The BEST WRITING on MATHEMATICS

2015

Mircea Pitici, Editor

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for Fangfang and our little Leo

Introduction

Mircea Pitici

This is the sixth anthology in our series of recent writings on mathematics selected from professional journals, general interest publications, and Internet sources. All pieces were first published in 2014, roughly in the form we reproduce (with one exception). Most of the volume is accessible to readers who do not have advanced training in mathematics but are curious to read well-informed commentaries about it.

What do I want by sending this book into the world? What kind of experience I want the readers to have? On previous occasions I answered these questions in detail. To summarize anew my intended goal and my vision underlying this series, I use an extension of Lev Vygotsky's concept of "zone of proximate development." Vygotsky thought that a child learns optimally in the twilight zone where knowing and not knowing meet—where she builds on already acquired knowledge and skills, through social interaction with adults who impart new knowledge and assist in honing new skills. Adapting this idea, I can say that I aspire to make the volumes in this series ripe for an optimal impact in the imaginary zone of proximal reception of their prospective audience. This means that the topics of some contributions included in these books might be familiar to some readers but novel and instructive for others. Every reader will find intriguing pieces here.

Besides offering a curated collection of articles, each book in this series doubles into a reference work of sorts, for the recent nontechnical writings on mathematics—with the caveat that I decline any claim to being comprehensive in this attempt. The list of book titles I give at the end of the introduction and the list of notable writings at the end of the volume contain a few entries published prior to the 2014 calendar year, in an acknowledgment that in previous volumes I overlooked materials worth mentioning. The same is surely the case for this year. The fast pace of the series, the immense quantity of literature I survey, and

the convention subtly ensconced in calling a "year" the interval from January 1 to December 31 not only make such lapses inevitable but to a high degree determine the content of the book(s). Were we to look at the same literature from July 1 of one year until June 30 of the next year, the books in this series would look very different from what you can read between these covers. That is why each volume should be seen in conjunction with the others, part of a serialized enterprise meant to facilitate the access to and exchange of ideas concerning diverse aspects of the mathematical experience.

In this volume a greater number of contributions than in the previous volumes concern mathematical games and puzzles. For many centuries and in many cultures, recreational mathematics used to be seen as a benign amusement of no immediate utility. That enduring but now old-fashioned perception has gradually changed over the past century because of at least two broad phenomena. First, the history of the most salient branches of contemporary mathematics (algebra, modern algebra, geometry, probability, number theory, graph theory, knot theory, topology, combinatorics, and even calculus) has been either rooted into or decisively influenced by "recreational" problems. Second, talented writers and popularizers of recreational mathematics (the most famous of whom was Martin Gardner) found a large audience in the public, enjoyed appreciation from select but remarkable mathematicians, and built a devoted following of like-minded authors who carry on working in the same vein, encouraged by the lasting impact of their predecessors. Recreational mathematics has a rich and sophisticated history studied in the past by a few authors who contributed brief works (notably David Singmaster); recently the scholarship is growing rapidly, as illustrated by a special issue of the journal *Historia Mathematica* dedicated entirely to recreational mathematics. Nowadays good recreational mathematics is placed midway between the intelligent but mathematically untrained public and the mathematics professionals, by virtue of linking easy-to-understand problems to serious mathematics. In other words the problems posed in high-quality recreational mathematics are comprehensible to the layperson, while pursuing and understanding the ideas developed in the solutions occasioned by the problems might require an independent learning effort the reader is free to undertake or not. Thus the context of good recreational mathematics straddles the popular and the pedagogical, having the dual value of intellectual

entertainment and optional instructional use. At its best, recreational mathematics illustrates the synergetic encounter between the ludic and the serious aspects of mathematics, as well as the one between the amateur and the professional strands of doing mathematics—just two of the many polarities of contrary/complementary features that meet and mesh in mathematical thinking and practice. I am glad that this year "recreational" mathematics is well represented in the anthology.

There is much more in this book. Some problems of recreational mathematics are at least two millennia old, representing well the nature of mathematics as *we have known it*—a type of thinking that endures forever, the timeless and unchanging mathematics of a realm at rest, in equilibrium. In that endurance we feel a flavor of the times when only a few million people lived on this planet, scattered over the Earth, and few of them thought mathematically—and those who did lived far apart, isolated from each other. But now we are in the billions, we instantly communicate with each other, and we put numbers on more things than can be measured. The mathematics adequate to this dynamic world in ceaseless disequilibrium has to have a basis different from the old mathematics, at least in some respects. It has to be a mathematics that treats the dynamic phenomena from the inside, not from the outside (just as Albert Einstein thought up relativity of motion by asking himself what would happen if one traveled as fast as light does). Glimpses of a more statistical bend in mathematical thinking are obvious in some contributions to this volume and attest not only to the ever-changing nature of mathematics but also to the merits of interpreting mathematics in broad personal and societal contexts.

Interpreting mathematics is a further stage of thinking mathematically—not a "higher" stage or a "lower" stage, just an essentially *different* one. Interpreting mathematics is not about mathematical truth (or any other truth); it is a *personal* take on mathematical facts, and in that it can be true or untrue, or it can even be fiction; it is vision, or it is rigorous reasoning, or it is pure speculation, all occasioned by mathematics; it is imagination on a mathematical theme; it goes back several millennia and it is flourishing today, as I hope this series of books lays clear. Fragments in Plato's philosophical dialogues qualify as interpreting mathematics, and so does Edwin Abbott's *Flatland*, and (for example) the work of our contemporaries Ian Stewart, Steven Strogatz, Edward Frenkel, Jordan Ellenberg, and many others, including the contributors

to this volume. Interpreting mathematics is a creative domain, inclusive but also competitive—and certainly potent, despite its neglect in academic circles. Interpreting mathematics points toward protean qualities of mathematics not immediately obvious in doing mathematics per se. An accepted mathematical result is merely the egalitarian premise from which each of us can part with the commonly shared view by interpreting it idiosyncratically, as we please or even as it suits us. While mathematics is a "great equalizer" in a sociological sense (as Jaime Escalante famously proclaims in the 1988 movie *Stand and Deliver*), interpreting mathematics is a subtle ability, as much competitive as it is differential. Interpretation sets each of us apart, even when we speak about the same mathematical fact. Interpreting mathematics applies to a part of human affairs where opportunities rule, not constraining mathematical rules. To speak about interpreting mathematics sounds odd, but it seems so only because the customary indoctrination served by our school system pervades the common views of mathematics, both among mathematicians and the lay public. For decades the penury of talented authors able to interpret mathematics in original ways has affected the interaction between mathematics and domains in which mathematical methods were coopted. Lack of reflection on the proper context of applied mathematical thinking perverted the humanities, the social sciences, and even the study and the practice of law—to name just a few areas.

This state of affairs is changing and *The Best Writing on Mathematics* series takes notice of the change. Criticisms of (ab)using mathematics and statistics crop up all the time, with a few included in this book. For me, this is payback of sorts. I once made the naive misstep to suggest, at a well-regarded business school, that mathematics relates to reality in subtler ways than is immediately apparent and that I would rather pursue my hunches than submit to the expensive dogma taught in the "mathematical finance" courses. For that I was not only disparaged but shown the door. Out I went, never to regret it—although, for a long while after that, I "pursued" only survival, as a lesson in what can happen if I take initiative in a place that nominally encourages it. I had been fooled by my preconceptions concerning the abstract notion of free inquiry; I had set up myself for the misadventures that come with resisting the enforcement of dogmatism. Those blunders did not shake my passion for inquiry, but it cured me of the tendency to speak my mind. (Other cures followed, also in the name of noble-sounding ideals; my favorite is

the one administered by courts under the slogan "in the best interest of the child.") Since then I became a lot more cautions with my suggestions concerning mathematics; I learn from my errors, the hard way.

Yet I still venture a bit in talking about mathematics, here and there—now prudent, aware that attempting to crack the thought establishments is fraught with dangers. As an example, I can say that the process of editing each volume in this series is a lesson in working with uncertainty while at the same time interpreting mathematics. The contrast between my limited knowledge and the limitless possibilities available to all the people who gloss on mathematics offers me palpable practice for a general mnemonic that serves well in other endeavors. I generalized it into a theoretical and practical principle, which I call "the paradox of reward." The paradox of reward says that in a competitive, fair, unpredictable, and infinitely complex environment, the most valuable knowledge is to know how to be rewarded for ignorance; in other words, more reward is available for taking advantage of ignorance (if one finds such a path to reward) than it is for taking advantage of knowledge. Of course I am *not* saying that ignorance is preferable to knowledge; it is *not*. I am saying that in certain environments harvesting rewards off ignorance is (by far) more valuable than seeking rewards for knowledge. This might seem to have little to do with mathematics; yet to my mind it is nothing else *but* interpreting mathematics, in a world so complex that ignorance is unavoidable but ignoring its benefits is avoidable. This subject is a lot vaster than I sketched in these few sentences, and it has consequences not only for learning and teaching mathematics but also for incorporating the private interpretation of mathematics in strategic thinking. Yet I am mindful of the dangers of venturing too far in speaking unconventionally about mathematics and interpreting mathematics, so I leave it for another occasion.

Overview of the Volume

I feel rewarded to collect in this book thoughts and perspectives on mathematics I could never think up myself.

Michael Barany and Donald MacKenzie locate the center of the mathematical activity done in institutional settings (and occasionally in private homes) at the blackboard; they note that blackboards are key objects that influence the organizing of the research and teaching

spaces, while chalk-writing on blackboards influences the logistics and the overall manner of mathematical communication.

Pradeep Mutalik finds that the repeated experience of feeling right when we suddenly comprehend the solution to a problem or a puzzle has had a positive evolutionary role in defining us as humans, both cognitively and emotionally.

Colm Mulcahy and Dana Richards write an informed centennial appreciation of the life and work of Martin Gardner, that remarkable polymath who inspired many mathematicians and laypersons to take up mathematical games and similar challenges.

Arthur Benjamin and Ethan Brown teach us how to construct an unlimited number of customized magic squares by improvising on a few ingenious templates.

Toby Walsh starts with the popular Candy Crush game as a guidepost for his discussion of the factors that determine the difficulty of solving computational problems in mathematics.

Marianne Freiberger takes us to the billiard room; she explains how the trajectory of a ball rolling on the pool table leads to mathematically complex problems related to chaos theory, the conductivity of metals, and other . . . infinite surprises.

Erik R. Tou models juggling numerically, to show that it is mathematically similar to the morphing game of transforming one word into a very different one by incremental steps that admit changes of only one letter at a time.

Scott Aaronson dissects the intricacies of the notion of randomness and connects it to the study of paradoxes, complexity, and quantum mechanics.

Dana Mackenzie describes how biologists, physicists, and mathematicians interact(ed) to overcome theoretical obstacles encountered in the birth and growth of synthetic biology.

In a similar vein, Natalie Wolchover describes the interdisciplinary efforts undertaken by researchers interested in the Tracy-Widom distribution associated with phase transitions in interactive systems of various types.

Eli Maor and Eugen Jost present (and illustrate beautifully) the basic geometric properties of the logarithmic spiral, cycloid, epicycloids, and hypocycloids—some of the best-known curves studied, over the centuries, in connection to natural phenomena and physical motions.

Burkard Polster analyzes the mathematical properties of several non-circular shapes of constant width and shows us how they have been applied to various gadgets and playful devices.

The quickest way to summarize the brief article by Annalisa Crannell, Marc Frantz, and Fumiko Futamura is to say that they look at Dürer's perspective drawing from several different perspectives!

Vi Hart and Henry Segerman ask whether there are groups of symmetries that can be visualized using real-life objects but have never been represented as such—and propose a novel modeling of the quaternion group.

John Conway and Alex Ryba use wordplay and a humorous ingenuity to discuss the merits of several different proofs they give to an old geometry problems that looks deceptively easy (until you try to solve it).

Gila Hanna and John Mason discuss the many facets, relative merits, and theoretical pedigrees of various terms used by mathematicians or mathematics educators to qualify the worthiness of proofs—with the main reference to a similar attempt by Timothy Gowers.

Jim Fey, Sol Garfunkel, and their coauthors formulate five tenets they consider important to be taken as guiding principles for the mathematics education reform at high school level (in the United States).

Guili Zhang and Miguel A. Padilla compare multiple aspects of mathematics instruction in China and the United States, based on previous theoretical and empirical studies.

Against commonly held wisdom, Benoît Rittaud and Albrecht Heeffer argue that the pigeonhole principle, usually attributed to Dirichlet, was stated in writing at least two centuries earlier in *Selectae Propositiones*, a book by Jean Leurechon.

Lisa Rougetet traces the earliest written descriptions of the popular game of Nim to a treatise written at the beginning of the sixteenth century by Luca Pacioli and follows the subsequent European developments of the game since then.

Jan von Plato considers the context of mathematical ideas and the personalities that shaped German mathematician Gerhard Gentzen's ordinal proof theory—and how this work relates (or does not!) with a theorem by Reuben Goodstein.

James Franklin illustrates with several well-chosen examples the local-global synergy in mathematics, one of the many conceptual polarities that characterize mathematical thinking.

Carlo Cellucci reviews many opinions on what constitutes mathematical beauty and its role in mathematics; he concludes that aesthetic factors play an indirect epistemological role in discovery via their selective role in choosing what hypotheses to consider.

Mark Balaguer argues that philosophers of mathematics are mainly concerned with the meaning of mathematical discourse and that the semantic theories they adhere to can lead to claims hardly acceptable for the mathematicians.

Steven Strogatz discerns three broad types of rapport with mathematics in the general public and tells us how he honed his talent for writing about mathematics by paying attention to the writing qualities of masters in similar trades.

Domenico Napoletani, Marco Panza, and Daniele C. Struppa examine the methodological underpinnings and the philosophical implications of using ever-more powerful computing techniques in the modeling of complex phenomena.

Andrew Gelman and Eric Loken caution that evidential claims of statistical significance in research journals are often spurious because of multiple factors related to the gathering of data and its interpretation; they give several suggestive examples.

Jeffrey S. Rosenthal tells the true story of how his statistical expertise led to the discovery and prosecution of fraudulent lottery winnings in Ontario, Canada.

David Hand explains a bias of expectations that precludes us from perceiving the increased likelihood of coincidences following the rapid combinatorial growth of possibilities that comes with the increase of the number of simple events.

More Writings on Mathematics

Every year I started this section by naming one book outstanding among all others. This time I cannot decide on only one; I give two, both excellent and badly needed reference books: Lizhen Ji's *Great Mathematics Books of the Twentieth Century* and *Encyclopedia of Mathematics Education*, edited by Stephen Lerman.

Now, as usual, I roughly group the other titles by theme (full references are at the end of the introduction); some of the books listed here are not easy to categorize, but I made ad hoc choices for the sake of expediency.

A handful of books blend mathematical ideas with describing the world from nonmathematical viewpoints, sometimes with a strong historical perspective—a growing trend I signaled previously in these pages and picking up steam lately. Thus are Jeff Suzuki's *Constitutional Calculus*, Anders Engberg-Pedersen's *Empire of Chance*, Keith Tribe's *The Economy of the World*, along with *The Norm Chronicles* by Michael Blastland and David Spiegelhalter, and *How Reason Almost Lost Its Mind* by Paul Erickson and colleagues. Less shy with giving mathematics a prominent role in daily life are *Grapes of Math* by Alex Bellos and *Mathematics and the Real World* by Zvi Artstein.

In the history of mathematics I note Joseph Mazur's *Enlightening Symbols*, Alexander Amir's *Infinitesimal*, Amir Aczel's *Finding Zero*, Jiri Hudecek's *Reviving Ancient Chinese Mathematics*, Vedveer Arya's *Indian Contributions to Mathematics and Astronomy,*David Reamer's *Count Like an Egyptian*, and, slightly more technical but with well-chosen historical vignettes, *From Mathematics to Generic Programming* by Alexander Stepanov and Daniel Rose. Books focused on personalities are *John Napier* by John Havil, *How Euler Did It Even More* by Edward Sandifer, *Beyond Banneker* by Erika Walker, and *Peter Lax, Mathematician* by Reuben Hersh. A broad swipe of history is covered in *Classical Mathematics from Al-Khwarizmi to Descartes* by Roshdi Rashed. Theme-based histories are *Whatever Happened to the Metric System?* by John Marciano, *The New Math* by Christopher Phillips, and *The Palgrave Centenary Companion to "Principia Mathematica"* edited by Nicholas Griffin and Bernard Linsky. Focused on the formerly communist countries are *Mathematics across the Iron Curtain* by Christopher Hollings and *Pearls from a Lost City* by Roman Duda. Other (auto)biographical or celebratory volumes are *Gordon Welchman, Bletchley Park's Architect of Ultra Intelligence* by Joel Greenberg, *Wearing Gauss's Jersey* by Dean Hathout, *My Life and Functions* by Walter Hayman, as well as the collective volumes *Arnold* edited by Boris Khesin and Serge Tabachnikov, *Alexandre Grothendieck* edited by Leila Schneps, and *Four Lives [of Raymond Smullyan]* edited by Jason Rosenhouse.

Some books on the interactions between mathematics and other disciplines: *The Oxford Handbook of Computational and Mathematical Psychology* edited by Jerome Busemeyer and his collaborators; *Biographical Encyclopedia of Astronomers* with Thomas Hockey as editor-in-chief; *Scientific Visualization* edited by Charles Hansen et al.; *Bond Math* by Donald Smith; *Measuring and Reasoning [in Life Sciences]* by Fred Bookstein; *Mathematics*

for the Life Sciences by Erin Bodine, Suzanne Lenhart, and Louis Gross; *Why You Hear What You Hear* by Eric Heller; *Classical Mechanics with Calculus of Variations and Optimal Control* by Mark Levi; and *The Sabermetric Revolution* by Benjamin Baumer and Andrew Zimbalist. Particularly on mathematics and art we have *Manifold Mirrors* by Felipe Cucker, *Mathematics in 20th Century Literature and Art* by Robert Tubs, *Explaining Beauty in Mathematics: An Esthetic Theory of Mathematics* by Ulianov Montano, *Crafting Conundrums* by Ellie Baker and Susan Goldstine, and *Geometrical Objects* edited by Anthony Gerbino.

Expository mathematical writing largely accessible to the general readers are *Mathematical Curiosities* by Alfred Posamentier and Ingmar Lehman; *How Not to Be Wrong* by Jordan Ellenberg; *Everyday Calculus* by Oscar Fernandez; *Beautiful Geometry* by Eli Maor and Eugen Jost; *Math Bytes* by Tim Chartier; *Explorations in Topology* by David Gay; *Mathematical Elegance* by Steven Goldberg; *Symmetry* by Ian Stewart; *The Fascinating World of Graph Theory* by Arthur Benjamin, Gary Chartrand, and Ping Zhang; *Things to Make and Do in the Fourth Dimension* by Matt Parter; *Mathematical Games, Abstract Games* by João Neto and Jorge Silva; *Paradoxes in Mathematics* by Stanley Farlow; and two books by Raymond Smullyan: *A Beginner's Guide to Mathematical Logic* and *The Gödelian Puzzle Book*. Well written and copiously illustrated are *Math in 100 Key Breakthroughs* by Richard Elwes, *The Mathematics Devotional* by Clifford Pickover, *A Curious History of Mathematics* by Joel Levy, and *Really Big Numbers* by Richard Schwartz.

Of the many books on mathematics education, I mention *Leaders in Mathematics Education* by Alexander Karp and David L. Roberts; *How to Study as a Mathematics Major* by Lara Alcock; *Effective Content Reading Strategies to Develop Mathematical and Scientific Literacy* by David Pugalee; the anthology *We Need Another Revolution* by Zalman Usiskin; *Questioning Numbers* by Karin Gwinn Wilkins; *Handbook on the History of Mathematics Education* edited by Alexander Karp and Gert Schubring; *Research Trends in Mathematics Teacher Education* edited by Jane-Jane Lo, Keith Leatham, and Laura Van Zoest; *Transforming Mathematics Instruction* edited by Yeping Li, Edward Silver, and Shiqi Li; *Exploring Mathematics through Play in the Early Childhood Classroom* by Amy Parks; *Mastering Basic Math Skills* by Bonnie Britt; *Putting Essential Understanding of Functions into Practice* by Robert Ronau and his collaborators; and the collective volumes *Using Research to Improve Instruction* edited by Karen Karp, *101*