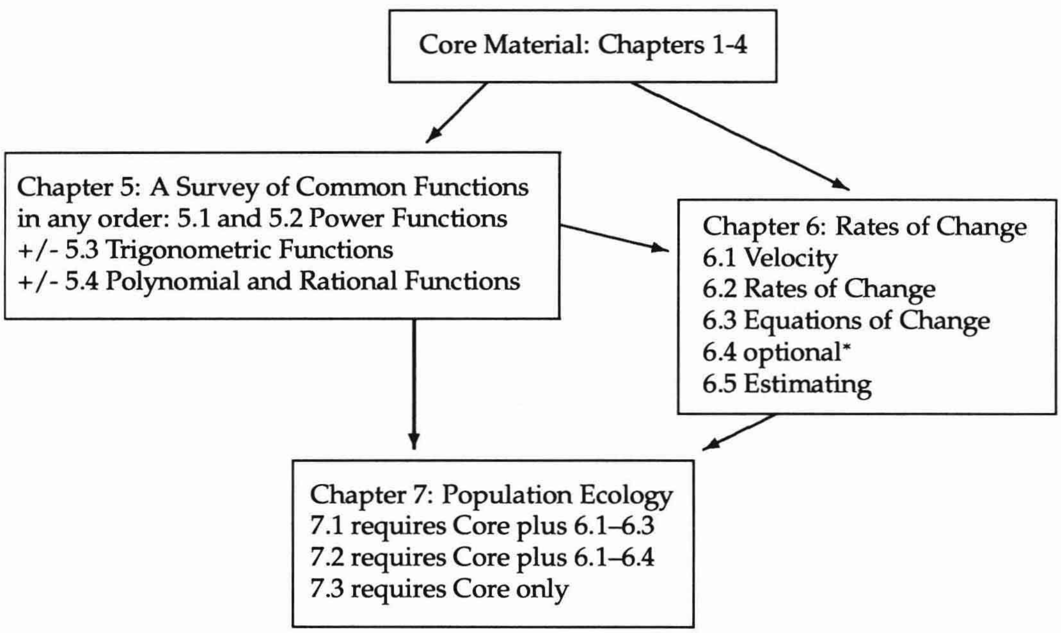
Graphing calculator use and reference

This text is designed to be used with a graphing calculator, and the calculator is an essential part of the presentation as well as the exercises. Throughout the text we employ TI-83 screens, but many graphing calculators and even some computer software can serve the purpose. Because the major graphing calculator producers have made their products remarkably powerful and easy to use, students should have no difficulty becoming proficient with the basic calculator operation. To ensure this, an accompanying *Keystroke Guide* provides TI-82 and

TI-83 keystrokes for creating tables and graphs, entering expressions, etc., in the Quick Reference section of the *Guide*. Keystrokes are cross-referenced using footnote boxes in the main text. For example, [3.4] on page 175 of the text indicates that the keystrokes needed to to create this graph are found in item number **3.4** of the Quick Reference pages of the *Guide*. We recommend removing the appropriate Quick Reference sheets from the *Keystroke Guide* for consultation while reading the text. The *Keystroke Guide* also includes generic instructions to help students become familiar with graphing calculators in general, along with related exercises.

Suggested paths through this text

The seven chapters in this text provide more than enough material for a one-semester course with three credit hours. Chapters 1 though 4 form the core material. In the context of rates of change, we examine functions from several points of view and study in some detail the important examples of linear and exponential functions. After covering the core material, instructors have some options, as shown below.



***Note:** The discussion of graphical solutions for equations of change (i.e., differential equations) in Section 6.4 is the most challenging material in this text. (The authors believe it may also be the most rewarding.) It is necessary for parts of the presentation in Section 7.2 but otherwise may be omitted at the instructor's discretion. In particular, Section 6.5 on calculating rates of change does not depend on this discussion of equations of change. At Oklahoma State University we have found that students respond well to Chapter 6, and examining the texts used in courses from ecology to economics shows how useful the development of rates of change can be to students from a diverse collection of backgrounds.

Supplementary materials

- *Graphing Calculator Keystroke Guide and Drill Exercise Supplement* includes:
  - generic instructions to help students become familiar with their graphing calculator, along with related exercises.
  - TI-82 and TI-83 keystrokes for creating tables and graphs, entering expressions, etc., in the Quick Reference section of the *Guide*. Keystrokes are cross-referenced using footnote boxes in the main text. For example, [3.4] on page 175 of the text indicates that the keystrokes needed to create this graph are found in item number **3.4** of the Quick Reference pages of the *Guide*.
  - supplementary drill exercises for each chapter for students needing extra practice.
- *Student Solutions Guide* includes complete solutions to all odd-numbered exercises.
- *Complete Solutions Guide* includes complete solutions to all exercises for instructors.
- *Instructor's Resource Guide with Tests* includes general teaching tips for adapting to this book's approach, specific section-by-section teaching tips, sample tests and quizzes, and transparency masters.

To Students

Every effort has been made to show mathematics as you are likely to encounter it in other courses as well as in daily life. Learning mathematics requires effort. But learning is also fun, and success in mathematics can be rewarding in terms of personal accomplishment. It can also facilitate understanding in other courses as well as in everyday experience, and it may be a key to attaining your career goals. We intend that you reap these and other benefits from your experience with this text.

How to learn with this text

Effective use of this text requires that you actively participate in the presentation. You should read with your graphing calculator turned on and with the Quick Reference pages from the *Keystroke Guide* handy. (Please see the description of this supplement above to see what it teaches and how it works.) It is not sufficient simply to read the examples in the text. Rather, you should work through each example yourself as it is presented, and when a calculator screen is shown, you should reproduce it on your own calculator.

As you begin, you will note that rarely is the final answer for an example presented simply as a number or a graph. Rather the answer is accompanied by sentences explaining how the answer was obtained and with appropriate conclusions. You should follow this pattern in solving the exercises. Your solution should include whatever calculations, graphs, or tables (copied from your calculator) you use as well as a clear statement of your conclusion accompanied by an explanation of your methods. A simple test of the clarity of your explanation is whether your peers can understand the solution by reading your work.

Our answers to the odd-numbered exercises are provided at the back of the text. They are of necessity brief and in general not acceptable as complete solutions. You should also be aware that for many of the exercises, there is no simple *right answer*. Instead, there is room for a number of conclusions, and any of them may be acceptable provided they are accompanied by a convincing argument. Sometimes you may have an answer that is different from that of one of your peers, and neither of your answers matches the one given in the back of the book, yet both of you may have correct solutions.

Mathematics is a tool that enhances your reasoning ability. It does not supplant that ability, and it is not a device that gives magical, unassailable answers. Whenever you are led to a conclusion that flies in the face of common sense, you should question the validity of your work and check carefully for a mistake.

A solicitation

This text is designed to be read by students, and while we are very much interested in input from instructors, the evolution of the text into its final form will be heavily influenced by what you have to say. We earnestly solicit any and all comments about the presentation. We would like your reactions to topics included or omitted as well as your estimation of the effectiveness of the presentation. We appreciate hearing about any errors, omissions, or inaccuracies in this preliminary edition. The best way to get information to us is through e-mail to `crauder`, `bevans`, or `noell`, each at `@math.okstate.edu`.

Thanks

Class Testers In addition to being used at our school, earlier drafts of *Functions and Change* were class tested at the following schools. We would like to thank the following instructors, some of whom have been using various drafts for several years, for being willing to try something different and for providing feedback based on their experiences. Thanks also to the students at these schools for their participation.

Bill Coberly	University of Tulsa
Doug Colbert	University of Nevada at Reno
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Mary Jane Sterling	Bradley University
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In writing this book, we have also relied on the help of other mathematicians as well as specialists from agriculture, biology, business, chemistry, ecology, economics, engineering, physics, political science, and zoology. We offer our thanks to Bruce Ackerson, Brian Adam, Robert Darcy, Joel Haack, Stanley Fox, Adrienne Hyle, Smith Holt, Jerry Johnson, Lionel Raff, Scott Turner, and Gary Young. Errors and inaccuracies in applications are due to the authors' misrepresentation of correct information provided by our able consultants. We are grateful to the National Science Foundation for its foresight and support of initial development and to Oklahoma State University for its support. We very much appreciate Charles Hartford's continuing patience and good humor through some trying times.

The most important participants in the development of this work are the students at Oklahoma State University, particularly those in the fall of 1995 and spring of 1996, who suffered through very early versions of this text, and whose input has shaped the current version. This book is written for entering mathematics students, and further student reaction will direct the evolution of this preliminary edition into a better product. Students and teachers at Oklahoma State University have had fun and learned with this material. We hope the same happens for others.

BRUCE CRAUDER

BENNY EVANS

ALAN NOELL

CONTENTS

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PROLOGUE *Calculator Arithmetic*

Graphing calculators provide powerful tools for mathematical analysis, and this power has profound effects on how modern mathematics and its applications are done. Many mathematical applications which traditionally required sophisticated mathematical development can now be successfully analyzed at an elementary level. Indeed, modern calculating power places entering students in a position to attack problems that in the past would have been considered too complicated. The first step is to become proficient with arithmetic on the calculator. In this chapter we discuss key mathematical ideas associated with calculator arithmetic. Chapter 1 of the *Keystroke Guide* is intended to provide additional help for those who may be new to the operation of the calculator, or who need a brief refresher on arithmetic operations.

Typing mathematical expressions

When we write expressions such as $\frac{71}{7} + 3^2 \times 5$ using pen and paper, the paper serves as a two-dimensional display, and we can express fractions by putting one number on top of another and exponents by using a superscript. When such expressions are entered on a computer, calculator, or typewriter, they must be written on a single line using special symbols and often additional parentheses. The *caret* symbol $\wedge$ is commonly used to denote an exponent, so in *typewriter notation* $\frac{71}{7} + 3^2 \times 5$ comes out as

$$71 \div 7 + 3 \wedge 2 \times 5\ .$$

In Figure 0.1, we have entered $\boxed{0.1}$ this expression, and the resulting answer 55.14285714 is shown in Figure 0.2. You should use your calculator to verify that we did it correctly. (Since this is its first occurrence, we will point out that the footnote symbol in a box $\boxed{0.1}$ indicates that the exact keystrokes for doing this are shown on the Quick Reference pages of the *Keystroke Guide*.)

Figure 0.1: *Entering* $\frac{71}{7} + 3^2 \times 5$

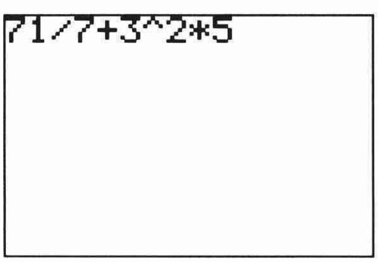
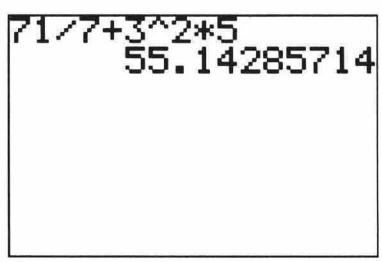


Figure 0.2: *The value of* $\frac{71}{7} + 3^2 \times 5$



Rounding

When a calculation yields a long answer such as 55.14285714 in Figure 0.2, we will commonly shorten it to a more manageable size by *rounding*. Rounding means that we keep a few of the digits after the decimal point, possibly changing the last one, and discard the rest. There is no set rule for how many digits after the decimal point you should keep; in practice it depends on how much accuracy you need in your answer as well as the accuracy of input data. As a general rule in this text we will round to two decimal places. Thus for

$$\frac{71}{7} + 3^2 \times 5 = 55.14285714$$

we would report the answer as 55.14.

In order to make the abbreviated answer more accurate, it is standard practice to increase the last decimal entry by one if the following entry is 5 or greater. Verify with your calculator that

$$\frac{58.7}{6.3} = 9.317460317\ .$$

In this answer the next digit after 1 is 7, which indicates that we should round up, and so we would report the answer rounded to two decimal places as 9.32. Note that in reporting 55.14 as the rounded answer above, we followed this same rule. The next digit after 4 in 55.14285714 is 2, which does not indicate that we should round up.

To provide additional emphasis for this idea, we have shown in Figure 0.3 a calculation where rounding does not change the last reported digit, and in Figure 0.4 a calculation where rounding requires that the last reported digit be changed.

Figure 0.3: An answer that will be reported as 1.71

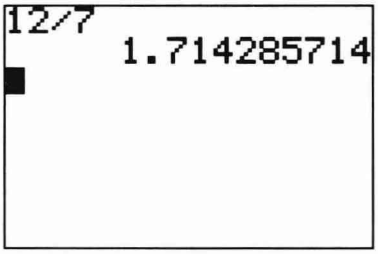
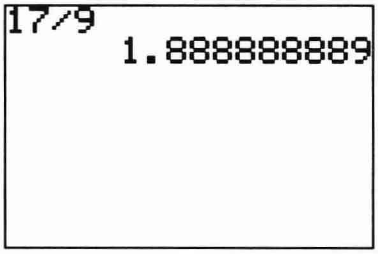


Figure 0.4: An answer that will be reported as 1.89



KEY IDEA 0.1: ROUNDING

When reporting complicated answers, we will adopt the convention of keeping two places beyond the decimal point. The last digit is increased by one if the next following number is 5 or greater.

Scientific notation

It is cumbersome to write down all the digits of some very large or very small numbers. A prime example of such a large number is *Avogadro's number*, which is the number of atoms in 12 grams of carbon 12. Its value is about

$$602,000,000,000,000,000,000,000\,.$$

An example of a small number which is awkward to write is the mass in grams of an electron:

0.000 000 000 000 000 000 000 000 000 000 911 grams.

Scientists and mathematicians usually express such numbers in a more compact form using *scientific notation*. In this notation, numbers are written in a form with one nonzero digit to the left of the decimal point times a power of 10. Examples of numbers written in scientific notation are 2.7×10^4 and 2.7×10^{-4}. The power of 10 tells how the decimal point should be moved in order to write out the number longhand. The 4 in 2.7×10^4 means that we should move the decimal point 4 places to the right. Thus

$$2.7 \times 10^4 = 27,000$$

since we move the decimal point four places to the right. When the exponent on 10 is negative, the decimal point should be moved to the left. Thus

$$2.7 \times 10^{-4} = 0.00027$$

since we move the decimal point four places to the left. With this notation, Avogadro's number comes out as 6.02×10^{23}, and the mass of an electron as 9.11×10^{-31} grams.

Many times calculators display numbers like this but use a different notation for the power of 10. For example, Avogadro's number 6.02×10^{23} is displayed as 6.02E23, and the mass in grams of an electron 9.11×10^{-31} is shown as 9.11E-31. In Figure 0.5 we have calculated 2^{50}. The answer reported by the calculator written in longhand is 1,125,899,907,000,000. In presenting the answer in scientific notation, it would in many settings be appropriate to round to two decimal places as 1.13×10^{15}. In Figure 0.6 we have calculated $\frac{7}{3^{20}}$. The answer reported there equals 0.000 000 002 007 580 394. If we write it in scientific notation and round to two decimal places, we get 2.01×10^{-9}.