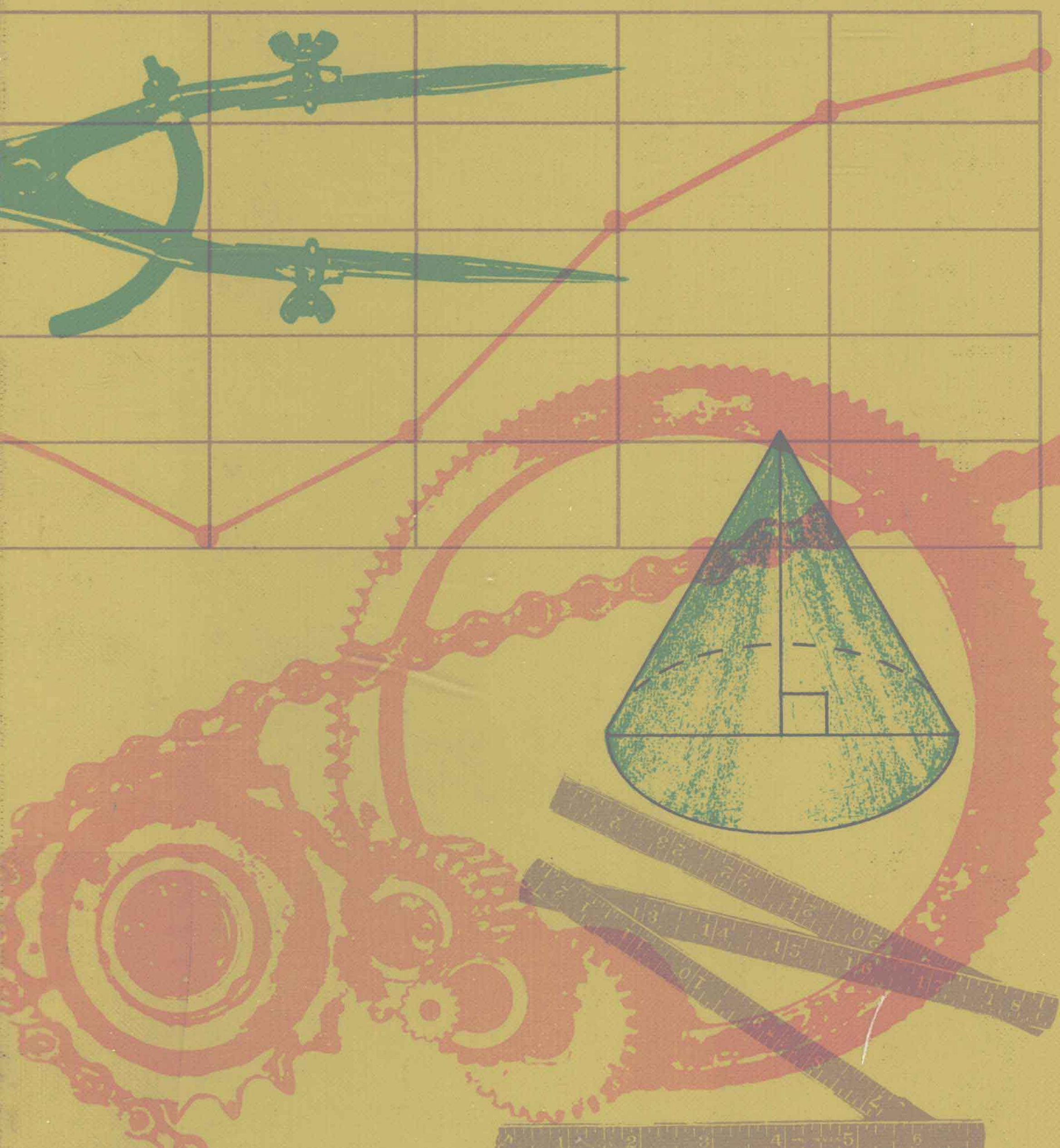
MODERN APPLIED MATHEMATICS

Gold · Carlberg



MATHEMATICS

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TEACHER'S MANUAL

MATHEMATICS

We are pleased to present *Modern Applied Mathematics*, a textbook in the Houghton Mifflin Basic Education Program.

Modern Applied Mathematics was developed to provide for those students who require a background of the essentials of mathematics for use in vocational or consumer situations. The authors of this textbook believe that students learn best when they are actively involved in what they are learning and are aware of its relevance. In demonstrating that belief, the authors have written a textbook which utilizes familiar examples of applied mathematics and directed experimentation by the student as integral components in the instruction. This approach makes the book particularly well-suited for use by students with non-academic interests. In general, *Modern Applied Mathematics* will provide students with the firm background needed for jobs in business and industry, as well as for everyday use.

Modern Applied Mathematics represents a full-year course in arithmetic, informal geometry, formulas, and trigonometry. Every facet of the book has been used successfully in the classroom. The authors are experienced teachers in the field of applied mathematics, and the editorial adviser, Albert E. Meder, is nationally known for his involvement in the various proposals for changes in the mathematics curriculum.

TEACHER'S MANUAL AND ANNOTATIONS

Basic Philosophy

The rapid growth of technological industries in recent years has broadened the range of endeavors to which mathematics can be properly applied and has increased the opportunities for technicians. This situation dramatically underlines the growing importance of providing for competence in the use of basic mathematical skills. *Modern Applied Mathematics* was written to provide the student with practice in those essential skills, while, at the same time, illustrating the importance of those skills in the fields of commerce and industry. Every effort has been made to create a book which will allow the student to:

1. find success through his own efforts;
2. progress at his own rate of speed; and
3. gain experience of typical everyday needs.

Modern Applied Mathematics is based on the universally accepted principle that a student learns best when he is actively engaged in what he is studying. Accordingly, the authors have attempted to strike a balance between discussion and discovery by augmenting the formal discussion of topics with experimentation by the student. Interest is sustained by keeping discussions brief and relatively nontechnical, and by explaining difficult concepts by means of examples rather than formal definitions. Variety in both the instructional and exercise material provides the necessary diversity of interests required by students with short spans of attention.

Throughout the book the authors have tried to present those concepts from arithmetic, geometry, trigonometry and formula solving in which the more complex applications of mathematics are rooted. Mastery of these skills will give the student the necessary background for work in more specialized areas.

Features of the Textbook

Modern Applied Mathematics was written for the specific purpose of maintaining and extending the student's mathematical skills through the study of many of the common applications of mathematics. To fulfill that purpose, the textbook has been written and designed to include the following features:

1. Emphasis is placed on the functional role of mathematics as a problem-solving tool in business and industry.
2. The use of technical terms and symbols to clarify discussions is kept to a minimum. Explanations are briefly stated in simple, crisp language supported by appropriate illustrative examples.

3. An abundance of carefully selected exercises, graded A or B, in order of increasing difficulty, provides for individual student differences.
4. Numerous activities are included as a motivating way to reinforce and extend the understanding of concepts through experimentation.
5. Self-Analysis Tests provide for periodic self-evaluation of the understanding of important ideas.
6. Optional sections, including several dealing with flow charts, may be used as enrichment material or as challenging projects for individual work by the abler students.
7. Chapter Summaries and Chapter Tests serve as a built-in evaluation program. A Glossary showing the pages on which important terms are introduced is also provided.
8. Maintaining Your Skills sections at the end of each chapter provide for students needing additional practice of basic computational skills.
9. Chapters are divided according to major topics, indicated by a heading preceded by a small colored square. Further subdivision of chapters into sections with numbered sideheads allows for easy identification and reference.
10. Every chapter opens with one or more eye-catching photographs which illustrate varied applications of mathematics in industrial and commercial settings. A concise explanation relates the photographs to the topics discussed in the chapter.
11. Clearly stated objectives preface each chapter to give positive direction to the student's investigation of the topics to be studied.
12. The functional use of color calls attention to significant terms, ideas, and rules. Clear and open typography gives pages an uncluttered, inviting appearance.
13. Numerous diagrams in discussion and exercise sections are an integral part of the instructional material.
14. Teaching aids to supplement this Teacher's Edition include a complete Solution Key and Progress Tests.

Suggestions for Teaching

The best way to use any particular textbook varies from teacher to teacher and from student to student. Therefore, any printed suggestions such as these cannot be expected to fit all situations. Nevertheless, the general comments here and the annotations on the text pages will serve to clarify the authors' point of view.

The material in *Modern Applied Mathematics* is arranged so that students have to concentrate on only one basic idea at a time. The division of each chapter into brief sections and subsections enables the teacher to plan the course to fit the needs and abilities of the class.

Teachers are urged to evaluate students essentially on the basis of their success in solving problems rather than on their ability to verbalize definitions, rules, and procedures.

Discuss with the students how to make effective use of the organization of the textbook. For example, call their attention to the Chapter Summaries and discuss possible ways of using them. As the course proceeds, show students how to locate topics and terms in the Contents, the Glossary, and the Index.

By emphasizing the importance of self-reliance and a questioning attitude, you will be developing your students' mathematical maturity.

USES OF THE EXERCISE MATERIAL

Exercises in this book are grouped into categories labeled "A" and "B" in order of increasing difficulty. Not all exercise sets contain "B" exercises, although many do. This organization will assist the teacher in making assignments at varying levels of difficulty. All students should be able to do the "A" exercises; "B" exercises are somewhat more challenging. In assigning exercises, teachers should consider, among other things, the following factors:

1. Exercises are usually closely related to the examples given in the discussion portion of the text. Those students who experience difficulty in solving a particular exercise should be encouraged to review the examples given in the text for assistance.

2. Some sets of exercises will require more time than others; therefore, it is suggested that the teacher preview each set of exercises before assigning homework. (That preview may be made easier with the complete Solution Key at hand.) Most exercise sections contain sufficient exercises to permit the teacher to use some for classroom demonstration, while reserving others for homework. Some teachers may choose to dispense with homework and to require all exercises to be done in class, either orally or as deskwork, depending on the nature and degree of difficulty of the exercises.

3. Care should be taken not to overload students with the assignments. A few exercises carefully done are generally of more benefit to the student than a greater number done hastily.

Answers to all exercises and problems are printed at the back of this Teacher's Edition; complete solutions for these are given in the separate Solution Key.

USES OF THE ACTIVITIES

This book contains numerous activity sections to provide the students with a "hands-on" experience of mathematics in applied situations. Some of the activities described in the textbook require the use of special materials and measuring instruments. Others require only a pencil, straightedge, compass, and paper. In most instances, however, the list of materials is given only as a suggestion. The teacher may wish to augment the materials needed or make appropriate substitutions for them as needs arise. It is suggested that the teacher make a thorough preview of each activity to ensure the effective use of classroom time in carrying out the experiment. The Assignment Guide at the end of this Manual gives advance notice to the teacher of upcoming experiments. A reading assignment for the students on the details of the experiment is also strongly recommended. The activities can be carried out either on a group or individual basis.

USES OF THE TESTS

Evaluation and testing material in this book falls in two categories:

1. Self-Analysis Tests; and
2. Chapter Summaries, Chapter Tests, and the Glossary.

Self-Analysis Tests appearing regularly throughout the text permit the student to test his own understanding of the material he has been studying. A time factor is suggested for the completion of each test as a means of evaluating both accuracy and speed. Answers to the Self-Analysis Tests are bound at the back of the text, beginning on page 407. Encourage the student to check his own answers and, in the event of errors, to refer to the discussions in the text for clarification. The teacher may find the Self-Analysis Tests adaptable for supplying programmed instruction for certain students.

Chapter Tests are designed to give a more comprehensive evaluation of the student's understanding of the topics presented in each chapter. Chapter Tests should be announced in advance to allow ample time for review. The Chapter Summary is well-suited for this purpose. The Glossary (pages 395–400) provides a convenient reference of important terms and fundamental principles of the mathematics being studied, and indicates the page on which they are introduced.

USES OF OPTIONAL MATERIAL

Also included in the text are a number of discussions, activities, and exercises designated as optional. This material is intended to allow for individual differences in ability. Optional activities, for example, can be used for independent study by abler students. Flow charts appear in the text in several places, and add a contemporary touch to the overall presentation. For further details on the use and application of flow charts as instructional tools, refer to page 6, **1–6**(1) of this Manual. A detailed outline showing the suggested usage of optional materials for an enriched curriculum is given in the Assignment Guide at the end of this Manual.

DETAILED COMMENTS ON THE TEXT

Chapter 1 Arithmetic Review

This chapter reviews the basic arithmetic operations involving whole numbers and common and decimal fractions needed to solve problems of applied mathematics. Many commonplace instances of applied mathematics, such as templates and invoices, are used to give the student concrete examples for practice in calculating. Flowcharting is introduced as an interesting aid in carrying out certain arithmetic operations. The "divide and average" method for determining square roots is introduced as an alternative to the traditional arithmetic procedure.

1–1 (1) The Self-Analysis Tests are designed to be used as self-evaluation tests, serving to point out to the student his strengths and weaknesses in understanding the material presented. A time factor is recommended in order to encourage speed in computations. Because much of Chapter 1 is a review of the basic operations of arithmetic, several of the Self-Analysis Tests precede the formal presentation of the material found in the tests. This arrangement is designed to allow those students who exhibit competence in one phase of the review to advance to later sections in the chapter. Beginning with Chapter 2 and continuing throughout the remaining chapters, the Self-Analysis Tests will always follow the formal discussion of the material being tested.

(2) The Exercises and Problems in this book are grouped into categories labeled "A" and "B." All students should be able to do the "A" exercises. "B" exercises are somewhat more challenging. Students who have successfully completed the "A" exercises should be encouraged to try the "B" exercises.

(3) Some students may recognize the associative and commutative properties of addition being used here. Wherever possible the authors have refrained from introducing technical mathematical terminology, feeling that, for the purposes of applied mathematics, practice in calculating should take precedence over analyzing the properties of a number system.

1–5 The authors have tried to eliminate much of the confusion surrounding the order of operations in an arithmetic expression lacking the usual symbols of inclusion. Notice that the exercises in this section ask only that the grouping symbols be inserted in the appropriate locations in order to establish the truth of the arithmetic statement. In the Exercises throughout the book care has been taken to delineate clearly for the student the elements to be grouped within a given expression.

1–6 (1) Students often enjoy flowcharting familiar processes such as the procedure used in starting a car or in buying merchandise. The teacher will find that many of the Activities and constructions treated later in the book can be readily adapted to flowcharting in order to facilitate the student's understanding. Making an odometer (page 54) and bisecting a line segment (page 256) are two examples suitable for flowcharting.

(2) The shapes used in the flow charts in this book are commonly employed. However, there is no general agreement as to what shape should be used for a particular operation. What matters is that the flow chart should be consistent in using the particular shapes to indicate particular kinds of operations.

1–8 Challenge Problems are located at several places in the book, and may be used profitably by students of varied abilities. Because their solutions entail a combination of arithmetic operations, they can serve as a review for those students who have encountered little difficulty in handling the material thus far. To accommodate students who have experienced difficulty with the material, the teacher can assign problems from this set which require using arithmetic operations related to the student's weaknesses.

1–11 Students often experience difficulty in determining whether a fraction is in lowest terms. Suggest that the student ask himself whether there is a number which is a factor of (that is, can be divided into) both the numerator and denominator. Have him use this number to form a fraction according to the Principle of 1, and follow the procedure shown in Example 2. Have the student test the resulting fraction as often as needed until no number can be found which is a factor of both the numerator and the denominator. With practice the student learns to choose progressively larger common factors, and eventually the greatest common factor.

1–13 The authors have purposefully avoided introducing the concept of cancellation in discussing the multiplication and division of fractions. The number of mathematical heresies propagated in the name of this "short cut" seems to militate against its inclusion in the text here. Students will appreciate knowing that the rules for operation with fractions in the panels on pages 27 and 28 are adequate for all purposes, and need no further qualifications placed upon them.

1–18 The authors prefer the "divide and average" method for determining square roots over the technique frequently taught in arithmetic for these reasons: First, it is a self-correcting procedure. Even though a numerical error may produce a relatively poor estimate for the square root of the given number, the effect of the error is dissipated after several repetitions of the method. Second, unlike the traditional method, which uses operations largely foreign to the student (pairing of digits, doubling of trial roots, and so on), the present method uses only the addition and division of decimals.

Chapter 2 Measurement

This chapter familiarizes the student with many of the common devices and methods used in measuring distance, area, volume, weight, time, and temperature. In addition, it will introduce the student to several less familiar kinds of measurements, such as those which record the amount of gas or electricity used in a certain period of time. Included in the chapter are ten Activities designed to acquaint the student with the instruments, methods, and difficulties involved in making accurate measurements.

In covering the material in this chapter, we suggest that the teacher try to stress the following facts about measurement:

1. Measurement is essentially the process of comparing a given magnitude (distance, area, volume, and so on) with an appropriate standard unit of measure. See especially the treatment of area measurement (pages 65–67) and of angle measurement (pages 77–79).

2. Most standard units of measure (the yard, the meter, the degree, and so on) originated either from convenience or from convention. This arbitrary nature of standard units is hinted at in the Activity on weight. See especially Exercise 8, page 77.

3. Measurement is always approximate. The results of the Activities on the stopwatch, the revolution counter, and the temperature scales (pages 87–89) will help the student verify this fact.

4. The accuracy of a measurement is jointly dependent upon, among other things, the type of measuring device being used and the thing being measured. The student should be brought to realize that, for example, a micrometer is more accurate than an odometer in measuring extremely small dimensions, whereas an odometer is more precise than a micrometer in measuring large distances.

2–1 **(1)** An Activity is designed to let the student learn by doing. Each of the ten Activities in this chapter is self-contained, to give the teacher flexibility in fixing the order of their presentation. The teacher may wish to organize the class into small groups and rotate the work on each experiment among the groups. A master chart may be helpful in recording each student's progress in handling the experiments. Encourage the students to work at their own rates of speed and to confer with one another in checking the results of their experiments. Because of the large number of manipulative devices needed for these Activities, we suggest careful pre-planning to ensure effective use of the time allowed for these experiments.

(2) To subtract C, place the compass at the end of B and measure back in the opposite direction.

The student will usually conclude that addition of line segments involves extension in the same direction, while subtraction requires extension in both directions. The teacher can profitably use this observation to strengthen the student's understanding of the inverse operations of addition and subtraction. Refer to Exercises 16–22, page 5, for some arithmetic counterparts of the Exercises here.

2–7 The authors have chosen to introduce the measurement of area through the use of the square unit. It is their belief that the student handles area problems more satisfactorily when asked to deal with the general case using the unit area, leaving aside for the moment any consideration of the dimensions of the unit or of the necessity of converting from one unit to another. The emphasis here is on understanding the concept of determining the area of a figure, which is essentially a process of comparison. Later, when this process is sufficiently clear to the student, he will be required to enunciate the specific dimensions involved and carry out certain specified conversions (Chapter 6). The same thinking applies to the treatment of volume in Section 2–9 of this chapter.

The teacher can provide additional help for students having difficulty in working with area by making area patterns from cardboard like those shown in the Exercises on pages 66–67.

2–14 Students may offer a variety of explanations to account for differences in the results, such as irregularities in the wood, inexperience in using the stopwatch, and so on. Reasonable explanations of the discrepancies are acceptable; highly technical reasons are not expected. It is more important that the students become aware of the inevitability of error in the process of measuring, and the consequences of this fact.

Chapter 3 Geometric Figures

This chapter acquaints the student with the basic figures of plane and solid geometry, and prepares for a fuller treatment of them in subsequent chapters. The study of their properties is informal, rather than deductive, and utilizes the discovery approach to facilitate understanding. The intent is to provide the necessary background for those whose work will require an understanding of geometric figures and relationships. Included in the chapter are several Activities in which the student will make models of prisms, pyramids, and other polyhedrons as a means of investigating the properties of these solids. The chapter also contains a thorough treatment of the formulas for determining the measure of certain angles associated with the circle.

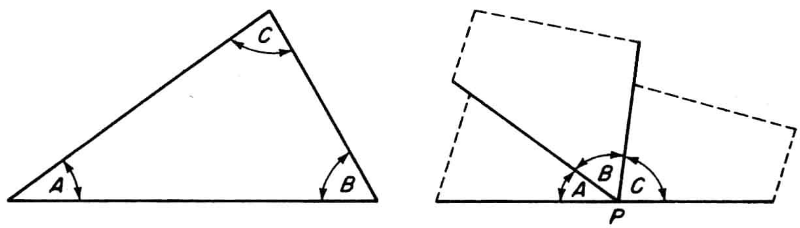
3–1 Students frequently experience difficulty in conceptualizing pure geometric forms. The teacher can assist the student in reaching an intuitive idea of each geometric figure by referring to familiar, physical objects having properties approximately similar to those of the geometric figure. The abler students will appreciate the difference between an object and its geometric form. It is unnecessary, however, to dwell upon theoretical aspects of the subject.

3–2 **(1)** You may wish to point out to the student that acute angles and obtuse angles, contrary to right angles, can exhibit a considerable range in their measures.

(2) Point out to the students that three or more angles are not called complementary even though the sum of their measures is 90°. This is also true in the case of supplementary angles. The definitions specify *two* angles.

3–4 The teacher may wish to suggest the following as an alternative method for determining the sum of the measures of the angles of a triangle.

1. On light cardboard draw a triangle and label the angles A, B, and C in their interiors.
2. Using scissors, cut out the triangle and then cut off each angle from the triangle.
3. Position the three angles so that they share a common point P as vertex, and their common sides lie flat, as in the diagram below.



4. If you place your protractor over the angles so that its center is at P, the outside edges of the angles should coincide with the straight edge of the protractor, indicating that the sum of the measures of angles A, B, and C is 180°.

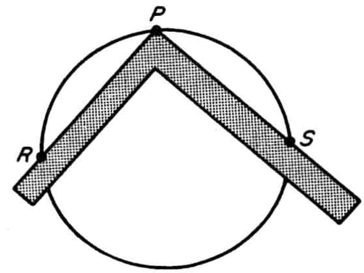
The teacher may wish to refer the student to page 108, Exercise 4 to review the meaning of a straight angle. As always, encourage those students whose protractors and angles do not align to analyze the possible causes of inaccuracy in their work.

3–6 The study of three-dimensional geometric figures is always more complicated when they are represented on a two-dimensional printed page. Accordingly, the authors have attempted in the next two sections to remove this obstacle to the student's understanding by having the student make actual models of the more common types of solid figures. Care should be exercised in the labeling of the different parts of these solids. Some students may wish to use other media in constructing these solids, such as wood, polystyrene, plastic, and so on. An interesting class project to complement the present study of polyhedrons is to construct a mobile using the models made by the students as weights.

3–8 **(1)** The students can demonstrate this definition by using a pencil with a piece of string tied to it. Point out that this simple device is commonly used by carpenters, gardeners, and others for drawing circles and arcs of sizeable radius.

(2) Students need to be made aware of the distinction between "the degree measure of an arc" and "the length of an arc." You can use the figure on page 133 to illustrate three arcs that have the same degree measure but unequal lengths. Emphasize that the *degree measure* of an arc does not depend upon the radius of the circle, whereas the *length* of an arc does. Thus, $m\widehat{AB} = m\widehat{CD} = m\widehat{EF}$, but length of $\widehat{EF}$ > length of $\widehat{CD}$ > length of $\widehat{AB}$.

3–9 Challenge the students to find a diameter of a circle with an unmarked carpenter's square, using what they know about the measure of an inscribed angle. To do so, set the square so that point P of the square lies on the circle. Mark intersection points R and S. Draw $\overline{RS}$. $\overline{RS}$ is the diameter since $\angle RPS$ is an inscribed right angle. The problem can be extended to finding the center of the circle. In this case, the student must draw two diameters. Their intersection is the center.



3–10 Point out that a diameter (or a radius) drawn to the point of tangency forms a right angle with the tangent.

3–12 The right circular cylinder is often referred to as a *cylinder of revolution;* the right circular cone is sometimes called a *cone of revolution*. Demonstrating with string and cardboard why these figures are so called can prove to be an interesting method for strengthening the students' ability to visualize three-dimensional figures. Students may wish to cut out other geometric shapes, rotate them in the manner above, and describe what they see.

Chapter 4 Geometric Formulas

Chapter 4 extends the discussion of the plane and solid figures from the previous chapter to cover many useful formulas related to them. The formulas for the perimeter and area of simple plane figures are given, as well as the area and volume formulas for the more common solid figures. Like Chapter 3, the purpose is to furnish the background necessary for understanding relationships among the geometric figures commonly put to industrial and household uses.

Much of the bewilderment and frustration which students regularly experience in working with formulas—especially formulas from geometry—can be eliminated by reminding the student of two important facts about formulas.

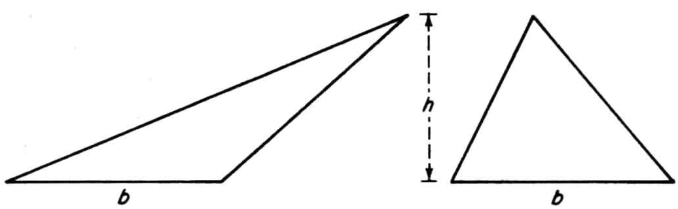
1. The terms used in describing a particular formula, as well as the symbols used to represent those terms, vary in meaning from formula to formula. Thus, for example, the word *base* used in describing the area of a triangle does not convey the same meaning as the word *base* used in describing the volume of a triangular prism. Accordingly, the variable b does not denote the same quantity as the variable B. Generally, lower case letters are used to refer to linear measure, while capital letters are used to refer to area or volume measure.

2. Many formulas are aggregates of simpler, more familiar terms. For instance, the formula for the total area of a right circular cone, $\pi rl + \pi r^2$, is composed of two terms, each of which is recognizable by itself. The student will learn that πrl represents the lateral area of the cone, while πr^2, of course, is the familiar

formula for the area of the circular base. The total area is, logically, the sum of these two quantities, as the formula clearly indicates. Although such composite formulas are often expressed in shorter, factored forms, the authors have chosen to retain, wherever possible, the expanded forms of these formulas in order to simplify the student's task of interpreting them.

4–1 (1) The definition of the altitude of a triangle uses the wording "perpendicular to the line containing the opposite side" in order to cover the case of the obtuse triangle, where two of the three altitudes must fall outside the triangle. Remind the student not to refer to the altitude as the "segment drawn from a vertex perpendicular to the opposite side," without, at least, adding the qualification "extended, if necessary."

(2) Students often find it difficult to accept the truth of this statement, especially in the case of an obtuse triangle and an acute triangle whose bases and altitudes are equal in measure, as in the figures below.



The use of a geoboard, if available, can be extremely helpful in demonstrating the truth of this formula.

4–2 (1) Emphasize that the altitude of a rectangle is one of the sides; thus, taking the product of the length of the two sides is a valid procedure for finding the area. Caution the student, however, against trying to apply this same method in attempting to find the area of a parallelogram or rhombus. See especially Exercise 5, page 159 for a good illustration of this distinction.

(2) This formula can be readily developed from the area formula for a triangle by drawing a diagonal in each of the figures on page 158. The result will show two triangles, each having an area equal to $\frac{1}{2}bh$. Thus, the area of each quadrilateral here will equal twice that of the triangles it contains, or $A = 2 \times \frac{1}{2}bh = bh$.

The division of polygons into a number of triangles as a means of determining the area of the polygon is a commonplace technique. Exercises 6 and 7, page 166, are good examples of the use of this method.

4–9 A cardboard model of a cylinder can be a useful device in discussing lateral area. As shown in the figure on page 180, when the cylinder is cut along a vertical line and laid flat, its lateral area becomes a rectangle whose base is equal to the circumference of the base of the cylinder ($2\pi r$) and whose altitude is equal to that of the cylinder (h). The area of the rectangle is determined by taking their product, $2\pi rh$.

Determining the lateral area (or total area) of a solid figure by reducing it to a plane figure or combination of plane figures is a method which many students find intriguing. The teacher may wish to have the students pursue this approach in analyzing other solid geometric figures such as prisms and cones, or in designing patterns for familiar objects such as containers and funnels.

Chapter 5 Formulas from Industry

In this chapter the student is introduced to many specialized, industrial formulas, particularly those related to the construction trade and the automobile industry. Each formula is presented directly, rather than derived, so that the student can focus his attention solely on using the formulas in several important applications. Also included in the chapter is a discussion of estimation, accompanied by a number of exercises and activities, to give the student practice in using this valuable skill.

5–1 (1) Point out to the students that the prices of lumber are generally based on the following thicknesses: 1″, $1\frac{1}{4}$″, $1\frac{1}{2}$″, $1\frac{3}{4}$″, 2″, 3″, and in even inches (4″, 6″, 8″ and so on). Since lumber is measured before it is seasoned and planed, finished boards are actually smaller than the dimensions indicated. For example, 4-inch boards actually measure $3\frac{5}{8}$ inches wide. In the exercises accompanying this section, consider the dimensions given to be the "rough" sizes so that calculating for board feet can be done directly with these dimensions.

(2) The teacher may wish to supplement these exercises by having the students determine the number of board feet in samples of scrap lumber. As mentioned in the previous note, the dimensions for the width and thickness are likely to involve fractions. Calculating for board feet using these dimensions can serve as a good review of the multiplication of fractions. The result of these calculations, however, should be referred to as the *actual* board feet. As an alternative method the students can solve for the *indicated* board feet. In this case the students would round off dimensions to the next greater size (see Note **5–1**(1)). For example, lumber that is $\frac{3}{4}$″ thick is considered to be 1″ thick, lumber that is $1\frac{1}{8}$″ thick is considered to be $1\frac{1}{4}$″ thick, lumber that is $2\frac{1}{2}$″ thick is considered to be 3″ thick, and so on.

5–4 Some students may be puzzled by this definition of work since it does not seem to relate to their everyday experience of work. Point out that in addition to the common meanings of work as exertion, a job, and so on, there is also the more technical meaning of work as the product of force and distance. In keeping with this definition of work as a product, remind the student that if either of the two factors—force or distance—is zero, the amount of work done is zero (recall the multiplicative property of zero on page 8). Thus, for example, a man who pushes against a large boulder, but fails to move it, has done no work.

5–5 Some students may find these formulas for horsepower somewhat remote. Having the student determine his own horsepower output during an everyday activity, such as climbing a flight of stairs, can be an interesting way to make this concept more familiar. If, for example, a student who weighs 150 pounds climbs a stairway 22-feet high in 15 seconds, the amount of horsepower expended in this activity can be calculated by reference to the formula on page 205. Thus,

$$\text{ft.-lb./sec.} = \frac{150 \times 22}{15} = 220$$

$$\text{hp} = \frac{220}{550} = \frac{4}{10}$$

The teacher may wish to carry out such an experiment in class, both to aid the students' understanding of horsepower, and to review the methods used in taking the different types of measurements involved in this experiment (distance, weight, and time).

5–7 Automotive literature showing the specifications of certain engines and other power train components is generally available upon request from automobile dealers. Such material can be used to supplement the discussion and exercises related to the topics covered in the next three sections.

5–8 In overhauling an engine it is a common practice to rebore the cylinders to a slightly larger diameter and equip the engine with oversize pistons. (An increase of $\frac{1}{8}''$ to the bore is generally a safe maximum.) The engine displacement is thereby increased. An interesting extension to the exercises given on page 210 consists in determining the increase in the cubic-inch displacement after a slight overbore. Thus, in the case of Exercise 1, if the bore were enlarged by $\frac{1}{8}''$ from 4.125″ to 4.250″, the engine displacement would be increased from approximately 401 cubic inches to about 426 cubic inches, for a gain of about 25 cubic inches. For students showing competence in this work, suggest lengthening the strokes given in Exercises 1–4. (An increase of $\frac{1}{4}''$ is usually a safe maximum when lengthening the stroke.)

5–12 A bicycle wheel mechanism like that shown on page 133 can be helpful in demonstrating the relationships between pulley sizes and pulley speeds. Attach a cardboard "snapper" to a spoke of the rear wheel, as was done in making the odometer (pages 54–55). Rotate the pedal mechanism slowly for a given number of times, say, 50 revolutions. At the same time count the number of snaps made by the rear wheel. (Do *not* allow the wheel to rotate freely.) Find the ratio of the number of snaps to the number of revolutions made by the pedal. Measure the diameter of the pedal gear and the diameter of the wheel gear, and find their ratio. Compare the two ratios. The ratios should be sufficiently close to substantiate the pulley formula, $DS = ds$. (Note. There is no need to consider a time factor in this experiment since both the pedal and the rear wheel are rotated for the same length of time.) Following the experiment, a discussion of the possible sources of error can be profitable.