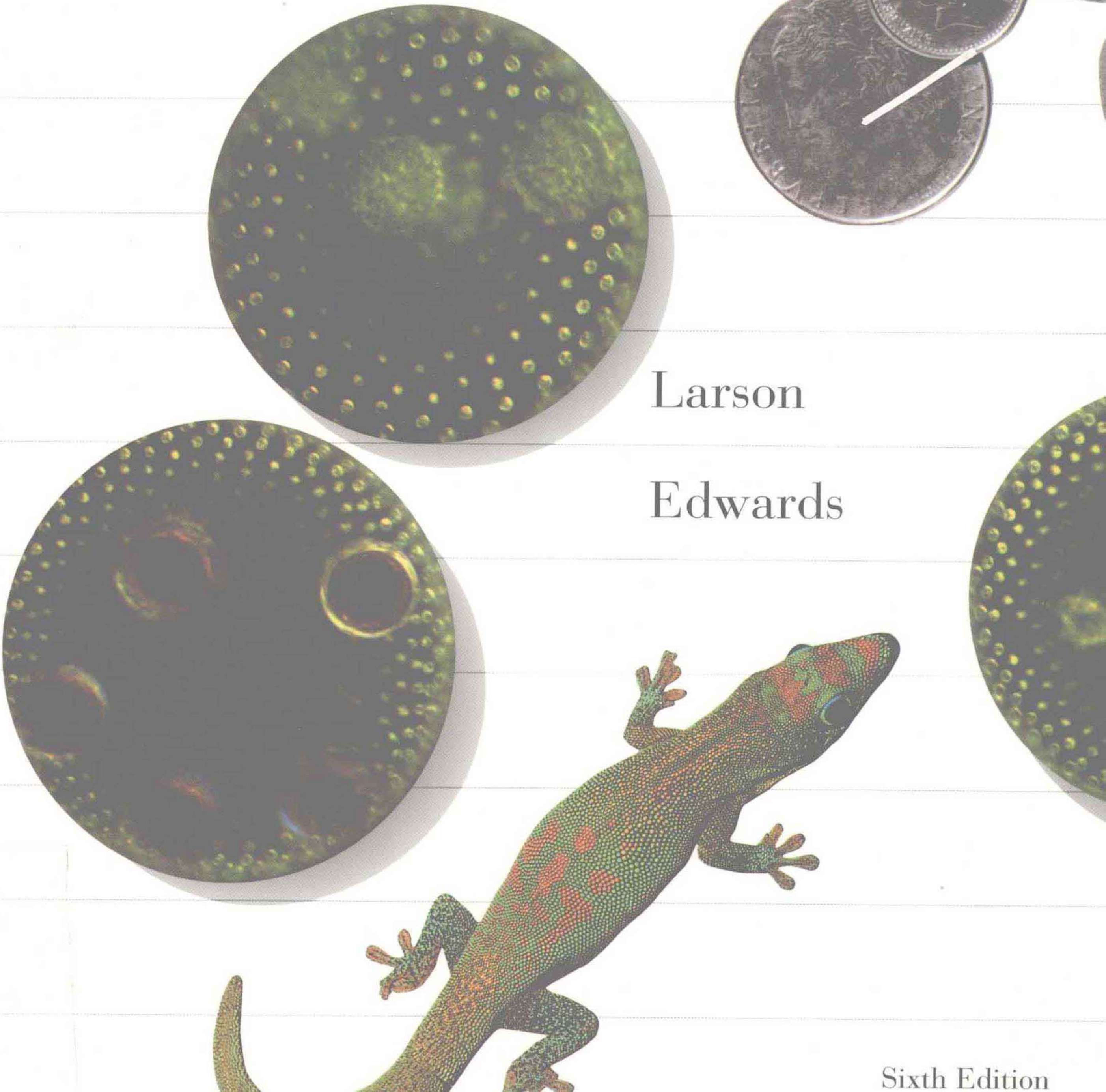
Brief Calculus

An Applied Approach



Larson

Edwards

Sixth Edition

BRIEF CALCULUS
An Applied Approach

SIXTH EDITION

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We have included examples and exercises that use real-life data as well as technology output from a variety of software. This would not have been possible without the help of many people and organizations. Our wholehearted thanks goes to all for their time and effort.

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BRIEF CALCULUS

An Applied Approach

SIXTH EDITION

A Word from the Authors

Welcome to *Brief Calculus: An Applied Approach,* Sixth Edition. In this revision, we have focused on making the text even more student-oriented. To encourage mastery and understanding, we have outlined a straightforward program of study with continual reinforcement and applicability to the real world.

New to the Sixth Edition:

Try It Appearing after every example, these new problems help students reinforce concepts right after they are presented. For example, see Example 7 on page 6.

Section Objectives This list appears at the beginning of each section, enabling students to identify and focus on the key points of the section. For example, see page 82.

Business Applications More business applications appear throughout the text to give a more complete coverage of calculus as used in economic and financial fields. For example, see Exercises 39 and 41 on page 271.

Chapter Openers Each chapter now begins with a list of key concepts that students will examine in the chapter. To help students see the wide applicability of the material, each chapter opener lists sample applications that appear throughout the chapter. For example, see page 171.

Technology Notes Several new sidebars on technology have been added, especially those that illustrate spreadsheet use. For example, see page 508.

Take Another Look Appearing just before each section exercise set, this feature asks students to look back at one or more concepts presented in the section, using questions designed to enhance understanding of key ideas. For example, see page 206.

eSolutions The text now comes with a CD entitled *Calculus: An Applied Approach Learning Tools Student CD-ROM*. This CD contains algebra review material as well as additional assistance with the calculus and algebra used in examples throughout the text. To help remind students of this review option, an icon appears next to those examples for which further help is available on the CD. For example, see page 126.

At the request of several users, Section 4.1 from the Fifth Edition has been divided into two sections in the Sixth Edition. See Section 4.1, Exponential Functions (page 258), and Section 4.2, Natural Exponential Functions (page 264).

Continuing Strong Pedagogy from the Fifth Edition:

A **Strategies for Success** feature is included with each chapter opener. In addition to outlining the key objectives of the chapter, this checklist provides page references for the various study tools in the chapter and notes additional available resources. The **Chapter Summary and Study Strategies** feature at the end of each chapter reinforces the Strategies for Success with a comprehensive list of skills covered in the chapter, section references, and a correlation to the **Review Exercises** for guided practice.

It is crucial for a student to understand an algebraic concept before attempting to master a related calculus concept. To help students in this area, **Algebra Tips** appear at point of use throughout the text. Many Algebra Tips correspond to the two-page **Algebra Review** at the end of each chapter, which emphasizes key algebraic concepts discussed in the chapter.

Throughout the text, **Study Tips** address special cases, expand on concepts, and help students avoid common errors. **Side Comments** help explain the steps of a solution. State-of-the-art graphics help students with visualization, especially when working with functions of several variables. **Topics in Calculus Videotapes** provide comprehensive coverage and explanation of core concepts.

Each chapter presents many opportunities for students to assess their progress, both at the end of each section (**Warm-Ups** and **Section Exercises**), and at the end of each chapter (**Chapter Summary, Study Strategies,** and **Review Exercises**). The test items in **Sample Post-Graduation Exam Questions** have been carefully crafted to build understanding as students progress through the course.

As always, we have paid careful attention to presentation, using precise mathematical language, innovative full-color design for emphasis and clarity, and a level of exposition that appeals to students to create an effective teaching and learning tool.

Application to the Changing World Around Us

Students studying calculus need to understand how the subject matter relates to the real world. In this edition, we have focused on increasing the variety of applications, especially in the life sciences, economics, and finance. All real-data applications have been revised to use the most current information available, and new ones have been added to increase exercise diversity. Exercises containing material from textbooks in other disciplines have been included to show the relevance of calculus in other areas. In addition, exercises involving the use of spreadsheets have been incorporated throughout. An **Additional Problems** supplement is also available to professors who would like a wider choice of exercises.

Advances in technology are helping to change the world around us. We have updated and increased technology coverage to be even more readily available at point of use. Students are encouraged to use a graphing utility, computer program, or spreadsheet software as a tool for exploration, discovery, and problemsolving.

Reference tables in the appendices have been revised to be more relevant to the real world, and graphing utility programs have been updated for use with the most common models on the market.

We hope you will enjoy the Sixth Edition. A readable text with a straightforward approach, it provides effective study tools and direct application to the lives and futures of calculus students.

Ron Larson

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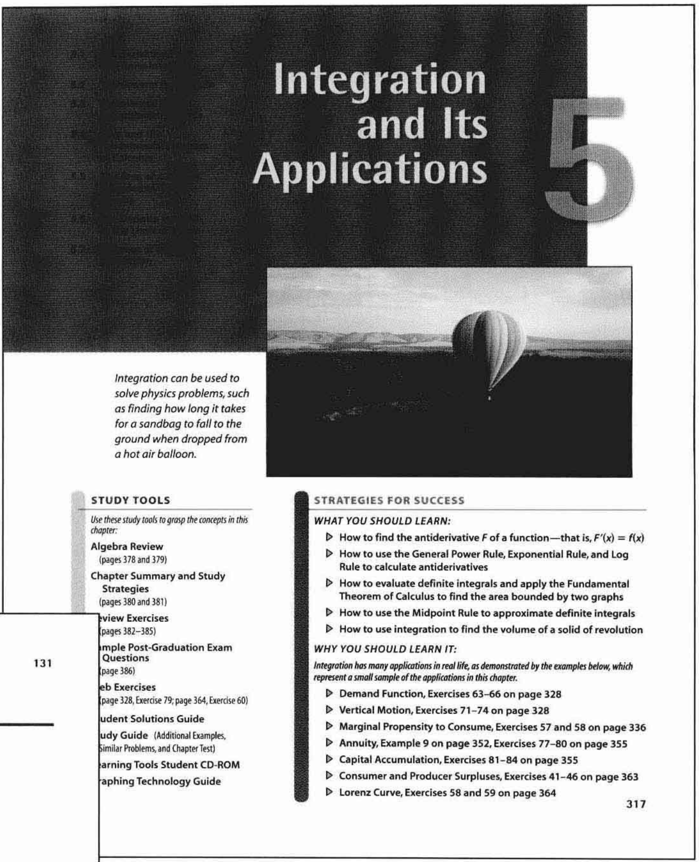
Features

Chapter Openers

Each chapter opens with *Strategies for Success,* a checklist that outlines what students should learn and lists several applications of those objectives. Also included are a list of study tools (with page references to help students achieve the key objectives) and additional resources that are available. Each chapter opener also contains a list of the section topics and a photo referring students to an interesting application in the section exercises.



Integration and Its Applications 5

Integration can be used to solve physics problems, such as finding how long it takes for a sandbag to fall to the ground when dropped from a hot air balloon.

STUDY TOOLS

Use these study tools to grasp the concepts in this chapter:

Algebra Review (pages 378 and 379)

Chapter Summary and Study Strategies (pages 380 and 381)

[Re]view Exercises (pages 382–385)

[Sa]mple Post-Graduation Exam Questions (page 386)

[W]eb Exercises (page 328, Exercise 79; page 364, Exercise 60)

[St]udent Solutions Guide

[St]udy Guide (Additional Examples, Similar Problems, and Chapter Test)

[Le]arning Tools Student CD-ROM

[Gr]aphing Technology Guide

STRATEGIES FOR SUCCESS

WHAT YOU SHOULD LEARN:

- How to find the antiderivative F of a function—that is, $F'(x) = f(x)$
- How to use the General Power Rule, Exponential Rule, and Log Rule to calculate antiderivatives
- How to evaluate definite integrals and apply the Fundamental Theorem of Calculus to find the area bounded by two graphs
- How to use the Midpoint Rule to approximate definite integrals
- How to use integration to find the volume of a solid of revolution

WHY YOU SHOULD LEARN IT:

Integration has many applications in real life, as demonstrated by the examples below, which represent a small sample of the applications in this chapter.

- Demand Function, Exercises 63–66 on page 328
- Vertical Motion, Exercises 71–74 on page 328
- Marginal Propensity to Consume, Exercises 57 and 58 on page 336
- Annuity, Example 9 on page 352, Exercises 77–80 on page 355
- Capital Accumulation, Exercises 81–84 on page 355
- Consumer and Producer Surpluses, Exercises 41–46 on page 363
- Lorenz Curve, Exercises 58 and 59 on page 364

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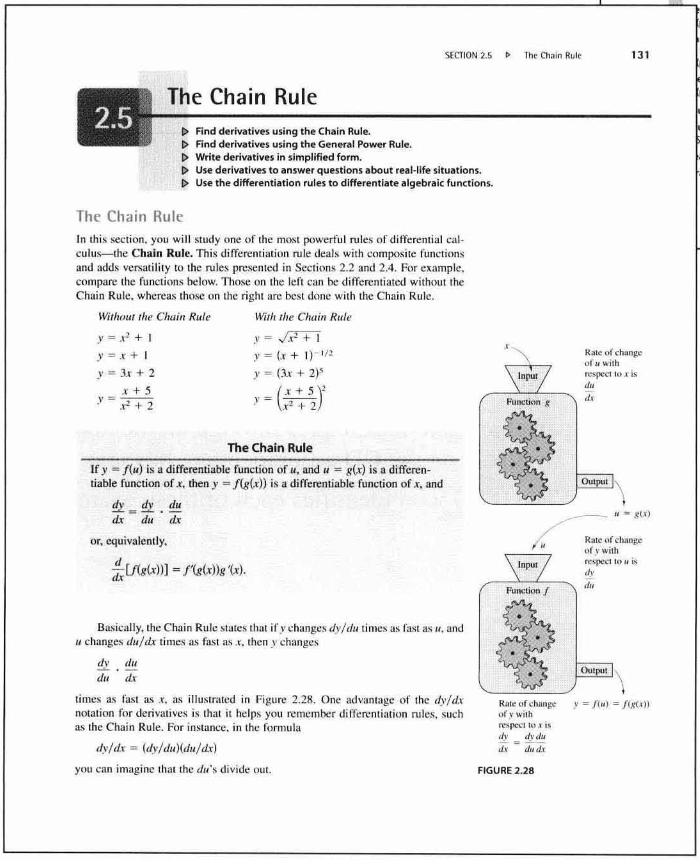


SECTION 2.5 ▷ The Chain Rule 131

2.5 **The Chain Rule**

- Find derivatives using the Chain Rule.
- Find derivatives using the General Power Rule.
- Write derivatives in simplified form.
- Use derivatives to answer questions about real-life situations.
- Use the differentiation rules to differentiate algebraic functions.

The Chain Rule

In this section, you will study one of the most powerful rules of differential calculus—the **Chain Rule.** This differentiation rule deals with composite functions and adds versatility to the rules presented in Sections 2.2 and 2.4. For example, compare the functions below. Those on the left can be differentiated without the Chain Rule, whereas those on the right are best done with the Chain Rule.

Without the Chain Rule	*With the Chain Rule*
$y = x^2 + 1$	$y = \sqrt{x^2 + 1}$
$y = x + 1$	$y = (x + 1)^{-1/2}$
$y = 3x + 2$	$y = (3x + 2)^5$
$y = \dfrac{x + 5}{x^2 + 2}$	$y = \left(\dfrac{x + 5}{x^2 + 2}\right)^2$

The Chain Rule

If $y = f(u)$ is a differentiable function of u, and $u = g(x)$ is a differentiable function of x, then $y = f(g(x))$ is a differentiable function of x, and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

or, equivalently,

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x).$$

Basically, the Chain Rule states that if y changes dy/du times as fast as u, and u changes du/dx times as fast as x, then y changes

$$\frac{dy}{du} \cdot \frac{du}{dx}$$

times as fast as x, as illustrated in Figure 2.28. One advantage of the dy/dx notation for derivatives is that it helps you remember differentiation rules, such as the Chain Rule. For instance, in the formula

$$dy/dx = (dy/du)(du/dx)$$

you can imagine that the du's divide out.

FIGURE 2.28

Section Objectives

Each section begins with a list of objectives covered in that section. This outline helps instructors with class planning and students in studying the material in the section.

Definitions and Theorems

All definitions and theorems are highlighted for emphasis and easy reference.

FEATURES

Examples

To increase the usefulness of the text as a study tool, the Sixth Edition presents a wide variety of examples, each titled for easy reference. Many of these detailed examples display solutions that are presented graphically, analytically, and/or numerically to provide further insight into mathematical concepts. Side comments clarify the steps of the solution as necessary. Examples using real-life data are identified with a globe icon and are accompanied by the types of illustrations that students are used to seeing in newspapers and magazines.

Marginal Analysis

Differentials are used in economics to approximate changes in revenue, cost, and profit. Suppose that $R = f(x)$ is the total revenue for selling x units of a product. When the number of units increases by one, the change in x is $\Delta x = 1$, and the change in R is

$$\Delta R = f(x + \Delta x) - f(x) \approx dR = \frac{dR}{dx}dx.$$

In other words, you can use the differential dR to approximate the change in the revenue that accompanies the sale of one additional unit. Similarly, the differentials dC and dP can be used to approximate the changes in cost and profit that accompany the sale (or production) of one additional unit.

TECHNOLOGY

Use a graphing utility to graph the revenue function $R(x) = x\sqrt{400 - x}$ and the tangent line approximation $y = \frac{4}{3}(x - 256) + 3072$ in the viewing window $250 \le x \le 260$, $3065 \le y \le 3080$. Explain why the two curves appear to be almost identical. Where do the curves intersect? Use the graphs to verify the solution to Example 2.

EXAMPLE 2 **Using Marginal Analysis**

The demand function for a product is modeled by

$$p = \sqrt{400 - x}, \quad 0 \le x \le 400.$$

Use differentials to approximate the change in revenue as sales increase from 256 units to 257 units. Compare this with the actual change in revenue.

Solution Begin by finding the marginal revenue, dR/dx.

$$R = xp \qquad \text{Formula for revenue}$$
$$= x\sqrt{400 - x} \qquad \text{Use } p = \sqrt{400 - x}$$
$$\frac{dR}{dx} = x\left(\frac{1}{2}\right)(400 - x)^{-1/2}(-1) + (400 - x)^{1/2}(1) \qquad \text{Product Rule}$$
$$= \frac{800 - 3x}{2\sqrt{400 - x}} \qquad \text{Simplify.}$$

When $x = 256$ and $dx = \Delta x = 1$, you can approximate the change in the revenue to be

$$\frac{800 - 3(256)}{2\sqrt{400 - 256}}(1) \approx \$1.33.$$

When x increases from 256 to 257, the actual change in revenue is

$$\Delta R = 257\sqrt{400 - 257} - 256\sqrt{400 - 256}$$
$$\approx 3073.27 - 3072.00$$
$$= \$1.27.$$

TRY IT ▷ 2 The demand function for a product is modeled by $p = \sqrt{200 - x}$. Use differentials to approximate the change in revenue as sales increase from 100 to 101 units. Compare this with the actual change in revenue.

Try Its

Appearing after every example, these new problems help students reinforce concepts right after they are presented.

Simplifying Derivatives

EXAMPLE 8 **Combining the Product and Quotient Rules**

Find the derivative of

$$y = \frac{(1 - 2x)(3x + 2)}{5x - 4}.$$

Solution This function contains a product within a quotient. You could first multiply the factors in the numerator and then apply the Quotient Rule. However, to gain practice in using the Product Rule within the Quotient Rule, try differentiating as follows.

$$y' = \frac{(5x - 4)\frac{d}{dx}[(1 - 2x)(3x + 2)] - (1 - 2x)(3x + 2)\frac{d}{dx}[5x - 4]}{(5x - 4)^2}$$
$$= \frac{(5x - 4)[(1 - 2x)(3) + (3x + 2)(-2)] - (1 - 2x)(3x + 2)(5)}{(5x - 4)^2}$$
$$= \frac{(5x - 4)(-12x - 1) - (1 - 2x)(15x + 10)}{(5x - 4)^2}$$
$$= \frac{(-60x^2 + 43x + 4) - (-30x^2 - 5x + 10)}{(5x - 4)^2}$$
$$= \frac{-30x^2 + 48x - 6}{(5x - 4)^2}$$

TRY IT ▷ 8 Find the derivative of $y = \dfrac{(1 + x)(2x - 1)}{x - 1}$.

In the examples in this section, much of the work in obtaining the final form of the derivative occurs *after* the differentiation. As summarized in the list below, direct application of differentiation rules often yields results that are not in simplified form. Note that two characteristics of simplified form are the absence of negative exponents and the combining of like terms.

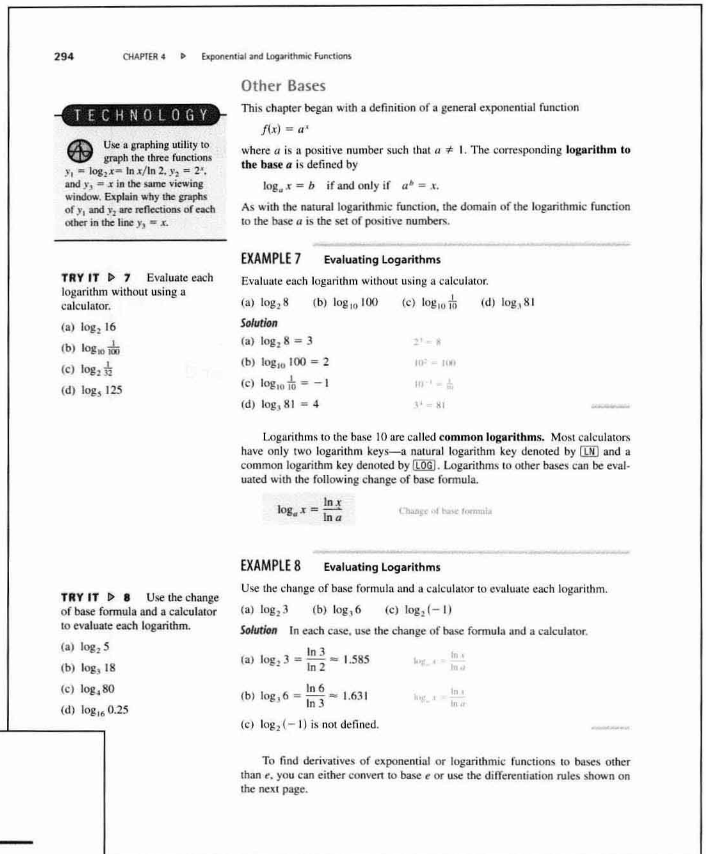
	f′(x) After Differentiating	*f′(x) After Simplifying*
Example 1	$(3x - 2x^2)(4) + (5 + 4x)(3 - 4x)$	$15 + 4x - 24x^2$
Example 2	$(x^{-1} + 1)(1) + (x - 1)(-x^{-2})$	$\dfrac{x^2 + 1}{x^2}$
Example 5	$\dfrac{(2 - 3x)(4x - 4) - (2x^2 - 4x + 3)(-3)}{(2 - 3x)^2}$	$\dfrac{-6x^2 + 8x + 1}{(2 - 3x)^2}$
Example 8	$\dfrac{(5x - 4)[(1 - 2x)(3) + (3x + 2)(-2)] - (1 - 2x)(3x + 2)(5)}{(5x - 4)^2}$	$\dfrac{-30x^2 + 48x - 6}{(5x - 4)^2}$

Learning Tools Student CD-ROM

Additional assistance with the calculus and algebra used in examples throughout the book is available on the CD accompanying this text. The (CD) icon identifies each of these examples.

Technology

Students are encouraged to use a graphing utility, computer algebra system, or spreadsheet software as a tool for exploration, discovery, and problem-solving. The text offers many opportunities to execute computations and programs, to visualize theoretical concepts, to discover alternative approaches, and to verify the results of other solution methods using technology. However, students are not required to have access to a graphing utility to use this text effectively. In addition to describing the benefits of using technology, the text also pays special attention to its possible misuse or misinterpretation.



294 CHAPTER 4 ▷ Exponential and Logarithmic Functions

Other Bases

This chapter began with a definition of a general exponential function

$f(x) = a^x$

where a is a positive number such that $a \neq 1$. The corresponding **logarithm to the base a** is defined by

$\log_a x = b$ if and only if $a^b = x$.

As with the natural logarithmic function, the domain of the logarithmic function to the base a is the set of positive numbers.

TECHNOLOGY

Use a graphing utility to graph the three functions $y_1 = \log_2 x = \ln x/\ln 2$, $y_2 = 2^x$, and $y_3 = x$ in the same viewing window. Explain why the graphs of y_1 and y_2 are reflections of each other in the line $y_3 = x$.

EXAMPLE 7 Evaluating Logarithms

Evaluate each logarithm without using a calculator.

(a) $\log_2 8$ (b) $\log_{10} 100$ (c) $\log_{10} \frac{1}{10}$ (d) $\log_3 81$

Solution

(a) $\log_2 8 = 3$ $2^3 = 8$

(b) $\log_{10} 100 = 2$ $10^2 = 100$

(c) $\log_{10} \frac{1}{10} = -1$ $10^{-1} = \frac{1}{10}$

(d) $\log_3 81 = 4$ $3^4 = 81$

TRY IT ▷ 7 Evaluate each logarithm without using a calculator.

(a) $\log_2 16$

(b) $\log_{10} \frac{1}{100}$

(c) $\log_2 \frac{1}{32}$

(d) $\log_5 125$

Logarithms to the base 10 are called **common logarithms.** Most calculators have only two logarithm keys—a natural logarithm key denoted by [LN] and a common logarithm key denoted by [LOG]. Logarithms to other bases can be evaluated with the following change of base formula.

$$\log_a x = \frac{\ln x}{\ln a}$$ Change of base formula

EXAMPLE 8 Evaluating Logarithms

Use the change of base formula and a calculator to evaluate each logarithm.

(a) $\log_2 3$ (b) $\log_3 6$ (c) $\log_2(-1)$

Solution In each case, use the change of base formula and a calculator.

(a) $\log_2 3 = \frac{\ln 3}{\ln 2} \approx 1.585$ $\log_a x = \frac{\ln x}{\ln a}$

(b) $\log_3 6 = \frac{\ln 6}{\ln 3} \approx 1.631$ $\log_a x = \frac{\ln x}{\ln a}$

(c) $\log_2(-1)$ is not defined.

TRY IT ▷ 8 Use the change of base formula and a calculator to evaluate each logarithm.

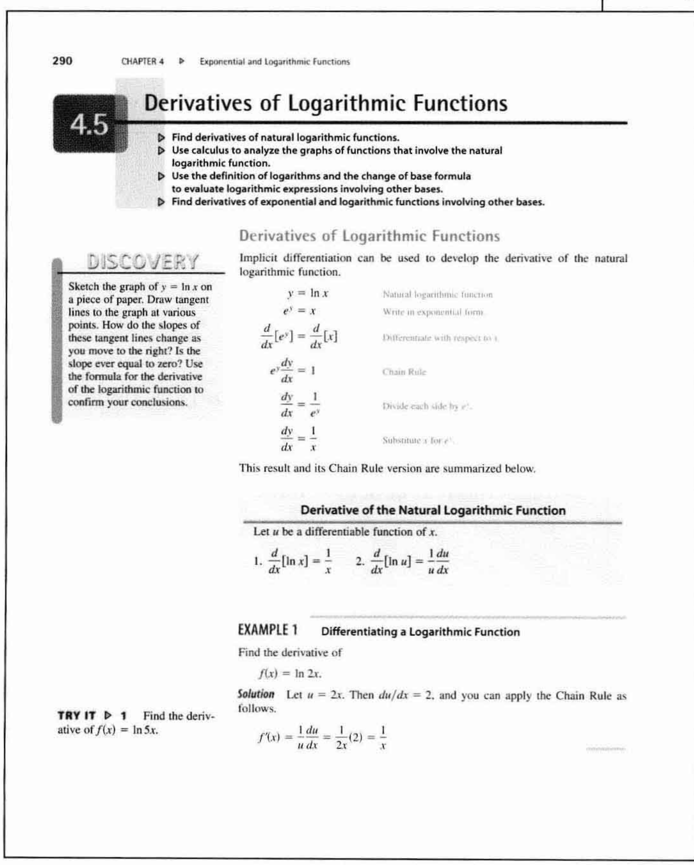
(a) $\log_2 5$

(b) $\log_3 18$

(c) $\log_4 80$

(d) $\log_{16} 0.25$

To find derivatives of exponential or logarithmic functions to bases other than e, you can either convert to base e or use the differentiation rules shown on the next page.



290 CHAPTER 4 ▷ Exponential and Logarithmic Functions

4.5 Derivatives of Logarithmic Functions

- ▷ Find derivatives of natural logarithmic functions.
- ▷ Use calculus to analyze the graphs of functions that involve the natural logarithmic function.
- ▷ Use the definition of logarithms and the change of base formula to evaluate logarithmic expressions involving other bases.
- ▷ Find derivatives of exponential and logarithmic functions involving other bases.

DISCOVERY

Sketch the graph of $y = \ln x$ on a piece of paper. Draw tangent lines to the graph at various points. How do the slopes of these tangent lines change as you move to the right? Is the slope ever equal to zero? Use the formula for the derivative of the logarithmic function to confirm your conclusions.

Derivatives of Logarithmic Functions

Implicit differentiation can be used to develop the derivative of the natural logarithmic function.

$y = \ln x$ Natural logarithmic function

$e^y = x$ Write in exponential form.

$\frac{d}{dx}[e^y] = \frac{d}{dx}[x]$ Differentiate with respect to x.

$e^y \frac{dy}{dx} = 1$ Chain Rule

$\frac{dy}{dx} = \frac{1}{e^y}$ Divide each side by e^y.

$\frac{dy}{dx} = \frac{1}{x}$ Substitute x for e^y.

This result and its Chain Rule version are summarized below.

Derivative of the Natural Logarithmic Function

Let u be a differentiable function of x.

1. $\frac{d}{dx}[\ln x] = \frac{1}{x}$ 2. $\frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx}$

EXAMPLE 1 Differentiating a Logarithmic Function

Find the derivative of

$f(x) = \ln 2x$.

Solution Let $u = 2x$. Then $du/dx = 2$, and you can apply the Chain Rule as follows.

$$f'(x) = \frac{1}{u}\frac{du}{dx} = \frac{1}{2x}(2) = \frac{1}{x}$$

TRY IT ▷ 1 Find the derivative of $f(x) = \ln 5x$.

Discovery

Before students are exposed to selected topics, *Discovery* projects allow them to explore concepts on their own, making them more likely to remember the results. These optional boxed features can be omitted, if the instructor desires, with no loss of continuity in the coverage of material.

Algebra Tips

Algebra Tips identified by the (x y) symbol offer students algebraic support at point of use. Many *Algebra Tips* are related to the *Algebra Review* at the end of the chapter, which provides additional details of examples with solutions and explanations.

Study Tips

Throughout the text, *Study Tips* help students avoid common errors, address special cases, and expand on theoretical concepts.

SECTION 3.3 Concavity and the Second-Derivative Test 193

EXAMPLE 2 Determining Concavity

Determine the intervals on which the graph of the function is concave upward or concave downward.

$$f(x) = \frac{6}{x^2+3}$$

Solution Begin by finding the second derivative of f.

$$f(x) = 6(x^2+3)^{-1} \quad \text{Rewrite original function.}$$

$$f'(x) = (-6)(2x)(x^2+3)^{-2} \quad \text{Chain Rule}$$

$$= \frac{-12x}{(x^2+3)^2} \quad \text{Simplify.}$$

$$f''(x) = \frac{(x^2+3)^2(-12) - (-12x)(2)(2x)(x^2+3)}{(x^2+3)^4} \quad \text{Quotient Rule}$$

$$= \frac{-12(x^2+3) + (48x^2)}{(x^2+3)^3} \quad \text{Simplify.}$$

$$= \frac{36(x^2-1)}{(x^2+3)^3} \quad \text{Simplify.}$$

From this, you can see that $f''(x)$ is defined for all real numbers and $f''(x) = 0$ when $x = \pm 1$. So, you can test the concavity of f by testing the intervals $(-\infty, -1)$, $(-1, 1)$, and $(1, \infty)$, as shown in the table. The graph of f is shown in Figure 3.23.

Interval	$-\infty < x < -1$	$-1 < x < 1$	$1 < x < \infty$
Test value	$x = -2$	$x = 0$	$x = 2$
Sign of $f''(x)$	$f''(-2) > 0$	$f''(0) < 0$	$f''(2) > 0$
Conclusion	Concave upward	Concave downward	Concave upward

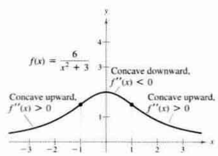


FIGURE 3.23

ALGEBRA TIP

For help on the algebra in Example 2, see Example 1a in the *Chapter 3 Algebra Review*, on page 248.

STUDY TIP

In Example 2, f' is increasing on the interval $(1, \infty)$ even though f is decreasing there. Be sure you see that the increasing or decreasing of f' does not necessarily correspond to the increasing or decreasing of f.

TRY IT ▷ 2 Determine the intervals on which the graph of the function is concave upward and concave downward.

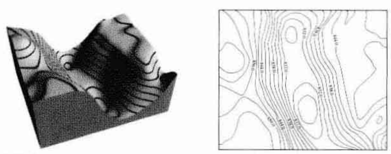
$$f(x) = \frac{24}{x^2+12}$$

476 CHAPTER 7 Functions of Several Variables

Level Curves and Contour Maps

A **contour map** of a surface is created by *projecting* traces, taken in evenly spaced planes that are parallel to the xy-plane, onto the xy-plane. Each projection is a **level curve** of the surface.

Contour maps are used to create weather maps, topographical maps, and population density maps. For instance, Figure 7.19(a) shows a graph of a "mountain and valley" surface given by $z = f(x, y)$. Each of the level curves in Figure 7.19(b) represents the intersection of the surface $z = f(x, y)$ with a plane $z = c$, where $c = 828, 830, \ldots, 854$.



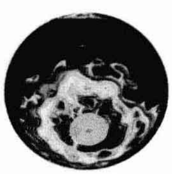
(a) Surface (b) Contour Map

FIGURE 7.19

EXAMPLE 3 Reading a Contour Map

The "contour map" in Figure 7.20 was computer generated using data collected by satellite instrumentation. The map uses color to represent levels of chlorine nitrate in the atmosphere. Chlorine nitrate contributes to the ozone depletion in the Earth's atmosphere. The red areas represent the highest level of chlorine nitrate and the dark blue areas represent the lowest level. Describe the areas that have the highest levels of chlorine nitrate. (*Source: Lockheed Missiles and Space Company*)

Solution The highest levels of chlorine nitrate are in the Antarctic Ocean, surrounding Antarctica. Although chlorine nitrate is not itself harmful to ozone, it has a tendency to convert to chlorine monoxide, which *is* harmful to ozone. Once the chlorine nitrate is converted to chlorine monoxide, it no longer shows on the contour map. So, Antarctica itself shows little chlorine nitrate—the nitrate has been converted to monoxide. If you have seen maps showing the "ozone hole" in Earth's atmosphere, you know that the hole occurs over Antarctica.



Lockheed Missiles and Space Company

FIGURE 7.20

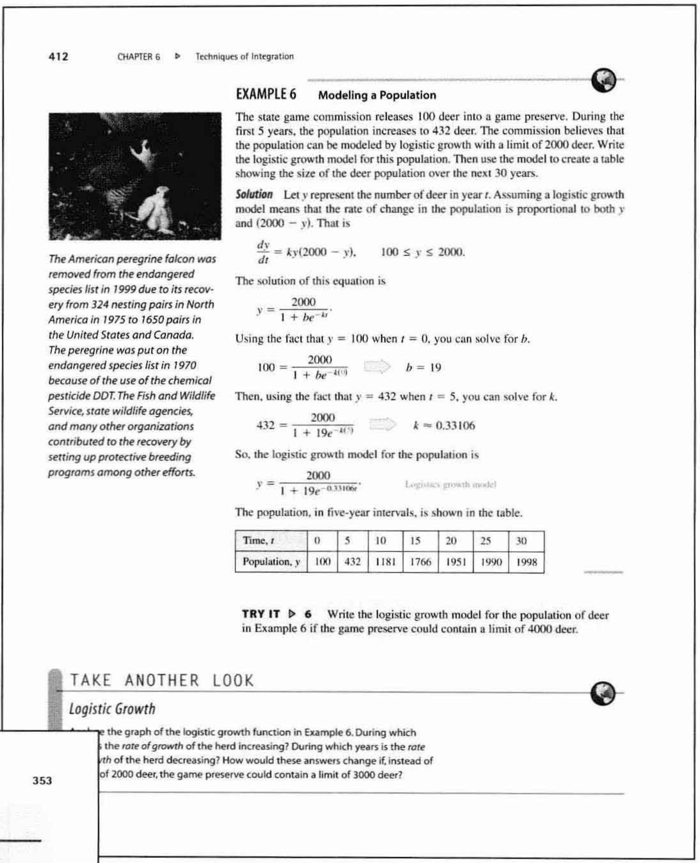
TRY IT ▷ 3 When the level curves of a contour map are close together, is the surface of the contour map steep or nearly level? When the level curves of a contour map are far apart, is the surface of the contour map steep or nearly level?

Graphics

The Sixth Edition has more than 1900 figures. Computer generated for accuracy, clarity, and realism, these illustrations help students visualize mathematical concepts more easily.

Take Another Look

Starting with Chapter 1, each section in the text closes with a *Take Another Look* problem asking students to look back at one or more concepts presented in the section, using questions designed to enhance understanding of key ideas. These problems can be completed as group projects in class or as homework assignments. Because these problems encourage students to think, reason, and write about calculus, they emphasize the synthesis or the further exploration of the concepts presented in the section.



412 CHAPTER 6 ▷ Techniques of Integration

EXAMPLE 6 **Modeling a Population**

The state game commission releases 100 deer into a game preserve. During the first 5 years, the population increases to 432 deer. The commission believes that the population can be modeled by logistic growth with a limit of 2000 deer. Write the logistic growth model for this population. Then use the model to create a table showing the size of the deer population over the next 30 years.

The American peregrine falcon was removed from the endangered species list in 1999 due to its recovery from 324 nesting pairs in North America in 1975 to 1650 pairs in the United States and Canada. The peregrine was put on the endangered species list in 1970 because of the use of the chemical pesticide DDT. The Fish and Wildlife Service, state wildlife agencies, and many other organizations contributed to the recovery by setting up protective breeding programs among other efforts.

Solution Let y represent the number of deer in year t. Assuming a logistic growth model means that the rate of change in the population is proportional to both y and $(2000 - y)$. That is

$$\frac{dy}{dt} = ky(2000 - y), \qquad 100 \le y \le 2000.$$

The solution of this equation is

$$y = \frac{2000}{1 + be^{-kt}}.$$

Using the fact that $y = 100$ when $t = 0$, you can solve for b.

$$100 = \frac{2000}{1 + be^{-k(0)}} \quad \Rightarrow \quad b = 19$$

Then, using the fact that $y = 432$ when $t = 5$, you can solve for k.

$$432 = \frac{2000}{1 + 19e^{-k(5)}} \quad \Rightarrow \quad k \approx 0.33106$$

So, the logistic growth model for the population is

$$y = \frac{2000}{1 + 19e^{-0.33106t}}. \qquad \text{Logistics growth model}$$

The population, in five-year intervals, is shown in the table.

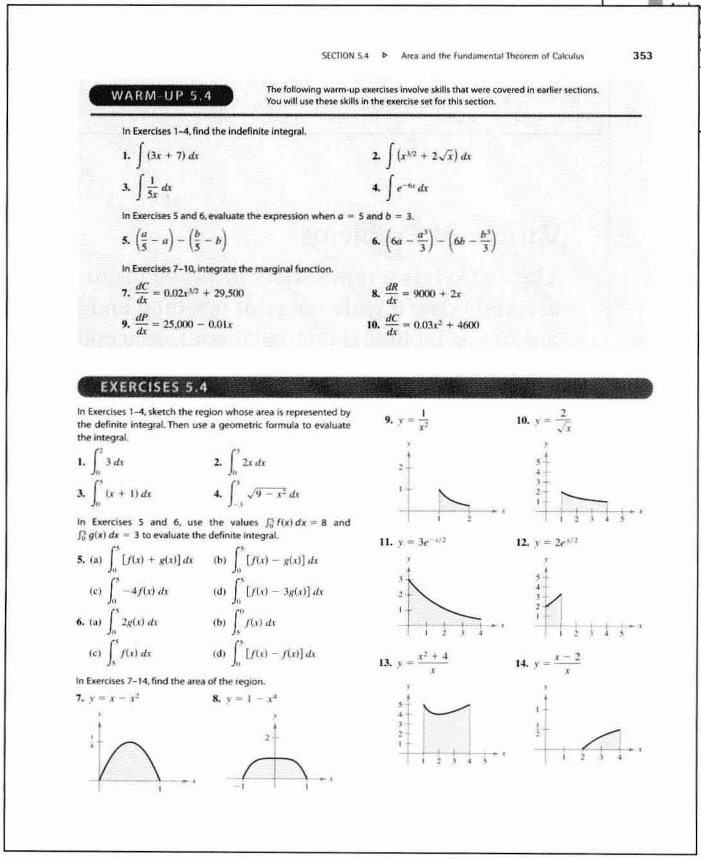
Time, t	0	5	10	15	20	25	30
Population, y	100	432	1181	1766	1951	1990	1998

TRY IT ▷ 6 Write the logistic growth model for the population of deer in Example 6 if the game preserve could contain a limit of 4000 deer.

TAKE ANOTHER LOOK

Logistic Growth

...e the graph of the logistic growth function in Example 6. During which ...s the *rate of growth* of the herd increasing? During which years is the *rate* ...th of the herd decreasing? How would these answers change if, instead of ...of 2000 deer, the game preserve could contain a limit of 3000 deer?



SECTION 5.4 ▷ Area and the Fundamental Theorem of Calculus 353

WARM-UP 5.4 The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, find the indefinite integral.

1. $\int (3x + 7)\,dx$
2. $\int (x^{3/2} + 2\sqrt{x})\,dx$
3. $\int \frac{1}{5x}\,dx$
4. $\int e^{-6x}\,dx$

In Exercises 5 and 6, evaluate the expression when $a = 5$ and $b = 3$.

5. $\left(\frac{a}{5} - a\right) - \left(\frac{b}{5} - b\right)$
6. $\left(6a - \frac{a^3}{3}\right) - \left(6b - \frac{b^3}{3}\right)$

In Exercises 7–10, integrate the marginal function.

7. $\frac{dC}{dx} = 0.02x^{3/2} + 29{,}500$
8. $\frac{dR}{dx} = 9000 + 2x$
9. $\frac{dP}{dx} = 25{,}000 - 0.01x$
10. $\frac{dC}{dx} = 0.03x^2 + 4600$

EXERCISES 5.4

In Exercises 1–4, sketch the region whose area is represented by the definite integral. Then use a geometric formula to evaluate the integral.

1. $\int_0^2 3\,dx$
2. $\int_0^3 2x\,dx$
3. $\int_0^5 (x + 1)\,dx$
4. $\int_{-3}^3 \sqrt{9 - x^2}\,dx$

In Exercises 5 and 6, use the values $\int_0^5 f(x)\,dx = 8$ and $\int_0^5 g(x)\,dx = 3$ to evaluate the definite integral.

5. (a) $\int_0^5 [f(x) + g(x)]\,dx$ (b) $\int_0^5 [f(x) - g(x)]\,dx$
   (c) $\int_0^5 -4f(x)\,dx$ (d) $\int_0^5 [f(x) - 3g(x)]\,dx$
6. (a) $\int_0^5 2g(x)\,dx$ (b) $\int_5^0 f(x)\,dx$
   (c) $\int_5^5 f(x)\,dx$ (d) $\int_0^5 [f(x) - f(x)]\,dx$

In Exercises 7–14, find the area of the region.

7. $y = x - x^2$
8. $y = 1 - x^4$
9. $y = \frac{1}{x^2}$
10. $y = \frac{2}{\sqrt{x}}$
11. $y = 3e^{-x/2}$
12. $y = 2e^{x/2}$
13. $y = \frac{x^2 + 4}{x}$
14. $y = \frac{x - 2}{x}$

Warm-Up Exercises

Starting with Chapter 1, each text section has a set of ten *Warm-Up* exercises. The *Warm-Ups* enable students to review and practice the previously learned skills necessary to master the new skills presented in the section. Answers to *Warm-Ups* appear in the back of the text.

Exercises

The text now contains almost 6000 exercises. Each exercise set is graded, progressing from skill-development problems to more challenging problems, to build confidence, skill, and understanding. The wide variety of types of exercises include many technology-oriented, real, and engaging problems. Answers to all odd-numbered exercises are included in the back of the text. To help instructors make homework assignments, many of the exercises in the text are labeled to indicate the area of application.

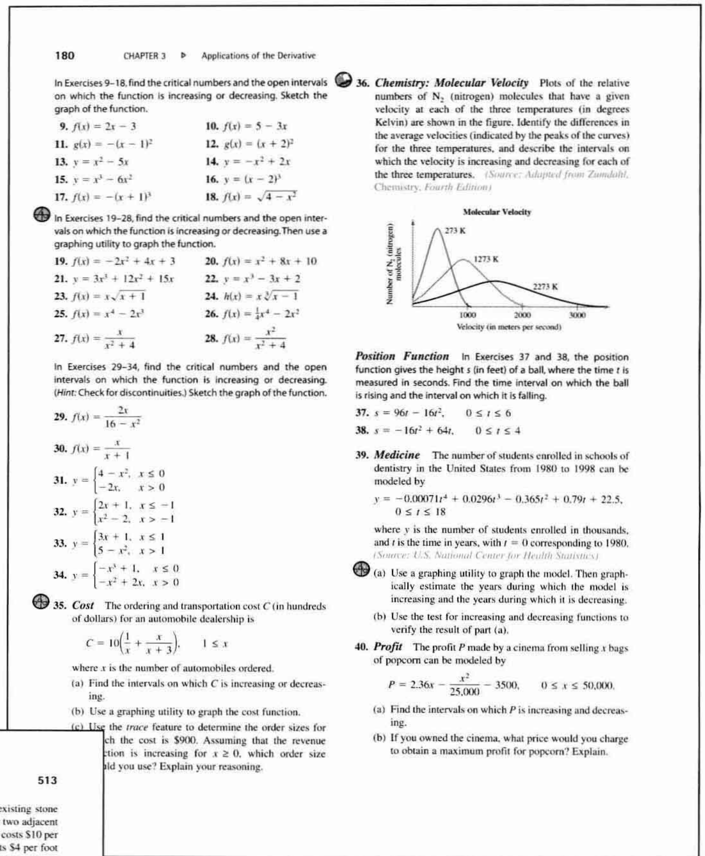
FEATURES

Graphing Utilities

Many exercises in the text can be solved using technology; however, the [graphing utility] symbol identifies all exercises for which students are specifically instructed to use a graphing utility or a computer algebra system.

Textbook Exercises

The Sixth Edition includes a number of exercises that contain material from textbooks in other disciplines, such as biology, chemistry, economics, finance, geology, physics, and psychology. These applications make the point to students that they will need to use calculus in future courses outside of the math curriculum. These exercises are identified by the [textbook] icon and are labeled to indicate the subject area.



180 CHAPTER 3 ▷ Applications of the Derivative

In Exercises 9–18, find the critical numbers and the open intervals on which the function is increasing or decreasing. Sketch the graph of the function.

9. $f(x) = 2x - 3$
10. $f(x) = 5 - 3x$
11. $g(x) = -(x-1)^2$
12. $g(x) = (x+2)^2$
13. $y = x^2 - 5x$
14. $y = -x^2 + 2x$
15. $y = x^3 - 6x^2$
16. $y = (x-2)^3$
17. $f(x) = -(x+1)^3$
18. $f(x) = \sqrt{4 - x^2}$

In Exercises 19–28, find the critical numbers and the open intervals on which the function is increasing or decreasing. Then use a graphing utility to graph the function.

19. $f(x) = -2x^2 + 4x + 3$
20. $f(x) = x^2 + 8x + 10$
21. $y = 3x^3 + 12x^2 + 15x$
22. $y = x^3 - 3x + 2$
23. $f(x) = x\sqrt{x+1}$
24. $h(x) = x\sqrt[3]{x-1}$
25. $f(x) = x^4 - 2x^3$
26. $f(x) = \frac{1}{4}x^4 - 2x^2$
27. $f(x) = \dfrac{x}{x^2+4}$
28. $f(x) = \dfrac{x^2}{x^2+4}$

In Exercises 29–34, find the critical numbers and the open intervals on which the function is increasing or decreasing. (*Hint:* Check for discontinuities.) Sketch the graph of the function.

29. $f(x) = \dfrac{2x}{16 - x^2}$
30. $f(x) = \dfrac{x}{x+1}$
31. $y = \begin{cases} 4 - x^2, & x \le 0 \\ -2x, & x > 0 \end{cases}$
32. $y = \begin{cases} 2x + 1, & x \le -1 \\ x^2 - 2, & x > -1 \end{cases}$
33. $y = \begin{cases} 3x + 1, & x \le 1 \\ 5 - x^2, & x > 1 \end{cases}$
34. $y = \begin{cases} -x^3 + 1, & x \le 0 \\ -x^2 + 2x, & x > 0 \end{cases}$

35. ***Cost*** The ordering and transportation cost C (in hundreds of dollars) for an automobile dealership is

$$C = 10\left(\frac{1}{x} + \frac{x}{x+3}\right), \quad 1 \le x$$

where x is the number of automobiles ordered.

(a) Find the intervals on which C is increasing or decreasing.

(b) Use a graphing utility to graph the cost function.

(c) Use the *trace* feature to determine the order sizes for ch the cost is $900. Assuming that the revenue ction is increasing for $x \ge 0$, which order size ld you use? Explain your reasoning.

36. ***Chemistry: Molecular Velocity*** Plots of the relative numbers of N_2 (nitrogen) molecules that have a given velocity at each of the three temperatures (in degrees Kelvin) are shown in the figure. Identify the differences in the average velocities (indicated by the peaks of the curves) for the three temperatures, and describe the intervals on which the velocity is increasing and decreasing for each of the three temperatures. (*Source: Adapted from Zumdahl,* Chemistry, *Fourth Edition*)

Position Function In Exercises 37 and 38, the position function gives the height s (in feet) of a ball, where the time t is measured in seconds. Find the time interval on which the ball is rising and the interval on which it is falling.

37. $s = 96t - 16t^2, \quad 0 \le t \le 6$
38. $s = -16t^2 + 64t, \quad 0 \le t \le 4$

39. ***Medicine*** The number of students enrolled in schools of dentistry in the United States from 1980 to 1998 can be modeled by

$$y = -0.00071t^4 + 0.0296t^3 - 0.365t^2 + 0.79t + 22.5, \quad 0 \le t \le 18$$

where y is the number of students enrolled in thousands, and t is the time in years, with $t = 0$ corresponding to 1980. (*Source: U.S. National Center for Health Statistics*)

(a) Use a graphing utility to graph the model. Then graphically estimate the years during which the model is increasing and the years during which it is decreasing.

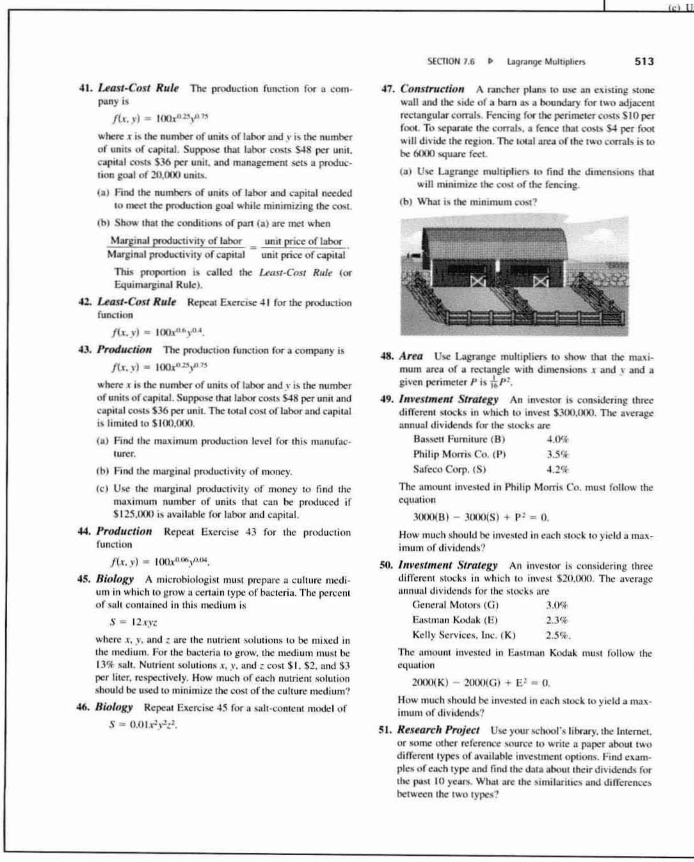
(b) Use the test for increasing and decreasing functions to verify the result of part (a).

40. ***Profit*** The profit P made by a cinema from selling x bags of popcorn can be modeled by

$$P = 2.36x - \frac{x^2}{25{,}000} - 3500, \quad 0 \le x \le 50{,}000.$$

(a) Find the intervals on which P is increasing and decreasing.

(b) If you owned the cinema, what price would you charge to obtain a maximum profit for popcorn? Explain.



SECTION 7.6 ▷ Lagrange Multipliers 513

41. ***Least-Cost Rule*** The production function for a company is

$$f(x, y) = 100x^{0.25}y^{0.75}$$

where x is the number of units of labor and y is the number of units of capital. Suppose that labor costs $48 per unit, capital costs $36 per unit, and management sets a production goal of 20,000 units.

(a) Find the numbers of units of labor and capital needed to meet the production goal while minimizing the cost.

(b) Show that the conditions of part (a) are met when

$$\frac{\text{Marginal productivity of labor}}{\text{Marginal productivity of capital}} = \frac{\text{unit price of labor}}{\text{unit price of capital}}.$$

This proportion is called the *Least-Cost Rule* (or Equimarginal Rule).

42. ***Least-Cost Rule*** Repeat Exercise 41 for the production function

$$f(x, y) = 100x^{0.6}y^{0.4}.$$

43. ***Production*** The production function for a company is

$$f(x, y) = 100x^{0.25}y^{0.75}$$

where x is the number of units of labor and y is the number of units of capital. Suppose that labor costs $48 per unit and capital costs $36 per unit. The total cost of labor and capital is limited to $100,000.

(a) Find the maximum production level for this manufacturer.

(b) Find the marginal productivity of money.

(c) Use the marginal productivity of money to find the maximum number of units that can be produced if $125,000 is available for labor and capital.

44. ***Production*** Repeat Exercise 43 for the production function

$$f(x, y) = 100x^{0.06}y^{0.04}.$$

45. ***Biology*** A microbiologist must prepare a culture medium in which to grow a certain type of bacteria. The percent of salt contained in this medium is

$$S = 12xyz$$

where x, y, and z are the nutrient solutions to be mixed in the medium. For the bacteria to grow, the medium must be 13% salt. Nutrient solutions x, y, and z cost $1, $2, and $3 per liter, respectively. How much of each nutrient solution should be used to minimize the cost of the culture medium?

46. ***Biology*** Repeat Exercise 45 for a salt-content model of

$$S = 0.01x^2y^2z^2.$$

47. ***Construction*** A rancher plans to use an existing stone wall and the side of a barn as a boundary for two adjacent rectangular corrals. Fencing for the perimeter costs $10 per foot. To separate the corrals, a fence that costs $4 per foot will divide the region. The total area of the two corrals is to be 6000 square feet.

(a) Use Lagrange multipliers to find the dimensions that will minimize the cost of the fencing.

(b) What is the minimum cost?

48. ***Area*** Use Lagrange multipliers to show that the maximum area of a rectangle with dimensions x and y and a given perimeter P is $\frac{1}{16}P^2$.

49. ***Investment Strategy*** An investor is considering three different stocks in which to invest $300,000. The average annual dividends for the stocks are

Bassett Furniture (B)	4.0%
Philip Morris Co. (P)	3.5%
Safeco Corp. (S)	4.2%

The amount invested in Philip Morris Co. must follow the equation

$$3000(B) - 3000(S) + P^2 = 0.$$

How much should be invested in each stock to yield a maximum of dividends?

50. ***Investment Strategy*** An investor is considering three different stocks in which to invest $20,000. The average annual dividends for the stocks are

General Motors (G)	3.0%
Eastman Kodak (E)	2.3%
Kelly Services, Inc. (K)	2.5%.

The amount invested in Eastman Kodak must follow the equation

$$2000(K) - 2000(G) + E^2 = 0.$$

How much should be invested in each stock to yield a maximum of dividends?

51. ***Research Project*** Use your school's library, the Internet, or some other reference source to write a paper about two different types of available investment options. Find examples of each type and find the data about their dividends for the past 10 years. What are the similarities and differences between the two types?

Variety of Problems

The text offers a rich variety of problems to accommodate a wide range of teaching and learning styles, including computational, conceptual, interpretational, matching (graphs to functions), true/false, essay, multipart, exploratory, and applied problems. A variety of relevant application problems from a wide range of disciplines — many of which use real data — clearly demonstrate the real-world usage of the mathematics. Problems are often labeled to indicate the subject of the application (as in *Break-Even Analysis* and *Lung Transplants*) or to identify a special type of question (as in *True or False* and *Writing*) for easy reference.

Spreadsheets

Students are encouraged to try using spreadsheet software to solve exercises that carry the [spreadsheet] icon.

60. ***Marginal Revenue*** At a baseball stadium, souvenir caps are sold at two locations. If x_1 and x_2 are the numbers of baseball caps sold at location 1 and location 2, respectively, then the total revenue for the caps is

$$R = 15x_1 + 16x_2 - \frac{1}{10}x_1^2 - \frac{1}{10}x_2^2 - \frac{1}{100}x_1x_2.$$

Given that $x_1 = 50$ and $x_2 = 40$, find the marginal revenue at location 1 and at location 2.

61. ***Medical Science*** The surface area A of an average human body in square centimeters can be approximated by the model

$$A(w, h) = 101.4w^{0.425}h^{0.725}$$

where w is the weight in pounds and h is the height in inches.

(a) Determine the partial derivatives of A with respect to w and with respect to h.

(b) Evaluate dA/dw at (180, 70). Explain your result.

62. ***Medicine*** In order to treat a certain bacterial infection, a combination of two drugs is being tested. Studies have shown that the duration of the infection in laboratory tests can be modeled by

$$D(x, y) = x^2 + 2y^2 - 18x - 24y + 2xy + 120$$

where x is the dose in hundreds of milligrams of the first drug and y is the dose in hundreds of milligrams of the second drug. Evaluate $D(5, 2.5)$ and $D(7.5, 8)$ and interpret your results.

In Exercises 63–70, find any critical points and relative extrema of the function.

63. $f(x, y) = x^2 + 2xy + y^2$

64. $f(x, y) = x^3 - 3xy + y^2$

65. $f(x, y) = x^2 + 6xy + 3y^2 + 6x + 8$

66. $f(x, y) = x + y^2 - e^x$

67. $f(x, y) = x^3 + y^2 - xy$

68. $f(x, y) = y^2 + xy + 3y - 2x + 5$

69. $f(x, y) = x^3 + y^3 - 3x - 3y + 2$

70. $f(x, y) = y^2 - x^2$

71. ***Revenue*** A company manufactures and sells two products. The demand functions for the products are

$$p_1 = 100 - x_1$$

and

$$p_2 = 200 - 0.5x_2.$$

d the total revenue functions for x_1 and x_2.

d x_1 and x_2 such that the revenue is maximized.

at is the maximum revenue?

72. ***Profit*** A company manufactures a product at two locations. The costs of manufacturing x_1 units at plant 1 and x_2 units at plant 2 are

$$C_1 = 0.03x_1^2 + 4x_1 + 300 \quad \text{and}$$
$$C_2 = 0.05x_2^2 + 7x_2 + 175$$

respectively. If the product sells for \$10 per unit, find x_1 and x_2 such that the profit, $P = 10(x_1 + x_2) - C_1 - C_2$, is maximized.

In Exercises 73–78, locate any extrema of the function by using Lagrange multipliers.

73. $f(x, y) = x^2y$
Constraint: $x + 2y = 2$

74. $f(x, y) = x^2 + y^2$
Constraint: $x + y = 4$

75. $f(x, y, z) = xyz$
Constraint: $x + 2y + z - 4 = 0$

76. $f(x, y, z) = xz + yz$
Constraint: $x + y + z = 6$

77. $f(x, y, z) = x^2 + y^2 + z^2$
Constraints: $x + z = 6, y + z = 8$

78. $f(x, y, z) = xyz$
Constraints: $x + y + z = 32, x - y + z = 0$

[spreadsheet] In Exercises 79 and 80, use a spreadsheet to find the indicated extremum. In each case, assume that x, y, and z are nonnegative.

79. Maximize $f(x, y, z) = xy$
Constraints: $x^2 + y^2 = 16, x - 2z = 0$

80. Minimize $f(x, y, z) = x^2 + y^2 + z^2$
Constraints: $x - 2z = 4, x + y = 8$

81. ***Maximum Production Level*** The production function for a manufacturer is $f(x, y) = 4x + xy + 2y$. Assume that the total amount available for labor x and capital y is \$2000 and that units of labor and capital cost \$20 and \$4, respectively. Find the maximum production level for this manufacturer.

82. ***Minimum Cost*** A manufacturer has an order for 1500 units that can be produced at two locations. Let x_1 and x_2 be the numbers of units produced at the two locations. Find the number that should be produced at each location to meet the order and minimize cost if the cost function is

$$C = 0.20x_1^2 + 10x_1 + 0.15x_2^2 + 12x_2.$$

In Exercises 83 and 84, (a) use the method of least squares to find the least squares regression line and (b) calculate the sum of the squared errors.

83. $(-2, -3), (-1, -1), (1, 2), (3, 2)$

84. $(-3, -1), (-2, -1), (0, 0), (1, 1), (2, 1)$

36. ***Minimum Cost*** The ordering and transportation cost C of the components used in manufacturing a certain product is

$$C = 100\left(\frac{200}{x^2} + \frac{x}{x + 30}\right), \quad x \geq 1$$

where C is measured in thousands of dollars and x is the order size in hundreds. Find the order size that minimizes the cost. (*Hint:* Use the *root* feature of a graphing utility.)

37. ***Revenue*** The demand for a car wash is

$$x = 600 - 50p$$

where the current price is \$5.00. Can revenue be increased by lowering the price and thus attracting more customers? Use price elasticity of demand to determine your answer.

38. ***Revenue*** Repeat Exercise 37 with a demand function of

$$x = 800 - 40p.$$

39. ***Demand*** A demand function is modeled by $x = a/p^m$, where a is a constant and $m > 1$. Show that $\eta = -m$. In other words, show that a 1% increase in price results in an m% decrease in the quantity demanded.

40. ***Revenue*** The revenue R (in billions of dollars per year) for Sara Lee Corporation for the years 1991–2000 can be modeled by

$$R = \frac{11 - 0.86t}{1 - 0.15t + 0.007t^2}$$

where $t = 1$ corresponds to 1991. (*Source: Sara Lee Corporation*)

(a) During which year, between 1991 and 2000, was Sara Lee's revenue the least?

(b) During which year was the revenue the greatest?

(c) Find the revenue for each year in parts (a) and (b).

(d) Use a graphing utility to graph the revenue function. Then use the *zoom* and *trace* features to confirm the results in parts (a), (b), and (c).

41. ***Revenue*** The revenue R (in billions of dollars per year) for Johnson & Johnson for the years 1991–2000 can be modeled by

$$R = -0.014t^3 + 0.30t^2 + 0.1t + 12$$

where $t = 1$ corresponds to 1991. (*Source: Johnson & Johnson*)

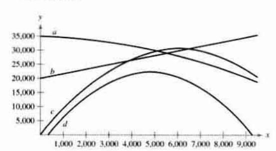
(a) During which year, between 1991 and 2000, was Johnson & Johnson's revenue increasing most rapidly?

(b) During which year was the revenue increasing at the slowest rate?

(c) Find the rate of increase or decrease for each year in parts (a) and (b).

(d) Use a graphing utility to graph the revenue function. Then use the *zoom* and *trace* features to confirm the results in parts (a), (b), and (c).

42. Match each graph with the function it best represents—a demand function, a revenue function, a cost function, or a profit function. Explain your reasoning. (The graphs are labeled a–d.)



BUSINESS CAPSULE

While graduate students in 1992, Liz Elting and Phil Shawe co-founded TransPerfect Translations. They used a rented computer and a \$5000 credit card cash advance to market their service-oriented translation firm, now one of the largest in the country. They have a full-time staff of 135 language professionals and, in 1999, they had a revenue of over \$12 million.

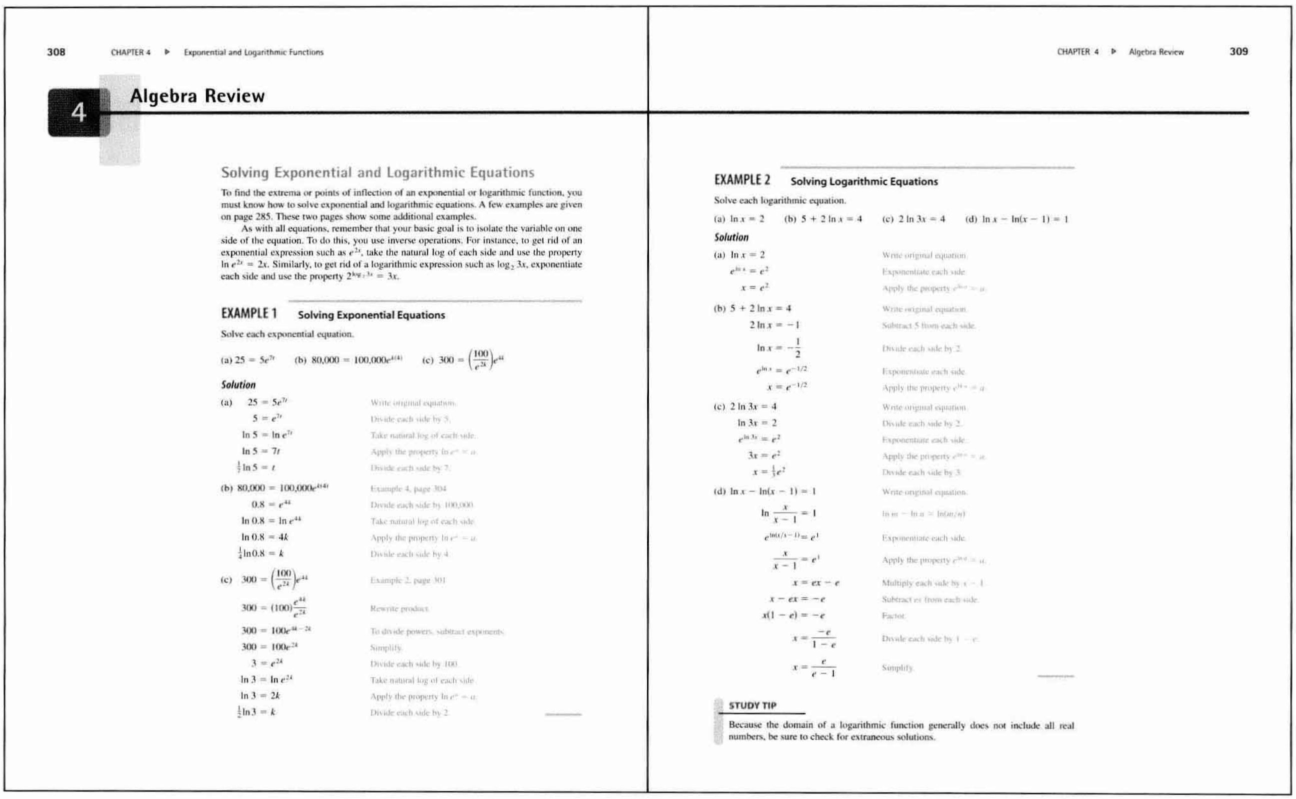
43. ***Research Project*** Choose an innovative product like the one described above. Use your school's library, the Internet, or some other reference source to research the history of the product or service. Collect data about the revenue that the product or service has generated, and find a mathematical model of the data. Summarize your findings.

Business Capsules

Business Capsules appear at the ends of numerous sections. These capsules and their accompanying exercises deal with business situations that are related to the mathematical concepts covered in the chapter.

Algebra Review

At the end of each chapter, the *Algebra Review* illustrates the key algebraic concepts used in the chapter. Often, rudimentary steps are provided in detail for selected examples from the chapter. This review offers additional support to those students who have trouble following examples as a result of poor algebra skills.



308 CHAPTER 4 ▶ Exponential and Logarithmic Functions

4 Algebra Review

Solving Exponential and Logarithmic Equations

To find the extrema or points of inflection of an exponential or logarithmic function, you must know how to solve exponential and logarithmic equations. A few examples are given on page 285. These two pages show some additional examples.

As with all equations, remember that your basic goal is to isolate the variable on one side of the equation. To do this, you use inverse operations. For instance, to get rid of an exponential expression such as e^{2x}, take the natural log of each side and use the property $\ln e^{2x} = 2x$. Similarly, to get rid of a logarithmic expression such as $\log_2 3x$, exponentiate each side and use the property $2^{\log_2 3x} = 3x$.

EXAMPLE 1 Solving Exponential Equations

Solve each exponential equation.

(a) $25 = 5e^{7t}$ (b) $80{,}000 = 100{,}000e^{k(4)}$ (c) $300 = \left(\frac{100}{e^{2k}}\right)e^{4k}$

Solution

(a) $25 = 5e^{7t}$ Write original equation.
$5 = e^{7t}$ Divide each side by 5.
$\ln 5 = \ln e^{7t}$ Take natural log of each side.
$\ln 5 = 7t$ Apply the property $\ln e^{a} = a$.
$\frac{1}{7}\ln 5 = t$ Divide each side by 7.

(b) $80{,}000 = 100{,}000e^{k(4)}$ Example 4, page 304
$0.8 = e^{4k}$ Divide each side by 100,000.
$\ln 0.8 = \ln e^{4k}$ Take natural log of each side.
$\ln 0.8 = 4k$ Apply the property $\ln e^{a} = a$.
$\frac{1}{4}\ln 0.8 = k$ Divide each side by 4.

(c) $300 = \left(\frac{100}{e^{2k}}\right)e^{4k}$ Example 2, page 301
$300 = (100)\frac{e^{4k}}{e^{2k}}$ Rewrite product.
$300 = 100e^{4k-2k}$ To divide powers, subtract exponents.
$300 = 100e^{2k}$ Simplify.
$3 = e^{2k}$ Divide each side by 100.
$\ln 3 = \ln e^{2k}$ Take natural log of each side.
$\ln 3 = 2k$ Apply the property $\ln e^{a} = a$.
$\frac{1}{2}\ln 3 = k$ Divide each side by 2.

CHAPTER 4 ▶ Algebra Review 309

EXAMPLE 2 Solving Logarithmic Equations

Solve each logarithmic equation.

(a) $\ln x = 2$ (b) $5 + 2\ln x = 4$ (c) $2\ln 3x = 4$ (d) $\ln x - \ln(x-1) = 1$

Solution

(a) $\ln x = 2$ Write original equation.
$e^{\ln x} = e^{2}$ Exponentiate each side.
$x = e^{2}$ Apply the property $e^{\ln a} = a$.

(b) $5 + 2\ln x = 4$ Write original equation.
$2\ln x = -1$ Subtract 5 from each side.
$\ln x = -\frac{1}{2}$ Divide each side by 2.
$e^{\ln x} = e^{-1/2}$ Exponentiate each side.
$x = e^{-1/2}$ Apply the property $e^{\ln a} = a$.

(c) $2\ln 3x = 4$ Write original equation.
$\ln 3x = 2$ Divide each side by 2.
$e^{\ln 3x} = e^{2}$ Exponentiate each side.
$3x = e^{2}$ Apply the property $e^{\ln a} = a$.
$x = \frac{1}{3}e^{2}$ Divide each side by 3.

(d) $\ln x - \ln(x-1) = 1$ Write original equation.
$\ln \frac{x}{x-1} = 1$ $\ln m - \ln n = \ln(m/n)$
$e^{\ln(x/x-1)} = e^{1}$ Exponentiate each side.
$\frac{x}{x-1} = e^{1}$ Apply the property $e^{\ln a} = a$.
$x = ex - e$ Multiply each side by $x - 1$.
$x - ex = -e$ Subtract ex from each side.
$x(1-e) = -e$ Factor.
$x = \frac{-e}{1-e}$ Divide each side by $1 - e$.
$x = \frac{e}{e-1}$ Simplify.

STUDY TIP

Because the domain of a logarithmic function generally does not include all real numbers, be sure to check for extraneous solutions.

Chapter Summary and Study Strategies

The *Chapter Summary* reviews the skills covered in the chapter and correlates each skill to the *Review Exercises* that test those skills. Following each *Chapter Summary* is a short list of *Study Strategies* for addressing topics or situations specific to the chapter.

3 Chapter Summary and Study Strategies

*After studying this chapter, you should have acquired the following skills. The exercise numbers are keyed to the Review Exercises that begin on page 252. Answers to odd-numbered Review Exercises are given in the back of the text.**

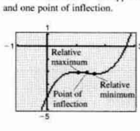
■ Find the critical numbers of a function. *(Section 3.1)* c is a critical number of f if $f'(c) = 0$ or $f'(c)$ is undefined.	*Review Exercises 1–4*
■ Find the open intervals on which a function is increasing or decreasing. *(Section 3.1)* Increasing if $f'(x) > 0$ Decreasing if $f'(x) < 0$	*Review Exercises 5–8*
■ Find intervals on which a real-life model is increasing or decreasing, and interpret the results in context. *(Section 3.1)*	*Review Exercises 9, 10, 95*
■ Use the First-Derivative Test to find the relative extrema of a function. *(Section 3.2)*	*Review Exercises 11–20*
■ Find the absolute extrema of a continuous function on a closed interval. *(Section 3.2)*	*Review Exercises 21–30*
■ Find minimum and maximum values of a real-life model and interpret the results in context. *(Section 3.2)*	*Review Exercises 31, 32*
■ Find the open intervals on which the graph of a function is concave upward or concave downward. *(Section 3.3)* Concave upward if $f''(x) > 0$ Concave downward if $f''(x) < 0$	*Review Exercises 33–36*
■ Find the points of inflection of the graph of a function. *(Section 3.3)*	*Review Exercises 37–40*
■ Use the Second-Derivative Test to find the relative extrema of a function. *(Section 3.3)*	*Review Exercises 41–44*
■ Find the point of diminishing returns of an input-output model. *(Section 3.3)*	*Review Exercises 45, 46*
■ Solve real-life optimization problems. *(Section 3.4)*	*Review Exercises 47–53, 96*
■ Solve business and economics optimization problems. *(Section 3.5)*	*Review Exercises 54–58, 99*
■ Find the price elasticity of demand for a demand function. *(Section 3.5)*	*Review Exercises 59–62*
■ Find the vertical and horizontal asymptotes of a function and sketch its graph. *(Section 3.6)*	*Review Exercises 63–68*
■ Find infinite limits and limits at infinity. *(Section 3.6)*	*Review Exercises 69-76*
■ Use asymptotes to answer questions about real life. *(Section 3.6)*	*Review Exercises 77, 78*
■ Analyze the graph of a function. *(Section 3.7)*	*Review Exercises 79–86*
■ Find the differential of a function. *(Section 3.8)*	*Review Exercises 87–90*
■ Use differentials to approximate changes in a function. *(Section 3.8)*	*Review Exercises 91–94*
■ Use differentials to approximate changes in real-life models. *(Section 3.8)*	*Review Exercises 97, 98*

* Use a wide range of valuable study aids to help you master the material in this chapter. The *Student Solutions Guide* includes step-by-step solutions to all odd-numbered exercises to help you review and prepare. The *Calculus: An Applied Approach Learning Tools Student CD-ROM* and *The Algebra of Calculus* help you brush up on your algebra skills. The *Graphing Technology Guide* offers step-by-step commands and instructions for a wide variety of graphing calculators, including the most recent models.

- ***Solve Problems Graphically, Analytically, and Numerically*** When analyzing the graph of a function, use a variety of problem-solving strategies. For instance, if you were asked to analyze the graph of

  $f(x) = x^3 - 4x^2 + 5x - 4$

  you could begin *graphically*. That is, you could use a graphing utility to find a viewing window that appears to show the important characteristics of the graph. From the graph shown below, the function appears to have one relative minimum, one relative maximum, and one point of inflection.



  Next, you could use calculus to *analyze* the graph. Because the derivative of f is

  $f'(x) = 3x^2 - 8x + 5 = (3x - 5)(x - 1)$

  the critical numbers of f are $x = \frac{5}{3}$ and $x = 1$. By the First-Derivative Test, you can conclude that $x = \frac{5}{3}$ yields a relative minimum and $x = 1$ yields a relative maximum. Because

  $f''(x) = 6x - 8$

  you can conclude that $x = \frac{4}{3}$ yields a point of inflection. Finally, you could analyze the graph *numerically*. For instance, you could construct a table of values and observe that f is increasing on the interval $(-\infty, 1)$, decreasing on the interval $\left(1, \frac{5}{3}\right)$, and increasing on the interval $\left(\frac{5}{3}, \infty\right)$.

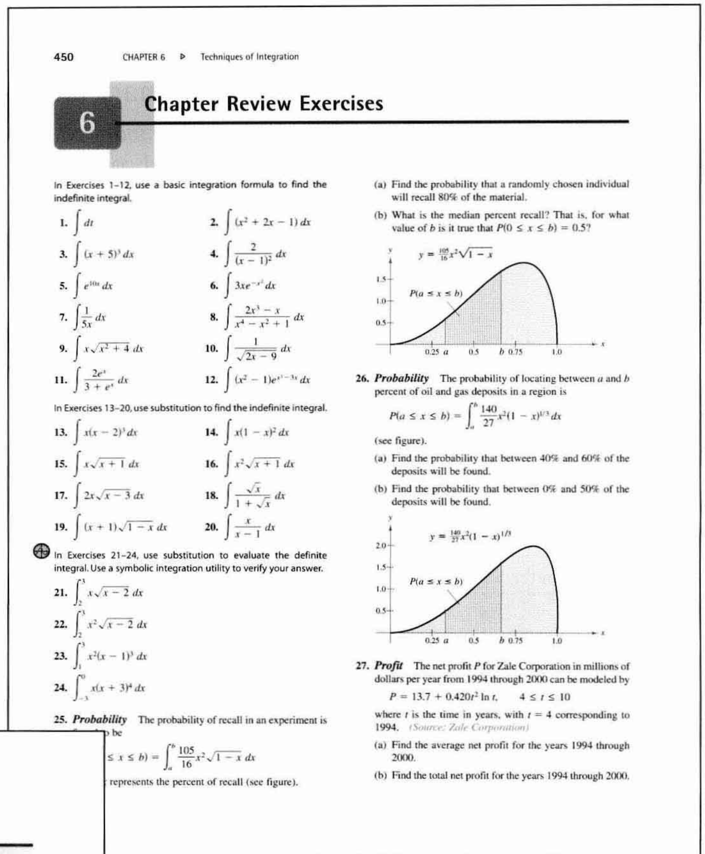
- ***Problem-Solving Strategies*** If you get stuck when trying to solve an optimization problem, consider the strategies that follow.

  1. *Draw a Diagram.* If feasible, draw a diagram that represents the problem. Label all known values and unknown values on the diagram.
  2. *Solve a Simpler Problem.* Simplify the problem, or write several simple examples of the problem. For instance, if you are asked to find the dimensions that will produce a maximum area, try calculating the areas of several examples.
  3. *Rewrite the Problem in Your Own Words.* Rewriting a problem can help you understand it better.
  4. *Guess and Check.* Try guessing the answer, then check your guess in the statement of the original problem. By refining your guesses, you may be able to think of a general strategy for solving the problem.

FEATURES

Review Exercises

The *Review Exercises* offer students opportunities for additional practice as they complete each chapter. Answers to all odd-numbered *Review Exercises* appear at the end of the text.



450 CHAPTER 6 ▷ Techniques of Integration

6 **Chapter Review Exercises**

In Exercises 1–12, use a basic integration formula to find the indefinite integral.

1. $\int dt$ 2. $\int (x^2 + 2x - 1)\,dx$
3. $\int (x+5)^3\,dx$ 4. $\int \frac{2}{(x-1)^2}\,dx$
5. $\int e^{10x}\,dx$ 6. $\int 3xe^{-x^2}\,dx$
7. $\int \frac{1}{5x}\,dx$ 8. $\int \frac{2x^3 - x}{x^4 - x^2 + 1}\,dx$
9. $\int x\sqrt{x^2+4}\,dx$ 10. $\int \frac{1}{\sqrt{2x-9}}\,dx$
11. $\int \frac{2e^x}{3+e^x}\,dx$ 12. $\int (x^2-1)e^{x^3-3x}\,dx$

In Exercises 13–20, use substitution to find the indefinite integral.

13. $\int x(x-2)^3\,dx$ 14. $\int x(1-x)^2\,dx$
15. $\int x\sqrt{x+1}\,dx$ 16. $\int x^2\sqrt{x+1}\,dx$
17. $\int 2x\sqrt{x-3}\,dx$ 18. $\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx$
19. $\int (x+1)\sqrt{1-x}\,dx$ 20. $\int \frac{x}{x-1}\,dx$

In Exercises 21–24, use substitution to evaluate the definite integral. Use a symbolic integration utility to verify your answer.

21. $\int_2^3 x\sqrt{x-2}\,dx$
22. $\int_2^3 x^2\sqrt{x-2}\,dx$
23. $\int_1^3 x^2(x-1)^3\,dx$
24. $\int_{-3}^0 x(x+3)^4\,dx$

25. ***Probability*** The probability of recall in an experiment is ... be

$\le x \le b) = \int_a^b \frac{105}{16}x^2\sqrt{1-x}\,dx$

... represents the percent of recall (see figure).

(a) Find the probability that a randomly chosen individual will recall 80% of the material.

(b) What is the median percent recall? That is, for what value of b is it true that $P(0 \le x \le b) = 0.5$?

$y = \frac{105}{16}x^2\sqrt{1-x}$

$P(a \le x \le b)$

26. ***Probability*** The probability of locating between a and b percent of oil and gas deposits in a region is

$$P(a \le x \le b) = \int_a^b \frac{140}{27}x^2(1-x)^{1/3}\,dx$$

(see figure).

(a) Find the probability that between 40% and 60% of the deposits will be found.

(b) Find the probability that between 0% and 50% of the deposits will be found.

$y = \frac{140}{27}x^2(1-x)^{1/3}$

$P(a \le x \le b)$

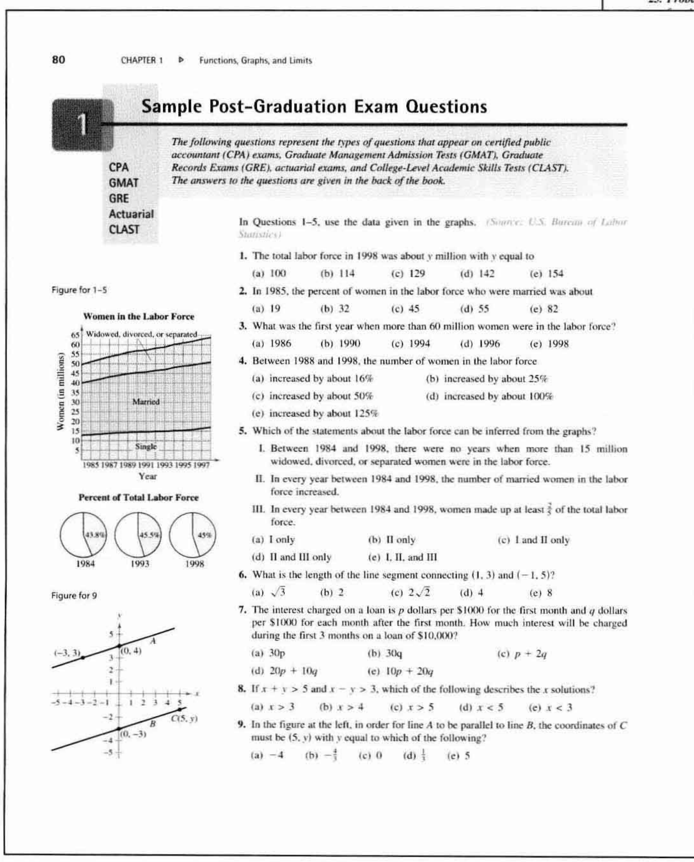
27. ***Profit*** The net profit P for Zale Corporation in millions of dollars per year from 1994 through 2000 can be modeled by

$$P = 13.7 + 0.420t^2 \ln t, \quad 4 \le t \le 10$$

where t is the time in years, with $t = 4$ corresponding to 1994. (Source: Zale Corporation)

(a) Find the average net profit for the years 1994 through 2000.

(b) Find the total net profit for the years 1994 through 2000.



80 CHAPTER 1 ▷ Functions, Graphs, and Limits

1 **Sample Post-Graduation Exam Questions**

CPA
GMAT
GRE
Actuarial
CLAST

The following questions represent the types of questions that appear on certified public accountant (CPA) exams, Graduate Management Admission Tests (GMAT), Graduate Records Exams (GRE), actuarial exams, and College-Level Academic Skills Tests (CLAST). The answers to the questions are given in the back of the book.

In Questions 1–5, use the data given in the graphs. (Source: U.S. Bureau of Labor Statistics)

Figure for 1–5

Women in the Labor Force

Widowed, divorced, or separated; Married; Single; Women (in millions); Year

Percent of Total Labor Force: 1984 43.8%; 1993 45.5%; 1998 45%

1. The total labor force in 1998 was about y million with y equal to
(a) 100 (b) 114 (c) 129 (d) 142 (e) 154

2. In 1985, the percent of women in the labor force who were married was about
(a) 19 (b) 32 (c) 45 (d) 55 (e) 82

3. What was the first year when more than 60 million women were in the labor force?
(a) 1986 (b) 1990 (c) 1994 (d) 1996 (e) 1998

4. Between 1988 and 1998, the number of women in the labor force
(a) increased by about 16% (b) increased by about 25%
(c) increased by about 50% (d) increased by about 100%
(e) increased by about 125%

5. Which of the statements about the labor force can be inferred from the graphs?
I. Between 1984 and 1998, there were no years when more than 15 million widowed, divorced, or separated women were in the labor force.
II. In every year between 1984 and 1998, the number of married women in the labor force increased.
III. In every year between 1984 and 1998, women made up at least $\frac{2}{5}$ of the total labor force.
(a) I only (b) II only (c) I and II only
(d) II and III only (e) I, II, and III

6. What is the length of the line segment connecting (1, 3) and (−1, 5)?
(a) $\sqrt{3}$ (b) 2 (c) $2\sqrt{2}$ (d) 4 (e) 8

Figure for 9

7. The interest charged on a loan is p dollars per \$1000 for the first month and q dollars per \$1000 for each month after the first month. How much interest will be charged during the first 3 months on a loan of \$10,000?
(a) $30p$ (b) $30q$ (c) $p + 2q$
(d) $20p + 10q$ (e) $10p + 20q$

8. If $x + y > 5$ and $x - y > 3$, which of the following describes the x solutions?
(a) $x > 3$ (b) $x > 4$ (c) $x > 5$ (d) $x < 5$ (e) $x < 3$

9. In the figure at the left, in order for line A to be parallel to line B, the coordinates of C must be $(5, y)$ with y equal to which of the following?
(a) −4 (b) $-\frac{4}{5}$ (c) 0 (d) $\frac{1}{5}$ (e) 5

Post-Graduation Exam Questions

To emphasize the relevance of calculus, every chapter concludes with sample questions representative of the types of questions on certified public accountant (CPA) exams, Graduate Management Admission Tests® (GMAT®), Graduate Record Examinations® (GRE®), actuarial exams, and College-Level Academic Skills Tests (CLAST). The answers to all *Post-Graduation Exam Questions* are given in the back of the text.

Supplements

Printed Resources for Instructors

Instructor's Resource Guide **by Bruce H. Edwards, University of Florida**

- Notes to the New Teacher
- Chapter summaries
- Detailed solutions to all even-numbered exercises
- Suggested solutions for all *Discovery, Technology,* and *Take Another Look* features

Test Item File

- Printed test bank
- More than 2500 multiple-choice and open-ended test items
- Mid-chapter quizzes, chapter tests, cumulative tests, and final exams
- Also available as test-generating software

Printed Resources for Students

Student Solutions Guide **by Bruce H. Edwards, University of Florida**

- Detailed solutions to all odd-numbered exercises
- Step-by-step solutions to illustrate how to arrive at the answer

Graphing Technology Guide **by Benjamin N. Levy and Laurel Technical Services**

- Detailed keystroke instructions for a wide variety of graphing calculators
- Examples with step-by-step solutions
- Extensive graphics screen output
- Technology tips

Excel Guide for Finite Math & Applied Calculus **by Eric J. Braude**

- Specialized, step-by-step instructions on how to use Excel to explore calculus concepts
- Brief introduction to Excel for those unfamiliar with the software

The Algebra of Calculus **by Revathi Narasimhan**

- Review of algebra, trigonometry, and analytic geometry required for calculus
- More than 200 examples with solutions
- Pretests and exercises with answers