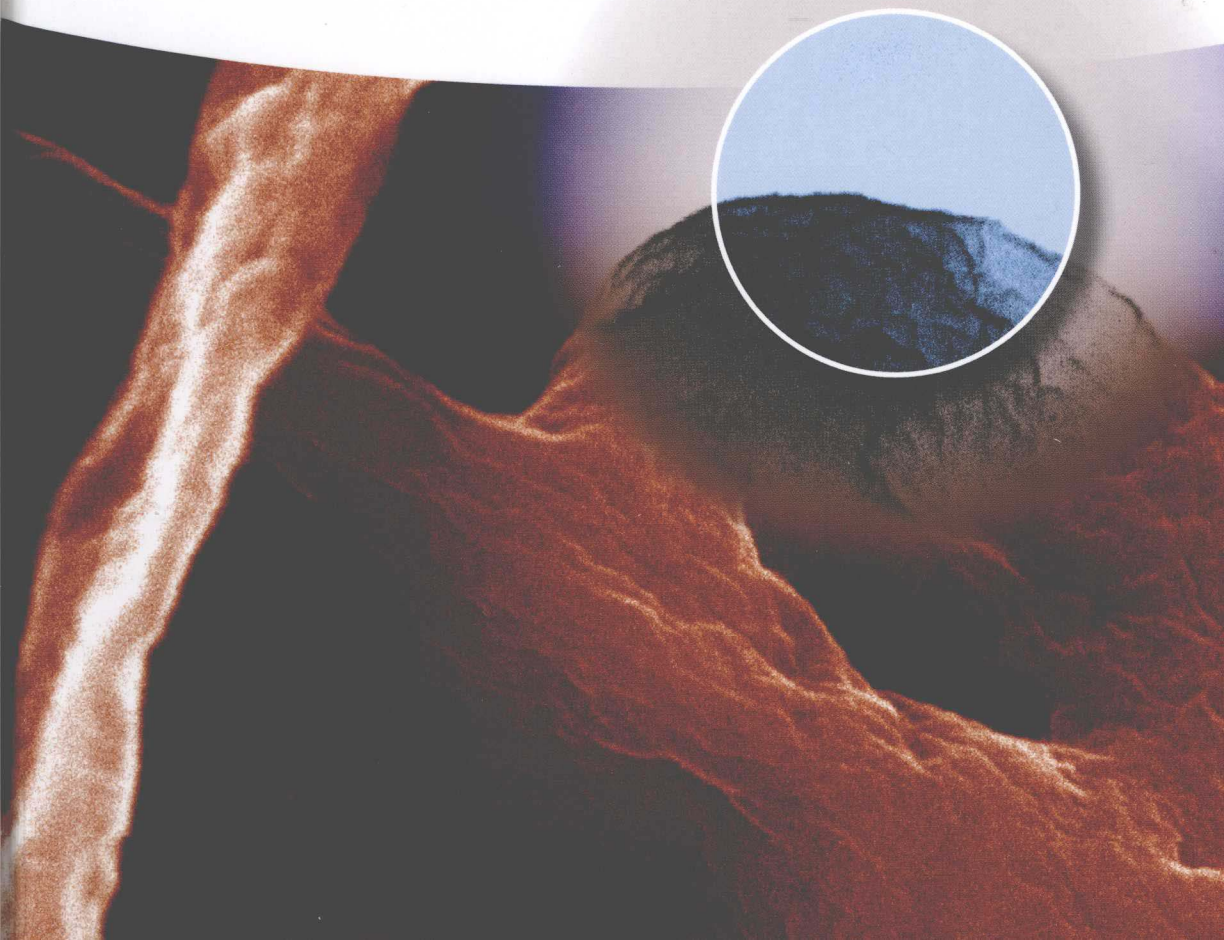


Yang Leng

# Materials Characterization

Introduction to Microscopic  
and Spectroscopic Methods

Second Edition



1282845

Yang Leng

TB30

LIBM2

2013

## Materials Characterization

Introduction to Microscopic and Spectroscopic Methods

*Second Edition*



**WILEY-VCH**  
Verlag GmbH & Co. KGaA

## The Author

### **Prof. Yang Leng**

The Hong Kong University of Science &  
Technology  
Department of Mechanical Engineering  
Clear Water Bay  
Kowloon  
Hong Kong

■ All books published by **Wiley-VCH** are carefully produced. Nevertheless, authors, editors, and publisher do not warrant the information contained in these books, including this book, to be free of errors. Readers are advised to keep in mind that statements, data, illustrations, procedural details or other items may inadvertently be inaccurate.

**Library of Congress Card No.:** applied for

### **British Library Cataloguing-in-Publication Data**

A catalogue record for this book is available from the British Library.

### **Bibliographic information published by the Deutsche Nationalbibliothek**

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available on the Internet at <<http://dnb.d-nb.de>>.

© 2013 Wiley-VCH Verlag GmbH & Co.  
KGaA, Boschstr. 12,  
69469 Weinheim, Germany

All rights reserved (including those of translation into other languages). No part of this book may be reproduced in any form – by photoprinting, microfilm, or any other means – nor transmitted or translated into a machine language without written permission from the publishers. Registered names, trademarks, etc. used in this book, even when not specifically marked as such, are not to be considered unprotected by law.

**Print ISBN:** 978-3-527-33463-6

**ePDF ISBN:** 978-3-527-67080-2

**ePub ISBN:** 978-3-527-67079-6

**mobi ISBN:** 978-3-527-67078-9

**oBook ISBN:** 978-3-527-67077-2

**Cover Design** Bluesea Design, Simone Benjamin, McLeese Lake, Canada

**Typesetting** Laserwords Private Ltd., Chennai

**Printing and Binding** Markono Print Media Pte Ltd, Singapore

Printed on acid-free paper  
Printed in Singapore

## Contents

<b>1</b>	<b>Light Microscopy</b>	<b>1</b>
1.1	Optical Principles	1
1.1.1	Image Formation	1
1.1.2	Resolution	3
1.1.2.1	Effective Magnification	5
1.1.2.2	Brightness and Contrast	5
1.1.3	Depth of Field	6
1.1.4	Aberrations	7
1.2	Instrumentation	9
1.2.1	Illumination System	9
1.2.2	Objective Lens and Eyepiece	13
1.2.2.1	Steps for Optimum Resolution	15
1.2.2.2	Steps to Improve Depth of Field	15
1.3	Specimen Preparation	15
1.3.1	Sectioning	16
1.3.1.1	Cutting	16
1.3.1.2	Microtomy	17
1.3.2	Mounting	17
1.3.3	Grinding and Polishing	19
1.3.3.1	Grinding	19
1.3.3.2	Polishing	21
1.3.4	Etching	23
1.4	Imaging Modes	26
1.4.1	Bright-Field and Dark-Field Imaging	26
1.4.2	Phase-Contrast Microscopy	27
1.4.3	Polarized-Light Microscopy	30
1.4.4	Nomarski Microscopy	35
1.4.5	Fluorescence Microscopy	37
1.5	Confocal Microscopy	39
1.5.1	Working Principles	39
1.5.2	Three-Dimensional Images	41

	References	45
	Further Reading	45
<b>2</b>	<b>X-Ray Diffraction Methods</b>	<b>47</b>
2.1	X-Ray Radiation	47
2.1.1	Generation of X-Rays	47
2.1.2	X-Ray Absorption	50
2.2	Theoretical Background of Diffraction	52
2.2.1	Diffraction Geometry	52
2.2.1.1	Bragg's Law	52
2.2.1.2	Reciprocal Lattice	53
2.2.1.3	Ewald Sphere	55
2.2.2	Diffraction Intensity	58
2.2.2.1	Structure Extinction	60
2.3	X-Ray Diffractometry	62
2.3.1	Instrumentation	62
2.3.1.1	System Aberrations	64
2.3.2	Samples and Data Acquisition	65
2.3.2.1	Sample Preparation	65
2.3.2.2	Acquisition and Treatment of Diffraction Data	65
2.3.3	Distortions of Diffraction Spectra	67
2.3.3.1	Preferential Orientation	67
2.3.3.2	Crystallite Size	68
2.3.3.3	Residual Stress	69
2.3.4	Applications	70
2.3.4.1	Crystal-Phase Identification	70
2.3.4.2	Quantitative Measurement	72
2.4	Wide-Angle X-Ray Diffraction and Scattering	75
2.4.1	Wide-Angle Diffraction	76
2.4.2	Wide-Angle Scattering	79
	References	82
	Further Reading	82
<b>3</b>	<b>Transmission Electron Microscopy</b>	<b>83</b>
3.1	Instrumentation	83
3.1.1	Electron Sources	84
3.1.1.1	Thermionic Emission Gun	85
3.1.1.2	Field Emission Gun	86
3.1.2	Electromagnetic Lenses	87
3.1.3	Specimen Stage	89
3.2	Specimen Preparation	90
3.2.1	Prethinning	91
3.2.2	Final Thinning	91
3.2.2.1	Electrolytic Thinning	91
3.2.2.2	Ion Milling	92

3.2.2.3	Ultramicrotomy	93
3.3	Image Modes	94
3.3.1	Mass–Density Contrast	95
3.3.2	Diffraction Contrast	96
3.3.3	Phase Contrast	101
3.3.3.1	Theoretical Aspects	102
3.3.3.2	Two-Beam and Multiple-Beam Imaging	105
3.4	Selected-Area Diffraction (SAD)	107
3.4.1	Selected-Area Diffraction Characteristics	107
3.4.2	Single-Crystal Diffraction	109
3.4.2.1	Indexing a Cubic Crystal Pattern	109
3.4.2.2	Identification of Crystal Phases	112
3.4.3	Multicrystal Diffraction	114
3.4.4	Kikuchi Lines	114
3.5	Images of Crystal Defects	117
3.5.1	Wedge Fringe	117
3.5.2	Bending Contours	120
3.5.3	Dislocations	122
	References	126
	Further Reading	126
<b>4</b>	<b>Scanning Electron Microscopy</b>	<b>127</b>
4.1	Instrumentation	127
4.1.1	Optical Arrangement	127
4.1.2	Signal Detection	129
4.1.2.1	Detector	130
4.1.3	Probe Size and Current	131
4.2	Contrast Formation	135
4.2.1	Electron–Specimen Interactions	135
4.2.2	Topographic Contrast	137
4.2.3	Compositional Contrast	139
4.3	Operational Variables	141
4.3.1	Working Distance and Aperture Size	141
4.3.2	Acceleration Voltage and Probe Current	144
4.3.3	Astigmatism	145
4.4	Specimen Preparation	145
4.4.1	Preparation for Topographic Examination	146
4.4.1.1	Charging and Its Prevention	147
4.4.2	Preparation for Microcomposition Examination	149
4.4.3	Dehydration	149
4.5	Electron Backscatter Diffraction	151
4.5.1	EBSD Pattern Formation	151
4.5.2	EBSD Indexing and Its Automation	153
4.5.3	Applications of EBSD	155
4.6	Environmental SEM	156

4.6.1	ESEM Working Principle	156
4.6.2	Applications	158
	References	160
	Further Reading	160
<b>5</b>	<b>Scanning Probe Microscopy</b>	<b>163</b>
5.1	Instrumentation	163
5.1.1	Probe and Scanner	165
5.1.2	Control and Vibration Isolation	165
5.2	Scanning Tunneling Microscopy	166
5.2.1	Tunneling Current	166
5.2.2	Probe Tips and Working Environments	167
5.2.3	Operational Modes	168
5.2.4	Typical Applications	169
5.3	Atomic Force Microscopy	170
5.3.1	Near-Field Forces	170
5.3.1.1	Short-Range Forces	171
5.3.1.2	van der Waals Forces	171
5.3.1.3	Electrostatic Forces	171
5.3.1.4	Capillary Forces	172
5.3.2	Force Sensors	172
5.3.3	Operational Modes	174
5.3.3.1	Static Contact Modes	176
5.3.3.2	Lateral Force Microscopy	177
5.3.3.3	Dynamic Operational Modes	177
5.3.4	Typical Applications	180
5.3.4.1	Static Mode	180
5.3.4.2	Dynamic Noncontact Mode	181
5.3.4.3	Tapping Mode	182
5.3.4.4	Force Modulation	183
5.4	Image Artifacts	183
5.4.1	Tip	183
5.4.2	Scanner	185
5.4.3	Vibration and Operation	187
	References	189
	Further Reading	189
<b>6</b>	<b>X-Ray Spectroscopy for Elemental Analysis</b>	<b>191</b>
6.1	Features of Characteristic X-Rays	191
6.1.1	Types of Characteristic X-Rays	193
6.1.1.1	Selection Rules	193
6.1.2	Comparison of K, L, and M Series	194
6.2	X-Ray Fluorescence Spectrometry	196
6.2.1	Wavelength Dispersive Spectroscopy	199
6.2.1.1	Analyzing Crystal	200

6.2.1.2	Wavelength Dispersive Spectra	201
6.2.2	Energy Dispersive Spectroscopy	203
6.2.2.1	Detector	203
6.2.2.2	Energy Dispersive Spectra	204
6.2.2.3	Advances in Energy Dispersive Spectroscopy	204
6.2.3	XRF Working Atmosphere and Sample Preparation	206
6.3	Energy Dispersive Spectroscopy in Electron Microscopes	207
6.3.1	Special Features	208
6.3.2	Scanning Modes	210
6.4	Qualitative and Quantitative Analysis	211
6.4.1	Qualitative Analysis	211
6.4.2	Quantitative Analysis	213
6.4.2.1	Quantitative Analysis by X-Ray Fluorescence	214
6.4.2.2	Fundamental Parameter Method	215
6.4.2.3	Quantitative Analysis in Electron Microscopy	216
	References	219
	Further Reading	219
<b>7</b>	<b>Electron Spectroscopy for Surface Analysis</b>	<b>221</b>
7.1	Basic Principles	221
7.1.1	X-Ray Photoelectron Spectroscopy	221
7.1.2	Auger Electron Spectroscopy	222
7.2	Instrumentation	225
7.2.1	Ultrahigh Vacuum System	225
7.2.2	Source Guns	227
7.2.2.1	X-Ray Gun	227
7.2.2.2	Electron Gun	228
7.2.2.3	Ion Gun	229
7.2.3	Electron Energy Analyzers	229
7.3	Characteristics of Electron Spectra	230
7.3.1	Photoelectron Spectra	230
7.3.2	Auger Electron Spectra	233
7.4	Qualitative and Quantitative Analysis	235
7.4.1	Qualitative Analysis	235
7.4.1.1	Peak Identification	239
7.4.1.2	Chemical Shifts	239
7.4.1.3	Problems with Insulating Materials	241
7.4.2	Quantitative Analysis	246
7.4.2.1	Peaks and Sensitivity Factors	246
7.4.3	Composition Depth Profiling	247
	References	250
	Further Reading	251



<b>8</b>	<b>Secondary Ion Mass Spectrometry for Surface Analysis</b>	<b>253</b>
8.1	Basic Principles	253
8.1.1	Secondary Ion Generation	254
8.1.2	Dynamic and Static SIMS	257
8.2	Instrumentation	258
8.2.1	Primary Ion System	258
8.2.1.1	Ion Sources	259
8.2.1.2	Wien Filter	262
8.2.2	Mass Analysis System	262
8.2.2.1	Magnetic Sector Analyzer	263
8.2.2.2	Quadrupole Mass Analyzer	264
8.2.2.3	Time-of-Flight Analyzer	264
8.3	Surface Structure Analysis	266
8.3.1	Experimental Aspects	266
8.3.1.1	Primary Ions	266
8.3.1.2	Flood Gun	266
8.3.1.3	Sample Handling	267
8.3.2	Spectrum Interpretation	268
8.3.2.1	Element Identification	269
8.4	SIMS Imaging	272
8.4.1	Generation of SIMS Images	274
8.4.2	Image Quality	275
8.5	SIMS Depth Profiling	275
8.5.1	Generation of Depth Profiles	276
8.5.2	Optimization of Depth Profiling	276
8.5.2.1	Primary Beam Energy	278
8.5.2.2	Incident Angle of Primary Beam	278
8.5.2.3	Analysis Area	279
	References	282
<b>9</b>	<b>Vibrational Spectroscopy for Molecular Analysis</b>	<b>283</b>
9.1	Theoretical Background	283
9.1.1	Electromagnetic Radiation	283
9.1.2	Origin of Molecular Vibrations	285
9.1.3	Principles of Vibrational Spectroscopy	286
9.1.3.1	Infrared Absorption	286
9.1.3.2	Raman Scattering	287
9.1.4	Normal Mode of Molecular Vibrations	289
9.1.4.1	Number of Normal Vibration Modes	291
9.1.4.2	Classification of Normal Vibration Modes	291
9.1.5	Infrared and Raman Activity	292
9.1.5.1	Infrared Activity	293
9.1.5.2	Raman Activity	295
9.2	Fourier Transform Infrared Spectroscopy	297
9.2.1	Working Principles	298

9.2.2	Instrumentation	300
9.2.2.1	Infrared Light Source	300
9.2.2.2	Beamsplitter	300
9.2.2.3	Infrared Detector	301
9.2.2.4	Fourier Transform Infrared Spectra	302
9.2.3	Examination Techniques	304
9.2.3.1	Transmittance	304
9.2.3.2	Solid Sample Preparation	304
9.2.3.3	Liquid and Gas Sample Preparation	304
9.2.3.4	Reflectance	305
9.2.4	Fourier Transform Infrared Microspectroscopy	307
9.2.4.1	Instrumentation	307
9.2.4.2	Applications	309
9.3	Raman Microscopy	310
9.3.1	Instrumentation	310
9.3.1.1	Laser Source	311
9.3.1.2	Microscope System	311
9.3.1.3	Prefilters	312
9.3.1.4	Diffraction Grating	313
9.3.1.5	Detector	314
9.3.2	Fluorescence Problem	314
9.3.3	Raman Imaging	315
9.3.4	Applications	316
9.3.4.1	Phase Identification	317
9.3.4.2	Polymer Identification	319
9.3.4.3	Composition Determination	319
9.3.4.4	Determination of Residual Strain	321
9.3.4.5	Determination of Crystallographic Orientation	322
9.4	Interpretation of Vibrational Spectra	323
9.4.1	Qualitative Methods	323
9.4.1.1	Spectrum Comparison	323
9.4.1.2	Identifying Characteristic Bands	324
9.4.1.3	Band Intensities	327
9.4.2	Quantitative Methods	327
9.4.2.1	Quantitative Analysis of Infrared Spectra	327
9.4.2.2	Quantitative Analysis of Raman Spectra	330
	References	331
	Further Reading	332

<b>10</b>	<b>Thermal Analysis</b>	<b>333</b>
10.1	Common Characteristics	333
10.1.1	Thermal Events	333
10.1.1.1	Enthalpy Change	335
10.1.2	Instrumentation	335
10.1.3	Experimental Parameters	336

10.2	Differential Thermal Analysis and Differential Scanning Calorimetry	337
10.2.1	Working Principles	337
10.2.1.1	Differential Thermal Analysis	337
10.2.1.2	Differential Scanning Calorimetry	338
10.2.1.3	Temperature-Modulated Differential Scanning Calorimetry	340
10.2.2	Experimental Aspects	342
10.2.2.1	Sample Requirements	342
10.2.2.2	Baseline Determination	343
10.2.2.3	Effects of Scanning Rate	344
10.2.3	Measurement of Temperature and Enthalpy Change	345
10.2.3.1	Transition Temperatures	345
10.2.3.2	Measurement of Enthalpy Change	347
10.2.3.3	Calibration of Temperature and Enthalpy Change	348
10.2.4	Applications	348
10.2.4.1	Determination of Heat Capacity	348
10.2.4.2	Determination of Phase Transformation and Phase Diagrams	350
10.2.4.3	Applications to Polymers	351
10.3	Thermogravimetry	353
10.3.1	Instrumentation	354
10.3.2	Experimental Aspects	355
10.3.2.1	Samples	355
10.3.2.2	Atmosphere	356
10.3.2.3	Temperature Calibration	358
10.3.2.4	Heating Rate	359
10.3.3	Interpretation of Thermogravimetric Curves	360
10.3.3.1	Types of Curves	360
10.3.3.2	Temperature Determination	362
10.3.4	Applications	362
	References	365
	Further Reading	365
	<b>Index</b>	367

## 1

## Light Microscopy

Light or optical microscopy is the primary means for scientists and engineers to examine the microstructure of materials. The history of using a light microscope for microstructural examination of materials can be traced back to the 1880s. Since then, light microscopy has been widely used by metallurgists to examine metallic materials. Light microscopy for metallurgists became a special field named *metallography*. The basic techniques developed in metallography are not only used for examining metals, but also are used for examining ceramics and polymers. In this chapter, light microscopy is introduced as a basic tool for microstructural examination of materials including metals, ceramics, and polymers.

### 1.1

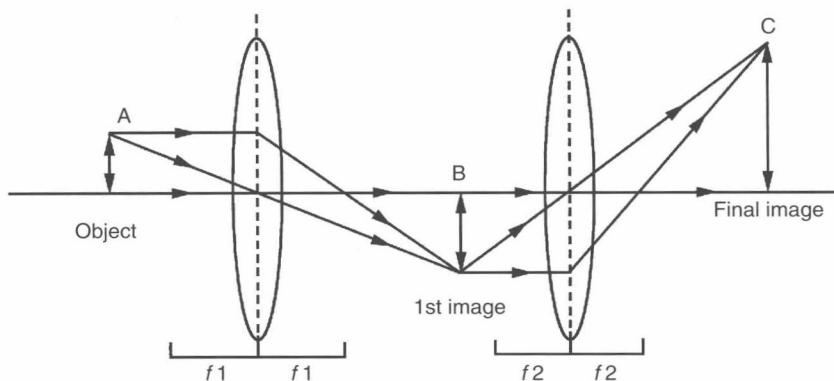
#### Optical Principles

##### 1.1.1

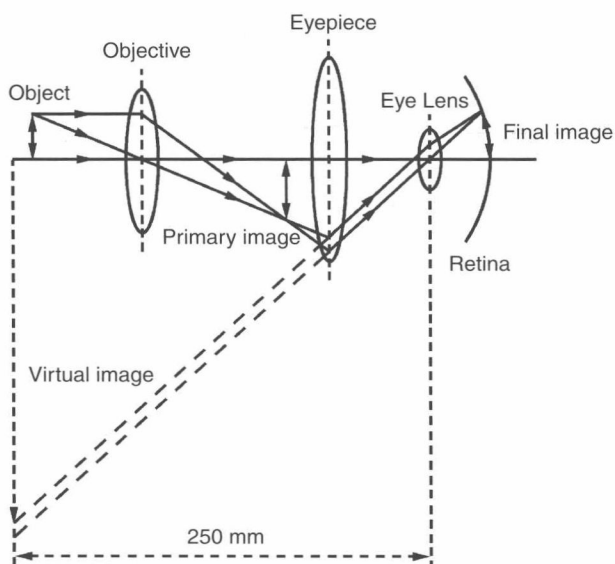
##### Image Formation

Reviewing the optical principles of microscopes should be the first step to understanding light microscopy. The optical principles of microscopes include image formation, magnification, and resolution. Image formation can be illustrated by the behavior of a light path in a compound light microscope as shown in Figure 1.1. A specimen (*object*) is placed at position A where it is between one and two focal lengths from an *objective lens*. Light rays from the object first converge at the objective lens and are then focused at position B to form a magnified inverted image. The light rays from the image are further converged by the second lens (*projector lens*) to form a final magnified image of an object at C.

The light path shown in Figure 1.1 generates the real image at C on a screen or camera film, which is not what we see with our eyes. Only a real image can be formed on a screen and photographed. When we examine microstructure with our eyes, the light path in a microscope goes through an *eyepiece* instead of projector lens to form a *virtual image* on the human eye retina, as shown in Figure 1.2. The virtual image is inverted with respect to the object. The virtual image is often adjusted to be located as the minimum distance of eye focus, which is conventionally taken



**Figure 1.1** Principles of magnification in a microscope.



**Figure 1.2** Schematic path of light in a microscope with an eyepiece. The virtual image is reviewed by a human eye composed of the eye lens and retina.

as 250 mm from the eyepiece. A modern microscope is commonly equipped with a device to switch from eyepiece to projector lens for either recording images on photographic film or sending images to a computer screen.

Advanced microscopes made since 1980 have a more complicated optical arrangement called “infinity-corrected” optics. The objective lens of these microscopes generates parallel beams from a point on the object. A tube lens is added between the objective and eyepiece to focus the parallel beams to form an image on a plane, which is further viewed and enlarged by the eyepiece.

The magnification of a microscope can be calculated by linear optics, which tells us the magnification of a convergent lens,  $M$ :

$$M = \frac{v - f}{f} \quad (1.1)$$

where  $f$  is the focal length of the lens and  $v$  is the distance between the image and lens. A higher magnification lens has a shorter focal length, as indicated by Eq. (1.1). The total magnification of a compound microscope as shown in Figure 1.1 should be the magnification of the objective lens multiplied by that of the projector lens.

$$M = M_1 M_2 \frac{(v_1 - f_1)(v_2 - f_2)}{f_1 f_2} \quad (1.2)$$

When an eyepiece is used, the total magnification should be the objective lens magnification multiplied by the eyepiece magnification.

### 1.1.2

#### Resolution

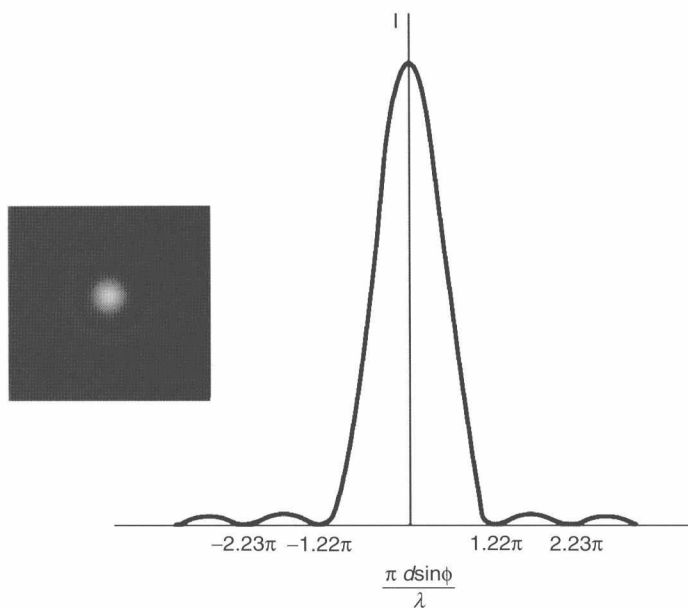
We naturally ask whether there is any limitation for magnification in light microscopes because Eq. (1.2) suggests there is no limitation. However, meaningful magnification of a light microscope is limited by its *resolution*. Resolution refers to the minimum distance between two points at which they can be visibly distinguished as two points. The resolution of a microscope is theoretically controlled by the diffraction of light.

Light diffraction controlling the resolution of microscope can be illustrated with the images of two self-luminous point objects. When the point object is magnified, its image is a central spot (the *Airy disk*) surrounded by a series of diffraction rings (Figure 1.3), not a single spot. To distinguish between two such point objects separated by a short distance, the Airy disks should not severely overlap each other. Thus, controlling the size of the Airy disk is the key to controlling resolution. The size of the Airy disk ( $d$ ) is related to the wavelength of light ( $\lambda$ ) and the angle of light coming into the lens. The *resolution of a microscope* ( $R$ ) is defined as the minimum distance between two Airy disks that can be distinguished (Figure 1.4). Resolution is a function of microscope parameters as shown in the following equation:

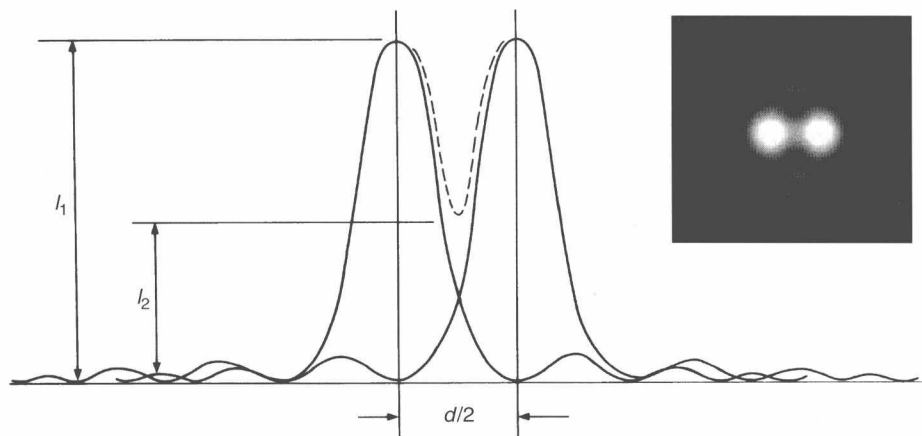
$$R = \frac{d}{2} = \frac{0.61\lambda}{\mu \sin \alpha} \quad (1.3)$$

where  $\mu$  is the refractive index of the medium between the object and objective lens and  $\alpha$  is the half-angle of the cone of light entering the objective lens (Figure 1.5). The product,  $\mu \sin \alpha$ , is called the *numerical aperture* (NA).

According to Eq. (1.3), to achieve higher resolution we should use shorter-wavelength light and larger NA. The shortest wavelength of visible light is about 400 nm, while the NA of the lens depends on  $\alpha$  and the medium between the

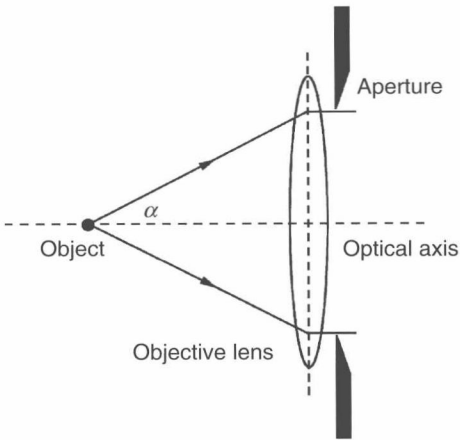


**Figure 1.3** A self-luminous point object and the light-intensity distribution along a line passing through its center.



**Figure 1.4** Intensity distribution of two airy disks with a distance  $d/2$ .  $I_1$  indicates the maximum intensity of each point and  $I_2$  represents the overlap intensity.

lens and object. Two media between object and objective lens are commonly used: either air for which  $\mu = 1$ , or oil for which  $\mu \approx 1.5$ . Thus, the maximum value of NA is about 1.5. We estimate the best resolution of a light microscope from Eq. (1.3) as about  $0.2 \mu\text{m}$ .



**Figure 1.5** The cone of light entering an objective lens showing  $\alpha$  is the half-angle.

#### 1.1.2.1 Effective Magnification

Magnification is meaningful only in so far as the human eye can see the features resolved by the microscope. Meaningful magnification is the magnification that is sufficient to allow the eyes to see the microscopic features resolved by the microscope. A microscope should enlarge features to about 0.2 mm, the resolution level of the human eye. This means that the microscope resolution multiplying the effective magnification should be equal to the eye resolution. Thus, the *effective magnification* of a light microscope should approximately be  $M_{\text{eff}} = 0.2 \div 0.2 \times 10^3 = 1.0 \times 10^3$ .

A higher magnification than the effective magnification only makes the image bigger, may make eyes more comfortable during observation, but does not provide more detail in an image.

#### 1.1.2.2 Brightness and Contrast

To make a microscale object in a material specimen visible, high magnification is not sufficient. A microscope should also generate sufficient *brightness* and *contrast* of light from the object. Brightness refers to the intensity of light. In a transmission light microscope the brightness is related to the numerical aperture (NA) and magnification ( $M$ ).

$$\text{Brightness} = \frac{(\text{NA})^2}{M^2} \quad (1.4)$$

In a reflected-light microscope the brightness is more highly dependent on NA.

$$\text{Brightness} = \frac{(\text{NA})^4}{M^2} \quad (1.5)$$

These relationships indicate that the brightness decreases rapidly with increasing magnification, and controlling NA is not only important for resolution but also for brightness, particularly in a reflected-light microscope.



*Contrast* is defined as the relative change in light intensity ( $I$ ) between an object and its background.

$$\text{Contrast} = \frac{I_{\text{object}} - I_{\text{background}}}{I_{\text{background}}} \quad (1.6)$$

Visibility requires that the contrast of an object exceeds a critical value called the *contrast threshold*. The contrast threshold of an object is not constant for all images but varies with image brightness. In bright light, the threshold can be as low as about 3%, while in dim light the threshold is greater than 200%.

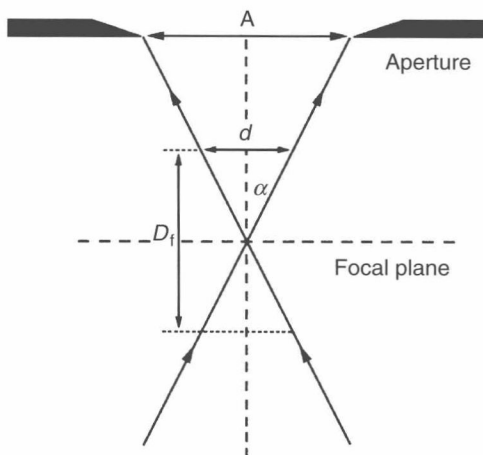
### 1.1.3

#### Depth of Field

*Depth of field* is an important concept when photographing an image. It refers to the range of position for an object in which image sharpness does not change. As illustrated in Figure 1.6, an object image is only accurately in focus when the object lies in a plane within a certain distance from the objective lens. The image is out of focus when the object lies either closer to or farther from the lens. Since the diffraction effect limits the resolution  $R$ , it does not make any difference to the sharpness of the image if the object is within the range of  $D_f$  shown in Figure 1.6. Thus, the depth of field can be calculated.

$$D_f = \frac{d}{\tan \alpha} = \frac{2R}{\tan \alpha} = \frac{1.22\lambda}{\mu \sin \alpha \tan \alpha} \quad (1.7)$$

Equation (1.7) indicates that a large depth of field and high resolution cannot be obtained simultaneously; thus, a larger  $D_f$  means a larger  $R$  and worse resolution.



**Figure 1.6** Geometric relation among the depth of field ( $D_f$ ), the half-angle entering the objective lens ( $\alpha$ ), and the size of the Airy disk ( $d$ ).