THE LOGIC BOOK

Sixth Edition

Merrie Bergmann | James Moor | Jack Nelson

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PREFACE

Our overall goal in the sixth edition of *The Logic Book* remains the same as in earlier versions: presenting deductive symbolic logic in an accessible yet formally rigorous way. To this end, we have extensively reorganized and rewritten several chapters. We have also condensed presentations throughout the book.

Chapter 1 now focuses almost exclusively on deductive logic. Chapter 2 presents and discusses the formal syntax for the language *SL* before turning to symbolizations. Chapter 4 presents all of the truth-tree rules in the first section, and Chapter 5 does the same for the derivation rules of *SD*. The discussion of the completeness proof in Chapter 6 has been rewritten to make the flow of the proof more apparent. Like Chapter 2, Chapter 7 now presents the formal syntax of *PL* before discussing symbolization, and the Aristotelian square of opposition figures less prominently than it did in previous editions. Chapter 8 begins with a presentation of the formal semantics for predicate logic, discussing the formal semantics at greater length and with more examples. (However, those who want to skip most of the formal semantics can do so—we indicate this in the middle of Section 8.1, and we continue to display interpretations in the style of symbolization keys in most of the remainder of the chapter.) All interpretations presented in Chapter 8, except for some exercises for the first section, now use the set of positive integers as the UD. Chapter 9 recovers only extensions of predicates, rather than English readings of those predicates, from completed open branches of truth-trees. Finally, we have added an appendix with some facts about the positive integers; this can serve as a refresher for students as they work through symbolization in Chapter 7 and the construction of interpretations in Chapter 8.

The Logic Book presupposes no previous training in logic, and because it covers sentential logic through the metatheory of first-order predicate logic, it is suitable for both introductory and intermediate courses in symbolic logic.

The instructor who does not want to emphasize metatheory can simply omit Chapters 6 and 11. The chapters on truth-trees and the chapters on derivations are independent, so it is possible to cover truth-trees but not derivations and vice versa. However, the chapters on truth-trees do depend on the chapters presenting semantics; that is, Chapter 4 depends on Chapter 3 and Chapter 9 depends on Chapter 8. In contrast, the derivation chapters can be covered without first covering semantics.

The Logic Book includes large exercise sets for all chapters. Answers to unstarred exercises appear in the *Student Solutions Manual*, available at www.mhhe.com/bergmann6e, while answers to starred exercises appear in the *Instructor's Manual*, which can be obtained by following the instructions on the same web page.

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Chapter 1

INTRODUCTION TO DEDUCTIVE LOGIC

1.1 INTRODUCTION

This is a text in deductive logic—more specifically, in formal or symbolic deductive logic. Chapters 1–5 are devoted to sentential logic, the branch of symbolic deductive logic that takes sentences as the fundamental units of logical analysis. Chapters 7–10 are devoted to predicate logic, the branch of symbolic deductive logic that takes predicates and individual terms as the fundamental units of logical analysis. Chapter 6 is devoted to the metatheory of sentential logic, while Chapter 11 is devoted to the metatheory of predicate logic.

The hallmark of deductive logic is **truth-preservation.** Reasoning that is acceptable by the standards of deductive logic is always truth-preserving; that is, it never takes one from truths to a falsehood. The following syllogism provides an example of such reasoning:

All mammals are vertebrates.
Some sea creatures are mammals.
_____
Some sea creatures are vertebrates.

If the first and second sentences (the **premises**) are true, then the third sentence (the **conclusion**) must also be true. In deductive logic, reasoning that is truth-preserving is said to be '**valid**'.[1]

Over the centuries, a variety of systems of deductive logic have been developed. One of the oldest is Euclid's axiomatization of plane geometry, developed around 300 BCE in classical Greece. All of the truths or theorems of plane geometry can be derived from the five fundamental assumptions or axioms of Euclid's system. Many have attempted to axiomatize other areas of knowledge, including many of the sciences and many areas of mathematics. Giuseppe Peano successfully axiomatized arithmetic in 1889. Aristotle (350 BCE), a near contemporary of Euclid, developed a system of deductive logic that is known as "categorical" or "syllogistic" logic. Our earlier example of valid deductive reasoning was an Aristotelian syllogism. Aristotle's system is built around the logic of terms that identify categories of things, fish, human beings, animals, and so on. Aristotelian logic is still taught today, and the Law School Admissions Test (LSAT) usually contains questions about Aristotelian logic. However, Aristotle's system is limited in some important ways. For example, every syllogism must have exactly two premises, and the premises and conclusion of a syllogism must be structured according to very restrictive rules. Aristotelian logic cannot accommodate such obviously valid reasoning as

> Either the maid or the butler killed Watson.
>
> If it was the maid, Watson was poisoned.
>
> Watson wasn't poisoned.
>
> ---
>
> The butler killed Watson.

because there are three premises, not two, and the first and second premises have more complex forms than can be accommodated in Aristotelian logic.

The systems of deductive logic that we present in this text have their foundations in the work of Gottlob Frege, David Hilbert, Bertrand Russell, and other logicians in the late nineteenth and early twentieth centuries. Unlike axiomatic systems, which are based on a (usually) small number of axioms, the deductive systems in this text are based on a small number of reasonably intuitive rules that govern how sentences can be derived from other sentences.

There are a variety of reasons for studying deductive logic. It is a well-developed discipline that many find interesting in its own right, a discipline that has a rich history and important current research programs and practical applications. Certainly, those who plan to major or do graduate work in areas such as philosophy, mathematics, and computer science should have a solid grounding in skills that are needed for presenting and evaluating arguments in

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[1] Deductive logic's requirement that good reasoning be truth-preserving sets a very high standard for acceptable reasoning. This stands in contrast to inductive logic, which sets a more modest standard for good reasoning, namely that if the claims with which one starts are true, then the claims one reaches by using inductive principles are likely to be true. A great deal of the reasoning used in the sciences and in ordinary life is judged by inductive rather than deductive standards.

any discipline. Another reason for studying symbolic logic is that, in learning to symbolize natural language sentences (in our case, English sentences) in a formal language, one becomes more aware and more appreciative of the importance of the structure and complexities of natural languages. The specific words that we use have a direct bearing on whether a piece of reasoning is valid or invalid. For example, it is essential to distinguish between 'Roberta will pass if she completes all the homework' and 'Roberta will pass only if she completes all the homework' if we want to reason well about Roberta's prospects for passing. Finally, the concepts that we explore in this text are abstract concepts. Learning to think about abstract concepts and the relations between them is an important skill that is useful in a wide range of theoretical and applied disciplines.

1.2 CORE CONCEPTS OF DEDUCTIVE LOGIC

Many—but not all—sentences of English are either true or false (this is true of any natural language). We will say that true sentences have the **truth-value T** and that false sentences have the **truth-value F.** Sentences that are true or false include

Canada is located in South America.

Beethoven composed nine symphonies.

The Boston Red Sox will win the next World Series.

On December 29, 1012, it rained in what is now Manhattan.

The first of these sentences is false and therefore has the truth-value **F**. The second sentence is true and has the truth-value **T**. We do now know whether the third and fourth sentences are true or false, but we do know that each is one or the other. Time will tell whether the third sentence has the truth-value **T** or the truth-value **F,** but we will probably never know the truth-value of the fourth sentence. However, regardless of the state of our knowledge, the fourth sentence does have either the truth-value **T** or the truth-value **F**. It is important not to confuse our inability to know which truth-value a sentence has with the sentence's lack of a truth-value. There are obviously many sentences whose truth-values we will never know but that do nevertheless have truth-values.

Examples of sentences that lack truth-values include

Do I really have to do the homework to do well in this course?

Lock the door when you leave.

Hurrah!

The first of these sentences is a question. The second is a request or command and the third is an exclamation. To have a truth-value, a sentence must assert something. These three sentences do not assert or claim anything and hence do not have truth-values. In this text, we will be concerned only with sentences that do have truth-values; when we refer to a sentence or sentences we are referring to sentences that do have truth-values.

When we are talking about expressions of English we will often use the **variables p, q, r**, and **s** to do so. We use these variables in the same way that mathematicians use **x** and **y** as variables when they are talking about positive integers, that is, the numbers 1, 2, 3, . . . For example, the claim 'If **x** is an even positive integer and **y** is an odd positive integer then **x** plus **y** is an odd positive integer' is a true claim of arithmetic. So too, where **p** and **q** are variables that take expressions of English as their values, the following is true:

> If **p** is a sentence of English and **q** is a sentence of English then
>
> Either **p** or **q**
>
> is also a sentence of English.

The use of variables provides a convenient way for us to make claims about all expressions of English of a certain sort.

We define an **argument** as follows:

> An *argument* is a set of two or more sentences, one of which is designated as the conclusion and the others as the premises.[2]

Note that this definition uses the concept of a set of sentences. Sets are abstract objects that have members (zero or more). We can specify a finite set by listing its members, separated by commas, within a set of curly brackets. Here, for example, is a set of three English sentences:

> {Helen is not very well educated if she believes there is intelligent life on Mars, Helen is very well educated, Helen does not believe there is intelligent life on Mars}

If we designate the first two members that we have listed as the premises and the third sentence as the conclusion, then we have the argument

> Helen is not very well educated if she believes there is intelligent life on Mars.
>
> Helen is very well educated.
>
> ---
>
> Helen does not believe there is intelligent life on Mars.

We adopt the convention of displaying arguments by listing the premises with a horizontal line under the last premise, followed by the conclusion. We will say that arguments displayed in this way are in **standard form.**

---

[2]This definition allows arguments to have *any* number of premises, including an infinite number. However, all the arguments that we use as examples in this text have only a finite number of premises.

We now have all the terminology we need to introduce the core concepts of deductive logic. The first concept is **logical validity,** a concept that applies to arguments:

> *Logically valid argument:* An argument is *logically valid* if and only if it is not possible for all the premises to be true and the conclusion false. An argument is *logically invalid* if and only if it is not logically valid.

A logically valid argument is truth-preserving. If the premises are true, then the conclusion must also be true. The previous argument about Helen is logically valid because it is impossible for the premises to be true and the conclusion false. That is, if the premises are all true, then the conclusion must be true as well. Note that to determine validity, we do not need to know whether the premises or conclusion are in fact true. All that we need to know is the logical relation between the premises and the conclusion.

The following argument is not logically valid:

> If Sara receives an A in her chemistry class, she will graduate with a 3.5 grade point average.
>
> If Sara graduates with a 3.5 grade point average, she will get into medical school.
>
> Sara will get into medical school
>
> ---
>
> Sara will receive an A in her chemistry class.

This argument is invalid because it is possible for all three premises to be true and the conclusion false. Perhaps Sara will only receive a B in her chemistry class but will nevertheless graduate with a 3.5 grade point average because her other grades are so high, or perhaps she'll get into medical school with less than a 3.5 average because her medical school admissions interview was exceptional.

An argument that is logically valid and that has true premises is said to be **logically sound:**

> *Logically sound argument:* An argument is *logically sound* if and only if it is logically valid and all of its premises are true. An argument is *logically unsound* if and only if it is not logically sound.

Obviously, all logically sound arguments are logically valid, but not all logically valid arguments are logically sound because not all logically valid arguments have premises that are all true. The following argument is logically valid but is not logically sound:

> Italy is a country that is located in North America.
>
> Every country that is located in North America uses the United States dollar as its currency.
>
> ---
>
> Italy uses the United States dollar as its currency.

This argument is logically valid because if the premises were both true, the conclusion would have to be true as well. Obviously, however, the premises are not both true; in fact, they are both false (as is the conclusion), and so the argument is not logically sound, On the other hand, the following logically valid argument is also logically sound, because both of its premises are true:

> The United States is a country that is located in North America.
>
> No country that is located in North America uses the euro as its currency.
>
> ---
>
> The United States does not use the euro as its currency.

Note that if an argument is logically sound, its conclusion will also be true. This is because if the premises of a logically valid argument are true, then, because it is impossible for the argument's premises to be true and its conclusion false, the conclusion must also be true.

Identifying passages of English that contain arguments, extracting those arguments, and presenting them in standard form are important skills that must be mastered before the techniques presented in this text can be used to evaluate English arguments. The following expressions often signal that the sentence that follows is the conclusion of an argument:

therefore
thus
it follows that
so
hence
consequently
as a result

We will call these 'conclusion indicator expressions'. Similarly, expressions such as

since
for
because
on account of
inasmuch as
for the reason that

often indicate that the sentences following these expressions are the premises of an argument, and we will call these 'premise indicator expressions'.