



Systems and Control \mathcal{E}_2

Daizhan Cheng
Xiaoming Hu
Tielong Shen

Analysis and Design of Nonlinear Control Systems

(非线性控制系统的分析与设计)

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Analysis and Design of Nonlinear Control Systems

Analysis and Design of Nonlinear Control Systems provides a comprehensive and up to date introduction to nonlinear control systems, including system analysis and major control design techniques. The book is self-contained, providing sufficient mathematical foundations for understanding the contents of each chapter. Scientists and engineers engaged in the field of Nonlinear Control Systems will find it an extremely useful handy reference book.

Dr. Daizhan Cheng, a professor at Institute of Systems Science, Chinese Academy of Sciences, has been working on the control of nonlinear systems for over 30 years and is currently a Fellow of IEEE and a Fellow of IFAC, he is also the chairman of Technical Committee on Control Theory, Chinese Association of Automation.

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Authors

Daizhan Cheng
Academy of Mathematics & Systems Science
Chinese Academy of Sciences
Beijing, 100190, China
Email: dcheng@iss.ac.cn

Xiaoming Hu
Optimization and Systems Theory
Royal Institute of Technology
100 44 Stockholm, Sweden
Email: hu@kth.se

Tielong Shen
Department of Engineering and Applied Sciences
Sophia University
Kiocho 7-1, Chiyoda-ku
Tokyo, 102-8554 Japan
Email: tetu-sin@sophia.ac.jp

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Preface

The purpose of this book is to present a comprehensive introduction to the theory and design technique of nonlinear control systems. It may serve as a standard reference of nonlinear control theory and applications for control scientists and control engineers as well as Ph.D students majoring in Automation or some related fields such as Operational Research, Management, Communication etc.

In the book we emphasize on the geometric approach to nonlinear control systems. In fact, we intend to put nonlinear control theory and its design techniques into a geometric framework as much as we can. The main motivation to write this book is to bring readers with basic engineering background promptly to the frontier of the modern geometric approach on the dynamic systems, particularly on the analysis and control design of nonlinear systems.

We have made a considerable effort on the following aspects:

First of all, we try to visualize the concepts. Certain concepts are defined over local coordinates, but in a coordinate free style. The purpose for this is to make them easily understandable, particularly at the first reading. Through this way a reader can understand a concept by just considering the case in \mathbb{R}^n . Later on, when the material has been digested, it is easy to lift them to general topological spaces or manifolds.

Secondly, we emphasize the numerical or computational aspect. We believe that making things computable is very useful not only for solving engineering problems but also for understanding the concepts and methods.

Thirdly, certain proofs have been simplified and some elementary proofs are presented to make the materials more readable for engineers or readers not specializing in mathematics. Finally, the topics which can be found easily in some other standard textbooks or references are briefly introduced and the corresponding references are included. Much attention has been put on new topics, new results, and new design techniques.

For convenience, a brief survey on linear control theory is included, which can be skipped for readers who are already familiar with the subject. For those who are not majoring in control theory, it provides a tutorial introduction to the field, which is sufficient for the further study of this book. The other mathematical pre-requirements are Calculus, Linear Algebra, Ordinal Differential Equation.

The materials in the book are selected and organized as self-sustained as possible. Chapter 1 is a general introduction to dynamic (control) systems. First, a brief survey for the theory of linear control systems is given. It provides certain necessary knowledge of systems and control, which will be used in the sequel. Then some special characteristics of nonlinear dynamics are discussed. Finally, some sample nonlinear control systems in practice are presented.

Chapter 2 gives an elementary introduction to topological spaces. It focuses primarily on those fundamental concepts of topological space. We emphasize on the second countable Hausdorff space, which is the enduring space of a manifold. Continuity of a mapping, homeomorphism, connectness, and quotient space etc. are also discussed.

Chapter 3 investigates differential manifolds. It is the basic tool for the geometric approach to nonlinear control systems. The concept of differential manifold is discussed first. Fiber bundle is then introduced. Vector fields, their integral curves and Lie brackets etc. are discussed subsequently. Then form, distribution and co-distribution are investigated. In fact, smooth function, vector field and co-vector field (one-form) are the three fundamental elements in nonlinear geometric control theory. Some important theorems such as Frobenius' Theorem, Lie series expansions and Chow's Theorem are presented with proofs. Finally, the tensor field is introduced, and based on it two important manifolds, namely, Riemannian manifold and symplectic manifold are investigated.

Chapter 4 gives a brief review on some basic concepts on abstract algebra, including group, ring and algebra. Then there is a brief introduction to some elementary concepts in algebraic topology, such as fundamental group, covering space etc. The main effort is focused on Lie group and Lie algebra. Particularly, the general linear group, general linear algebra and their sub-groups and sub-algebras are investigated.

In brief summary, Chapters 2–4 provide the mathematical foundation for nonlinear control theory that should suffice for the study of the rest of the book.

Chapter 5 considers controllability and observability of nonlinear systems. Similar to linear systems, the “controllable” and “observable” sub-manifolds are obtained. Based on them the Kalman decomposition is obtained. Algorithms for some control related distributions are provided.

Chapter 6 provides some further discussion for the global controllability of nonlinear systems. It is based on the properties of the integral orbits of a set of vector fields and under a general framework of switched systems. First, a heuristic sufficient condition is provided, which shows the geometric insight of the controllability. Then it is generalized to the case of nested distributions. Finally, it is shown that under certain circumstances the sufficient condition becomes necessary too.

Chapter 7 considers stability of nonlinear dynamic systems and feedback stabilization of nonlinear control systems. The Lyapunov theory and its generalization – LaSalle's invariance principle and their applications are discussed first. Converse theorems to Lyapunov's stability theorems are investigated. Stability of invariant set is then considered. Both input-to-output and input-to-state stabilities and stabilizations via control Lyapunov function are also discussed. Finally, for an asymptotically stable equilibrium the region of attraction is investigated.

Chapter 8 discusses the decoupling problems. First, as a generalization of (A, B) -invariant subspace, the (f, g) -invariant sub-distribution is defined. It is used to solve

the disturbance decoupling problem. Then the controlled invariant distribution is introduced and it is used for block decomposition of state space by state coordinate transformation with or without controls.

Chapter 9 considers the input-output structure of nonlinear systems. First, the decoupling matrix for affine nonlinear systems is introduced. In the light of it, the input-output decoupling problem (also called the Morgan's problem), the invertibility of nonlinear systems, and the dynamic feedback decoupling problems are investigated. The normal form is introduced, and then by using the point relative degree (vector), the generalized normal form is presented. Finally, the Fliess functional expansion is introduced to describe the input-output mapping of affine nonlinear systems. Following it, the application of Fliess expansion to tracking problem is discussed.

Chapter 10 discusses different linearizations of nonlinear control systems. First, linearization without control, called the Poincaré linearization, is considered. Then regular state feedback linearization with or without outputs is considered. The results for regular state feedback linearization have a fundamental meaning. Then global linearization is considered. Finally linearization via non-regular state feedback is discussed.

Chapters 5–10 may be considered as the kernel of nonlinear control theory.

Chapter 11 gives an introduction for the theory of center manifold first. Then the center manifold is used for the design of stabilizers for nonlinear systems with either minimum or non-minimum phase. For non-minimum phase systems a new tool, called the Lyapunov function with homogeneous derivative is proposed. It is then applied to stabilizing zero dynamics on the center manifold of nonlinear systems with both zero center and oscillatory center separately. The technique developed is applied to stabilization of systems in generalized normal form. Some stabilizer design methods are presented.

Chapter 12 is an elementary introduction to output regulation. First, the internal model principle of linear systems is introduced. Then the local output regulation problem is considered in detail. Finally, the results obtained for local output regulation are adopted to solving the robust local output regulation problem.

Chapter 13 discusses dissipative systems. First, definition and some useful properties of dissipative systems are introduced. Then, the passivity, as a special case of dissipativity, is investigated, where the main attention is focused on the Kalman-Yakubovich-Popov lemma. Passivity-based controller design methods are introduced for general nonlinear systems. Finally, some further applications of the design technique are discussed for Lagrange systems and Hamiltonian systems from different physical backgrounds.

Chapter 14 considers \mathcal{L}_2 -gain synthesis problems. Fundamentals of \mathcal{L}_2 -gain, and H_∞ norm in linear systems, are discussed. The H_∞ control problem for linear systems is considered and the extension to nonlinear systems is investigated. Finally, a control system synthesis technique with \mathcal{L}_2 -gain, mainly disturbance attenuation, is provided using the method of storage function construction.

Chapter 15 is about switched systems. Based on the fact that a switched linear system is also a nonlinear system, we consider equally the switched linear and nonlinear systems. Common quadratic Lyapunov functions are considered first. Based on common Lyapunov functions, the stabilization of switched systems is consid-

ered. Then the controllability of linear and bilinear switched systems is investigated in detail. A LaSalle's invariance principle for switched systems has been developed. Finally, the methods developed for switched systems are applied to investigate the consensus problem of different types of multi-agent systems.

Chapter 16 provides a tutorial introduction to discontinuous dynamical systems. The Filippov-framework for differential equations with discontinuous right hand side is discussed firstly. Then the non-smooth analysis method, which is motivated by the design problem for non-smooth Lyapunov functions, is reviewed. Stability conditions for the Filippov solutions are discussed and the control design is also investigated for a class of discontinuous systems. Finally, the behavior of sliding mode control systems is revisited in term of Filippov-framework.

The issues discussed in Chapters 11–16 are some special systems and special control techniques.

Parts of the contents were originally prepared for teaching the course of nonlinear control systems at the Institute of Systems Science, Academy of Mathematics and Systems Science, Chinese Academy of Sciences.

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March 1, 2010
Daizhan Cheng
Xiaoming Hu
Tielong Shen

Symbols

| | |
|----------------------------|---|
| \mathbb{C} | Set of complex numbers |
| \mathbb{R} | Set of real numbers |
| \mathbb{Q} | Set of rational numbers |
| \mathbb{Z} | Set of integers |
| \mathbb{N} | Set of natural numbers |
| $:=$ | Defined as |
| $M_{m \times n}$ | Set of $m \times n$ real matrices |
| M_n | Set of $n \times n$ real matrices |
| $\sigma(A)$ | Set of eigenvalues of matrix A |
| $\text{Im}(A)$ | Subspace spanned by the columns of A |
| $\text{Ker}(C)$ | Subspace generated by $\{X \mid XC = 0\}$ |
| \mathcal{C} | Controllable subspace |
| \mathcal{O} | Observable subspace |
| $\langle A \mid W \rangle$ | Smallest subspace containing W and A invariant |
| (M, \mathcal{T}) | Topological space M with topology \mathcal{T} |
| $B_r(x)$ | Open ball with center x and radius r |
| $\overset{\circ}{A}$ | Interior of A |
| \bar{A} | Closure of A |
| $T(M)$ | Tangent space on manifold M |
| $T^*(M)$ | Cotangent space on manifold M |
| $C^\infty(M)$ | Set of C^∞ functions on M |
| $C^\omega(M)$ | Set of analytic functions on M |
| $V(M)$ | Set of smooth vector fields on manifold M |
| $V^\omega(M)$ | Set of analytic vector fields on manifold M |
| $e_t^X(x_0)$ | Integral curve of X with initial value x_0 |
| $L_f h$ | Lie derivative of h (function or form) w.r.t. f |
| $\text{ad}_f g$ | Lie derivative of vector field g w.r.t. f |
| $\text{ad}_f^k g$ | k -th Lie derivative of vector field g w.r.t. f |
| $L_f^k h$ | k -th Lie derivative of h (function or form) w.r.t. f |
| $[\cdot, \cdot]$ | Lie bracket |
| $\{\dots\}_{LA}$ | Lie algebra generated by \dots |
| $\{\cdot, \cdot\}$ | Poisson bracket |

| | |
|--------------------------|--|
| $\text{Hess}(h(x))$ | Hessian matrix of $h(x)$ |
| $H < G$ | H is a sub-group of G |
| $H \triangleleft G$ | H is a normal sub-group of G |
| $GL(n, \mathbb{R})$ | General linear group |
| $gl(n, \mathbb{R})$ | General linear algebra |
| $SO(n, \mathbb{R})$ | Special orthogonal linear group |
| $o(n, \mathbb{R})$ | Orthogonal linear algebra |
| $Sp(n, \mathbb{R})$ | Symplectic group |
| $sp(n, \mathbb{R})$ | Symplectic algebra |
| S_n | Symmetric group |
| $T_s^r(V)$ | Set of tensors over V with co-variant r and contra-variant s |
| DF | Differential of F |
| ∇F | Gradient of F |
| $l_\Delta(x_0)$ | Integral manifold of Δ passing x_0 |
| $\overline{\Delta}$ | Involutive closure of a distribution |
| $RH_\infty^{n \times m}$ | Set of proper stable rational matrices of size $n \times m$ |
| RH_∞ | Set of proper stable rational functions |

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Chapter 1

Introduction

In this chapter we give an introduction to control theory and nonlinear control systems. In Section 1.1 we briefly review some basic concepts and results for linear control systems. Section 1.2 describes some basic characteristics of nonlinear dynamics. A few typical nonlinear control systems are presented in Section 1.3.

1.1 Linear Control Systems

A control system can be described as a black box in Fig. 1.1 (a), with input (or control) u and output y . In this sense, a control system is considered as a mapping from the input space to the output space. Before 1950's, primarily due to the nature of many electrical and electronic engineering problems then, control problems were largely treated as filtering problems and in the frequency domain, which is particularly suitable for single-input and single-output systems.

During 1950's Rudolf E. Kalman proposed a state space description for control systems. A set of state variables were introduced to describe the box. Intuitively, the black box is split into two parts: the first part is a set of differential (or difference) equations, which are used to describe the dynamics from control u to state variables x , and then a static equation is used to describe the mapping from state variables x to output y . See Fig. 1.1 (b).

There are many different state space descriptions that realize the same input-output mapping. These are called the state space realizations of the input-output mapping. A realization is minimum if there is no other realization that has less dimension of the state space.

Feedback is perhaps the most fundamental concept in automatic control and has a long history. Feedback means that the control strategy relies on the current status of the system. Depending on what information on the current status is available, it can be classified as state feedback control, see Fig. 1.1 (c), and output feedback control, see Fig. 1.1 (d).

A nonlinear control system considered throughout this book is described by

$$\begin{cases} \dot{x} = F(x, u), & x \in M, u \in U \\ y = h(x), & y \in N, \end{cases} \quad (1.1)$$

where M , U , and N are manifolds of dimensions n , m , p respectively. $F(x, u)$ and $h(x)$ are smooth mappings (C^∞ mappings unless elsewhere stated). In fact, we con-

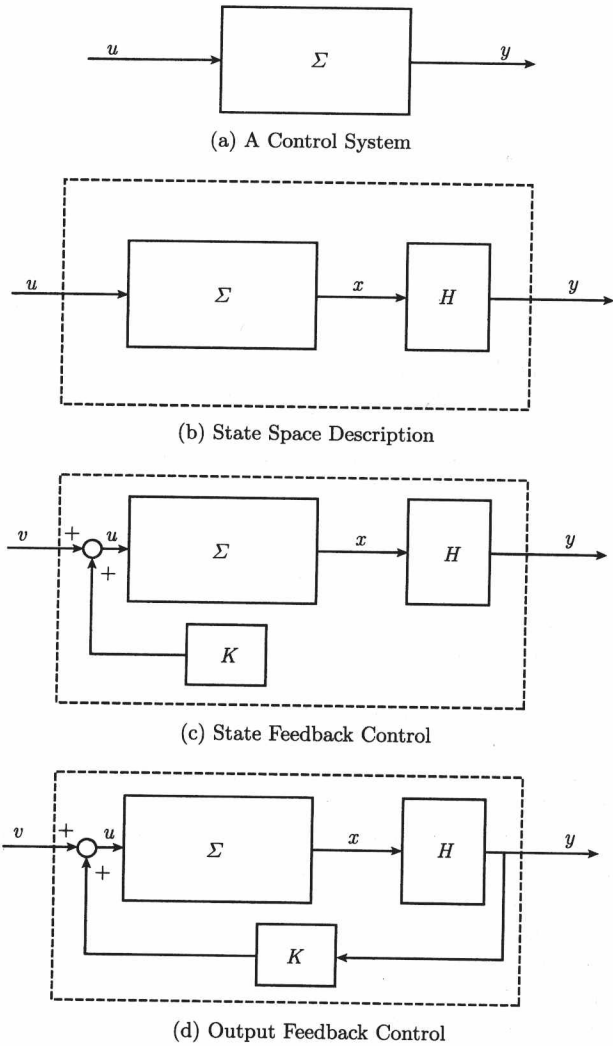


Fig. 1.1 A Control System

sider (1.1) as an expression of the system in a coordinate chart, and under the local coordinates x . In many applications we simply have $M = \mathbb{R}^n$, $U = \mathbb{R}^m$, and $N = \mathbb{R}^p$.

Particularly, when $F(x, u)$ is affine with respect to u , system (1.1) becomes an affine nonlinear system. It is commonly expressed as

$$\begin{cases} \dot{x} = f(x) + \sum_{i=1}^m g_i(x) u_i := f(x) + g(x)u, & x \in M, u \in U \\ y = h(x), & y \in N. \end{cases} \quad (1.2)$$

System (1.2) becomes a linear control system if $f(x) = Ax$, $h(x) = Cx$ are linear and $g_i(x)$ are constant. That is