Intermediate Algebra

second edition

Larson · Hostetler

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Second Edition

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Preface

The primary goals of *Intermediate Algebra,* Second Edition, are to encourage students to develop their proficiency in algebra and to show how algebra is a modern modeling language for real-life problems.

New to the Second Edition

In the Second Edition, all text elements were considered for revision, and many new examples, exercises, and applications were added to the text. Following are the major changes in the Second Edition.

Improved Coverage The new Second Edition was designed to be flexible with respect to the order of coverage of core algebra topics, adapting easily to a wide variety of course syllabi and teaching styles. This text begins with Prerequisites, a review chapter. All or part of this material may be covered or it can be omitted. Graphing is now introduced in Chapter 2, earlier than in the previous edition. The use of graphs encourages visualization to offer an opportunity for more conceptual understanding, strengthens graph-reading skills, and supports a smoother transition to math courses students may take in the future. Throughout the text, greater emphasis is given to geometry, collecting and interpreting data and statistics, updated data analysis, and creating models, as well as to the NCTM Standards and Addenda and the AMATYC Guidelines.

Problem Solving A general problem-solving process for applied problems is stressed throughout the text: form a verbal model, label terms, create a mathematical model, solve, and check the answer in the original statement of the problem (see page 74). This problem-solving process helps students understand the problem, organize their work, and develop facility with verbal, analytical, graphical, and numerical approaches to problem solving. Students are also reminded of specific problem-solving strategies (see page 74) that are reinforced throughout the text in the exercises (see Exercises 31–34 on page 365 and Exercise 125 on page 651).

Exercises The exercise sets were completely revised and expanded—by nearly 40%—for the Second Edition. These comprehensive exercise sets offer students ample opportunity to practice algebraic techniques (see pages 280–282 and 387–389) and develop their conceptual and critical-thinking skills (see Exercises 79 and 80 on page 144, Exercises 55 and 56 on page 178, Exercise 117 on page 216, Exercises 89 and 90 on page 294, Exercise 41 on page 461, and Exercise 104 on page 573). The broad range of computational, conceptual, and applied problems in each exercise set is carefully graded to provide a smooth transition from routine to more challenging problems. Section and review exercises—as well as mid-chapter quizzes (see page 155) and chapter tests (see page 196)—consistently encourage student mastery of algebraic skills and concepts through practice and self-assessment.

Technology Recognizing that graphing technology is becoming increasingly available, the Second Edition offers the opportunity to use graphing utilities throughout, but without requiring their use. This is achieved through a combination of features, including—at point of use—discovery opportunities that require scientific or graphing calculators (see pages 401 and 578), graphing utility instructions (see pages 306 and 383), and clearly labeled exercises that require the use of a graphing utility (see Exercise 64 on page 431 and Exercises 67–78 on pages 505 and 506).

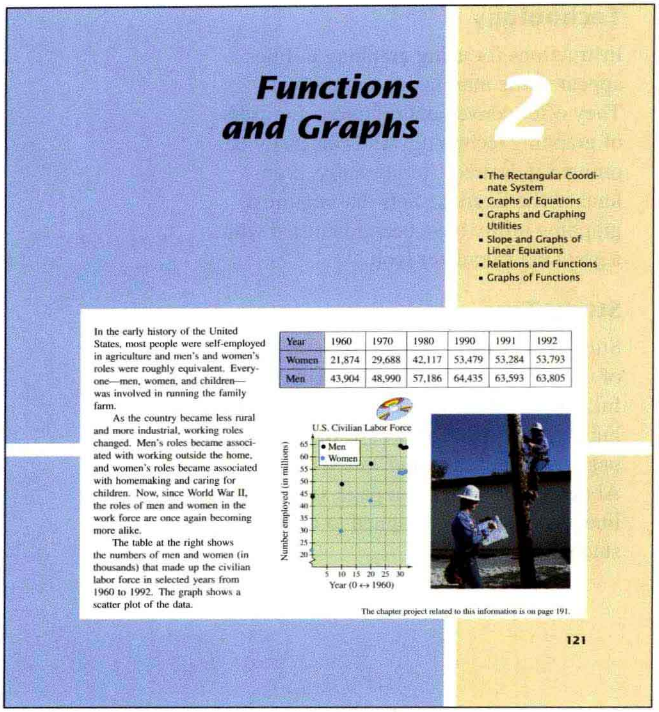
Group Activities Each section ends with a Group Activity. This exercise reinforces students' understanding by exploring mathematical concepts in a variety of ways: You Be the Instructor, Extending the Concept, Problem Solving, Exploring with Technology, and Communicating Mathematically. Some Group Activities encourage interpretation or discovery of mathematical concepts and results (see pages 202, 513, and 568); some provide opportunities for problem posing and error analysis (see pages 101, 261, 402, and 580); and others reinforce methods of interpreting and constructing mathematical models, tables, and graphs (see pages 386, 442, 459, and 602). Designed to be completed in class or as homework assignments, the Group Activities give students the opportunity to work cooperatively as they think, talk, and write about mathematics.

Data Analysis/Modeling Throughout the Second Edition, students are offered more opportunities to collect and interpret data, make conjectures, and construct mathematical models. Students are exposed to combining mathematical models to make related models (see Exercise 101 on page 206 and Exercise 39 on page 263); encouraged to use mathematical models to make predictions and estimates from real data (see Exercise 40 on page 143, Exercise 43 on page 312, and Exercise 95 on page 446); invited to compare two or more models or compare actual data with a model (see Exercise 103 on page 573 and Exercise 97 on page 625); and asked to use curve-fitting techniques to write their own models from data (see Exercise 97 on page 446, Exercises 37–39 on page 540, and Exercises 39–41 on page 553). This edition encourages greater use of charts, tables, scatter plots, and graphs to summarize, analyze, and interpret data.

Applications To emphasize for students the connection between mathematical concepts and real-world situations, up-to-date, real-life applications are integrated throughout the text. Appearing as examples (see pages 141, 362, 501, and 602), exercises (see Exercise 74 on page 134, Exercise 98 on page 190, Exercise 41 on page 515, and Exercise 116 on page 614), group activities (see pages 74 and 525), and projects (see pages 315 and 556), these applications help students validate the material they are learning and offer them frequent opportunities to use and review their problem-solving skills. A wide range of disciplines is represented by the applications—including such areas as physics, chemistry, electronics, the social sciences, biology, and business—as well as the career interviews, covering areas such as insurance, real estate, architecture, engineering, graphic arts, business, education, scuba diving, biochemistry, and economics.

Connections In addition to highlighting the connections between algebra and areas outside mathematics through real-world applications, this text also emphasizes the connections between algebra and other branches of mathematics, such as probability (see pages 273 and 668), geometry (see page 50), logic (see Appendix A), and statistics (see Appendix B). Too, many examples and exercises throughout the text reinforce the connections among graphical, numerical, and algebraic representations of important algebraic concepts (see Exercises 27 and 28 on page 414).

There are many other new features of the Second Edition as well, including Discovery, Chapter Opening Applications, Study Tips, Historical Notes, Mid-Chapter Quizzes, Chapter Summaries, Career Interviews, and Chapter Projects. These and other features of the Second Edition are described in greater detail on the following pages.


Functions and Graphs

2

- The Rectangular Coordinate System
- Graphs of Equations
- Graphs and Graphing Utilities
- Slope and Graphs of Linear Equations
- Relations and Functions
- Graphs of Functions

In the early history of the United States, most people were self-employed in agriculture and men's and women's roles were roughly equivalent. Everyone—men, women, and children—was involved in running the family farm.

As the country became less rural and more industrial, working roles changed. Men's roles became associated with working outside the home, and women's roles became associated with homemaking and caring for children. Now, since World War II, the roles of men and women in the work force are once again becoming more alike.

The table at the right shows the numbers of men and women (in thousands) that made up the civilian labor force in selected years from 1960 to 1992. The graph shows a scatter plot of the data.

Year	1960	1970	1980	1990	1991	1992
Women	21,874	29,688	42,117	53,479	53,284	53,793
Men	43,904	48,990	57,186	64,435	63,593	63,805

The chapter project related to this information is on page 191.

121

Chapter Opener

Each chapter opens with a look at a real-life application that is explored in depth in the Chapter Project at the end of the chapter. Real data is manipulated using graphical, numerical, and algebraic techniques. In addition, a list of the section titles shows students how the topics fit into the overall development of algebra.

Section Outline

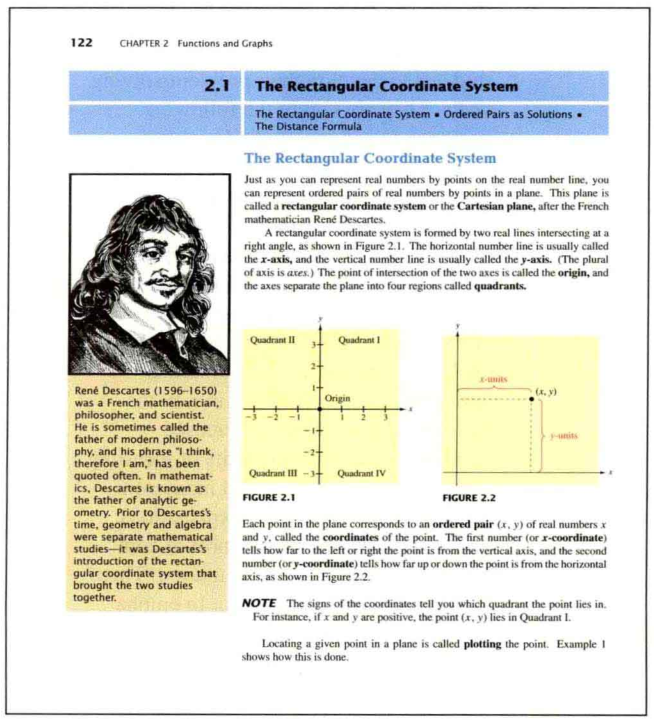
Each section begins with a list of the major topics covered in that section. These topics are also the subsection titles and can be used for easy reference and review by students.

Historical Notes

To help students understand that algebra has a past, historical notes featuring mathematical artifacts or mathematicians and their work are included in each chapter.

Notes

Notes anticipate students' needs by offering additional insight, pointing out common errors, and describing generalizations.


122 CHAPTER 2 Functions and Graphs

2.1 The Rectangular Coordinate System

The Rectangular Coordinate System • Ordered Pairs as Solutions • The Distance Formula

The Rectangular Coordinate System

René Descartes (1596–1650) was a French mathematician, philosopher, and scientist. He is sometimes called the father of modern philosophy, and his phrase "I think, therefore I am," has been quoted often. In mathematics, Descartes is known as the father of analytic geometry. Prior to Descartes's time, geometry and algebra were separate mathematical studies—it was Descartes's introduction of the rectangular coordinate system that brought the two studies together.

Just as you can represent real numbers by points on the real number line, you can represent ordered pairs of real numbers by points in a plane. This plane is called a **rectangular coordinate system** or the **Cartesian plane,** after the French mathematician René Descartes.

A rectangular coordinate system is formed by two real lines intersecting at a right angle, as shown in Figure 2.1. The horizontal number line is usually called the **x-axis,** and the vertical number line is usually called the **y-axis.** (The plural of axis is *axes*.) The point of intersection of the two axes is called the **origin,** and the axes separate the plane into four regions called **quadrants.**

FIGURE 2.1

FIGURE 2.2

Each point in the plane corresponds to an **ordered pair** (x, y) of real numbers x and y, called the **coordinates** of the point. The first number (or **x-coordinate**) tells how far to the left or right the point is from the vertical axis, and the second number (or **y-coordinate**) tells how far up or down the point is from the horizontal axis, as shown in Figure 2.2.

NOTE The signs of the coordinates tell you which quadrant the point lies in. For instance, if x and y are positive, the point (x, y) lies in Quadrant I.

Locating a given point in a plane is called **plotting** the point. Example 1 shows how this is done.

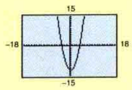
SECTION 3.5 Solving Polynomial Equations **239**

Technology

You can use a graphing utility to estimate the solutions of an equation. For instance, to estimate the solutions of the equation in Example 1, sketch the graph of

$y = x^2 - x - 12$

as shown below. The two solutions correspond to the x-intercepts of the graph: $x = -3$ and $x = 4$.



Solving Quadratic Equations by Factoring

A **quadratic equation** is an equation of the form $ax^2 + bx + c = 0$. Here are some examples.

$$x^2 - x - 6 = 0, \quad 3x^2 + 2x - 1 = 0, \quad \text{and} \quad x^2 + 3x = 0$$

In the next four examples, note how you can combine your factoring skills with the Zero-Factor Property to solve quadratic equations.

EXAMPLE 1 Using Factoring to Solve a Quadratic Equation

Solve $x^2 - x - 12 = 0$.

Solution

First, check to see that the right side of the equation is zero. Next, factor the left side of the equation. Finally, apply the Zero-Factor Property to find the solutions.

$x^2 - x - 12 = 0$ Original equation

$(x + 3)(x - 4) = 0$ Factor left side of equation.

$x + 3 = 0 \Rightarrow x = -3$ Set 1st factor equal to 0.

$x - 4 = 0 \Rightarrow x = 4$ Set 2nd factor equal to 0.

The equation has two solutions: -3 and 4.

Check

$x^2 - x - 12 = 0$ Original equation

$(-3)^2 - (-3) - 12 \stackrel{?}{=} 0$ Substitute -3 for x.

$9 + 3 - 12 \stackrel{?}{=} 0$ Simplify.

$0 = 0$ Solution checks. ✓

Check

$x^2 - x - 12 = 0$ Original equation

$(4)^2 - 4 - 12 \stackrel{?}{=} 0$ Substitute 4 for x.

$16 - 4 - 12 \stackrel{?}{=} 0$ Simplify.

$0 = 0$ Solution checks. ✓

Factoring and the Zero-Factor Property allow you to solve a quadratic equation by converting it into two *linear* equations, which you already know how to solve. This is a common strategy in algebra—breaking down a given problem into simpler parts, each of which can be solved by previously learned methods.

Technology

Instructions for using graphing utilities appear in the margin at point of use. They offer convenient reference for users of graphing technology and can easily be omitted if desired. Additionally, problems in the Exercise Sets that require a graphing utility have been identified with a graphing calculator icon.

Study Tips

Study Tips appear in the margin at point of use. They offer students specific, helpful, and insightful suggestions for studying algebra. "How to Study Algebra" on page xxvi and "Reading and Writing About Mathematics" on page xxix outline a general plan designed to improve student study skills.

Problem Solving

The text provides ample opportunity for students to develop their problem-solving skills. They are taught the following approach to solving applied problems: (1) Construct a verbal model; (2) Label variable and constant terms; (3) Construct an algebraic model; (4) Using the model, solve the problem; and (5) Check the answer in the original statement of the problem. This process has wide applicability, and it is used with verbal, analytical, graphical, and numerical approaches to problem solving. In the Second Edition, there is increased emphasis on identifying units of measure and checking solutions, and many solutions were rewritten with explanations and additional help in the form of comments adjacent to the computation. There is also increased use of color to emphasize and clarify the solution steps.

500 CHAPTER 8 Systems of Equations

Applications

To model a real-life situation with a system of equations, you can use the same basic problem-solving strategy that has been used throughout the text.

Write a verbal model. → Assign labels. → Write an algebraic model. → Solve the algebraic model. → Answer the question.

After answering the question, remember to check the answer in the original statement of the problem.

EXAMPLE 5 An Application

A roofing contractor bought 30 bundles of shingles and four rolls of roofing paper for \$528. A second purchase (at the same prices) cost \$140 for eight bundles of shingles and one roll of roofing paper. Find the price per bundle of shingles and the price per roll of roofing paper.

In 1990, there were about 90 thousand construction companies that specialized in single-family construction. These companies had an average of 4 employees each.

Solution

Verbal Model: [Cost of 30 bundles] + [Cost of 4 rolls] = [528]

[Cost of 8 bundles] + [Cost of 1 roll] = [140]

Labels: Price of bundle of shingles $= x$ (dollars)

Price of roll of roofing paper $= y$ (dollars)

System: $30x + 4y = 528$ Equation 1

$8x + y = 140$ Equation 2

Solving the second equation for y produces $y = 140 - 8x$, and substituting this expression into the first equation produces the following.

$$30x + 4(140 - 8x) = 528$$
$$30x + 560 - 32x = 528$$
$$-2x = -32$$
$$x = 16$$

Back-substituting $x = 16$ into the revised second equation produces

$$y = 140 - 8(16)$$
$$= 12.$$

Thus, you can conclude that the price of shingles is \$16 per bundle and the price of roofing paper is \$12 per roll. Check this in the original statement of the problem.

618 CHAPTER 9 Exponential and Logarithmic Functions

One common application of exponential growth is in modeling the growth of a population, as shown in Example 5.

EXAMPLE 5 Population Growth

A country's population was 2 million in 1980 and 3 million in 1990. What would you predict the population of the country to be in the year 2000?

Solution

If you assumed a *linear growth model*, you would simply predict the population in the year 2000 to be 4 million. If, however, you assumed an *exponential growth model*, the model would have the form

$$y = Ce^{kt}.$$

In this model, let $t = 0$ represent the year 1980. The given information about the population can be described by the following table.

t (years)	0	10	20
Ce^{kt} (million)	$Ce^{k(0)} = 2$	$Ce^{k(10)} = 3$	$Ce^{k(20)} = ?$

To find the population when $t = 20$, you must first find the values of C and k. From the table, you can use the fact that $Ce^{k(0)} = Ce^0 = 2$ to conclude that $C = 2$. Then, using this value of C, you solve for k as follows.

$Ce^{k(10)} = 3$ From table

$2e^{10k} = 3$ Substitute value of C.

$e^{10k} = \frac{3}{2}$ Divide both sides by 2.

$10k = \ln\frac{3}{2}$ Inverse property

$k = \frac{1}{10}\ln\frac{3}{2}$ Divide both sides by 10.

$k \approx 0.0405$ Simplify.

Finally, you can use this value of k to conclude that the population in the year 2000 is given by

$$2e^{0.0405(20)} \approx 2(2.25) = 4.5 \text{ million.}$$

Figure 9.16 graphically compares the exponential growth model with a linear growth model.

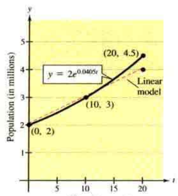


FIGURE 9.16 Population Models

Applications

Real-life applications are integrated throughout the text in examples and exercises. These applications offer students constant review of problem-solving skills and emphasize the relevance of the mathematics. Many of the applications use recent, real data, and all are titled for easy reference. Photographs with captions throughout the text also encourage students to see the link between mathematics and real life.

Examples

Each of the text examples was carefully chosen to illustrate a particular mathematical concept, problem-solving approach, or computational technique, and to enhance students' understanding. The examples in the text cover a wide variety of problem types, including computational, real-life applications (many with real data), and those requiring the use of graphing technology. Each example is titled for easy reference, and real-life applications are labeled. Many examples include side comments in color, which clarify the key steps of the solution process.

Discovery

Throughout the text, Discovery notes encourage active participation by students, often taking advantage of the power of technology (graphing calculators and scientific calculators) to explore mathematical concepts and discover mathematical patterns. Using a variety of approaches, including visualization, verification, pattern recognition, and modeling, students develop an intuitive understanding of algebraic topics.

Definitions and Rules

All of the important rules, formulas, guidelines, properties, definitions, and summaries are highlighted for emphasis. Each is also titled for easy reference.

184 CHAPTER 2 Functions and Graphs

DISCOVERY

Use a graphing utility to display the graphs of $y = x^2 + c$ where c is equal to -2, 0, 2, and 4. What conclusions can you make?

Transformations of Graphs of Functions

Many functions have graphs that are simple transformations of the basic graphs shown in Figure 2.43. The following list summarizes the various types of **horizontal** and **vertical shifts** of the graphs of functions.

Vertical and Horizontal Shifts

Let c be a positive real number. **Vertical** and **horizontal shifts** of the graph of the function $y = f(x)$ are represented as follows.

1. Vertical shift c units **upward**: $h(x) = f(x) + c$
2. Vertical shift c units **downward**: $h(x) = f(x) - c$
3. Horizontal shift c units to the **right**: $h(x) = f(x - c)$
4. Horizontal shift c units to the **left**: $h(x) = f(x + c)$

EXAMPLE 3 Shifts of the Graphs of Functions

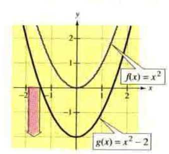
Use the graph of $f(x) = x^2$ to sketch the graph of each function.

a. $g(x) = x^2 - 2$ **b.** $h(x) = (x + 3)^2$

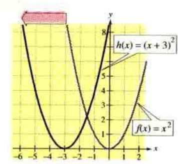
Solution

a. Relative to the graph of $f(x) = x^2$, the graph of $g(x) = x^2 - 2$ represents a *downward shift* of two units, as shown in Figure 2.44.

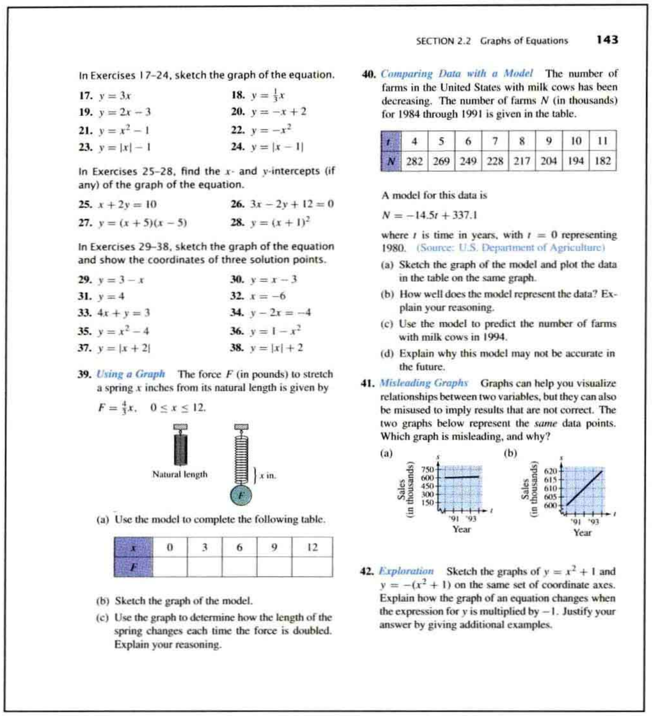
b. Relative to the graph of $f(x) = x^2$, the graph of $h(x) = (x + 3)^2$ represents a *left shift* of three units, as shown in Figure 2.45.



Vertical Shift: Two Units Down
FIGURE 2.44



Horizontal Shift: Three Units Left
FIGURE 2.45



SECTION 2.2 Graphs of Equations 143

In Exercises 17–24, sketch the graph of the equation.

17. $y = 3x$
18. $y = \frac{1}{3}x$
19. $y = 2x - 3$
20. $y = -x + 2$
21. $y = x^2 - 1$
22. $y = -x^2$
23. $y = |x| - 1$
24. $y = |x - 1|$

In Exercises 25–28, find the x- and y-intercepts (if any) of the graph of the equation.

25. $x + 2y = 10$
26. $3x - 2y + 12 = 0$
27. $y = (x + 5)(x - 5)$
28. $y = (x + 1)^2$

In Exercises 29–38, sketch the graph of the equation and show the coordinates of three solution points.

29. $y = 3 - x$
30. $y = x - 3$
31. $y = 4$
32. $x = -6$
33. $4x + y = 3$
34. $y - 2x = -4$
35. $y = x^2 - 4$
36. $y = 1 - x^2$
37. $y = |x + 2|$
38. $y = |x| + 2$

39. *Using a Graph* The force F (in pounds) to stretch a spring x inches from its natural length is given by

$F = \frac{4}{3}x, \quad 0 \le x \le 12.$

(a) Use the model to complete the following table.

x	0	3	6	9	12
F					

(b) Sketch the graph of the model.

(c) Use the graph to determine how the length of the spring changes each time the force is doubled. Explain your reasoning.

40. *Comparing Data with a Model* The number of farms in the United States with milk cows has been decreasing. The number of farms N (in thousands) for 1984 through 1991 is given in the table.

t	4	5	6	7	8	9	10	11
N	282	269	249	228	217	204	194	182

A model for this data is

$N = -14.5t + 337.1$

where t is time in years, with $t = 0$ representing 1980. (Source: U.S. Department of Agriculture)

(a) Sketch the graph of the model and plot the data in the table on the same graph.

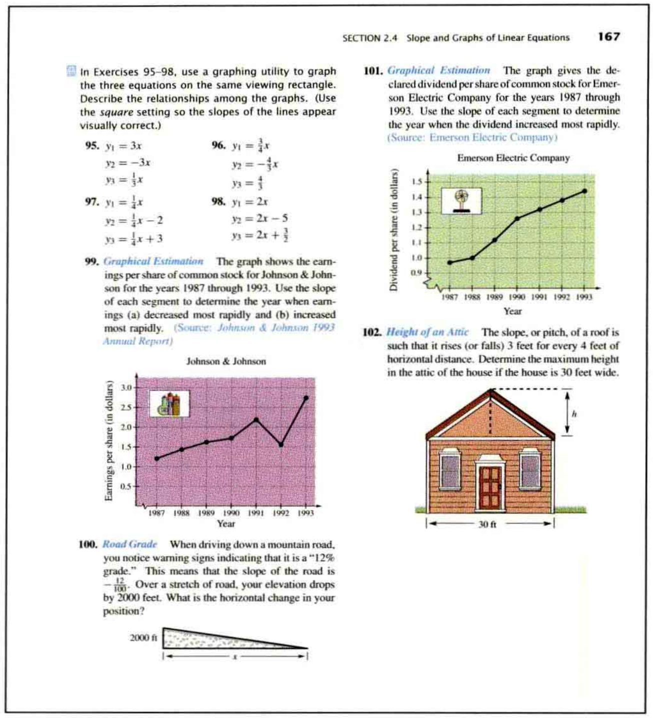
(b) How well does the model represent the data? Explain your reasoning.

(c) Use the model to predict the number of farms with milk cows in 1994.

(d) Explain why this model may not be accurate in the future.

41. *Misleading Graphs* Graphs can help you visualize relationships between two variables, but they can also be misused to imply results that are not correct. The two graphs below represent the *same* data points. Which graph is misleading, and why?

42. *Exploration* Sketch the graphs of $y = x^2 + 1$ and $y = -(x^2 + 1)$ on the same set of coordinate axes. Explain how the graph of an equation changes when the expression for y is multiplied by -1. Justify your answer by giving additional examples.



SECTION 2.4 Slope and Graphs of Linear Equations 167

In Exercises 95–98, use a graphing utility to graph the three equations on the same viewing rectangle. Describe the relationships among the graphs. (Use the *square* setting so the slopes of the lines appear visually correct.)

95. $y_1 = 3x$, $y_2 = -3x$, $y_3 = \frac{1}{3}x$
96. $y_1 = \frac{3}{4}x$, $y_2 = -\frac{4}{3}x$, $y_3 = \frac{4}{3}$
97. $y_1 = \frac{1}{4}x$, $y_2 = \frac{1}{4}x - 2$, $y_3 = \frac{1}{4}x + 3$
98. $y_1 = 2x$, $y_2 = 2x - 5$, $y_3 = 2x + \frac{3}{2}$

99. *Graphical Estimation* The graph shows the earnings per share of common stock for Johnson & Johnson for the years 1987 through 1993. Use the slope of each segment to determine the year when earnings (a) decreased most rapidly and (b) increased most rapidly. (Source: *Johnson & Johnson 1993 Annual Report*)

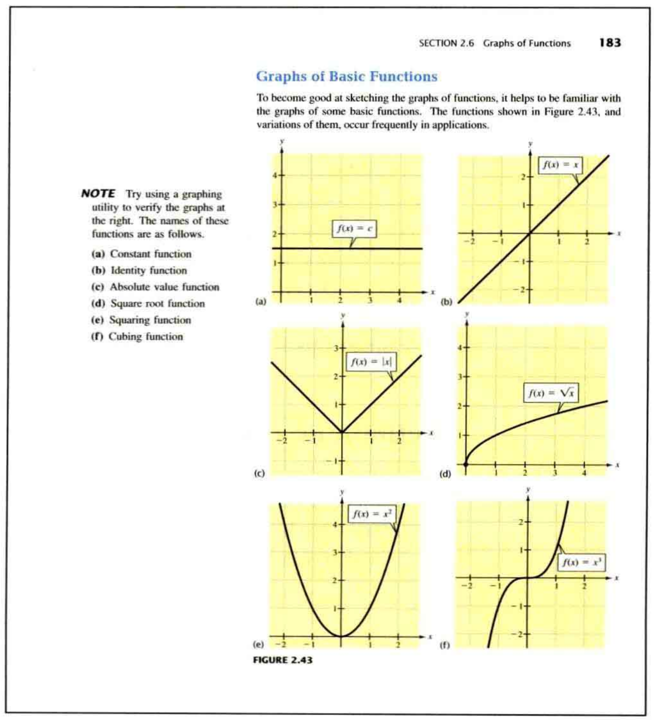
100. *Road Grade* When driving down a mountain road, you notice warning signs indicating that it is a "12% grade." This means that the slope of the road is $-\frac{12}{100}$. Over a stretch of road, your elevation drops by 2000 feet. What is the horizontal change in your position?

101. *Graphical Estimation* The graph gives the declared dividend per share of common stock for Emerson Electric Company for the years 1987 through 1993. Use the slope of each segment to determine the year when the dividend increased most rapidly. (Source: Emerson Electric Company)

102. *Height of an Attic* The slope, or pitch, of a roof is such that it rises (or falls) 3 feet for every 4 feet of horizontal distance. Determine the maximum height in the attic of the house if the house is 30 feet wide.

Graphics

Visualization is a critical problem-solving skill. To encourage the development of this ability, the text has numerous figures in examples, exercises, and answers to odd-numbered exercises. Included are graphs of equations and functions, geometric figures, displays of statistical information, scatter plots, and numerous screen outputs from graphing technology. All graphs of equations and functions, computer- or calculator-generated for accuracy, are designed to resemble students' actual screen outputs as closely as possible. Graphics are also used to emphasize graphical interpretation, comparison, and estimation.



SECTION 2.6 Graphs of Functions 183

Graphs of Basic Functions

To become good at sketching the graphs of functions, it helps to be familiar with the graphs of some basic functions. The functions shown in Figure 2.43, and variations of them, occur frequently in applications.

NOTE Try using a graphing utility to verify the graphs at the right. The names of these functions are as follows.

(a) Constant function
(b) Identity function
(c) Absolute value function
(d) Square root function
(e) Squaring function
(f) Cubing function

FIGURE 2.43

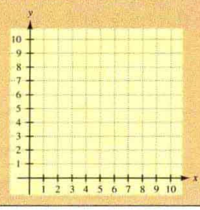
Group Activities — Communicating Mathematically

Translating a Formula Use the information provided in the following statement to write a mathematical formula for the 10-second pulse count.

"The target heart rate is the heartbeat rate a person should have during aerobic exercise to get the full benefit of the exercise for cardiovascular conditioning. . . . Using the American College of Sports Medicine Method to calculate one's target heart rate, an individual should subtract his or her age from 220, then multiply by the desired intensity level (as a percent—sedentary persons may want to use 60% and highly fit individuals may want to use 85 to 95%) of the workout. Then divide the answer by 6 for a 10-second pulse count. (The 10-second pulse count is useful for checking whether the target heart rate is being achieved during the workout. One can easily check one's pulse—at the wrist or side of the neck—counting the number of beats in 10 seconds.)" (Source: Aerobic Fitness Association of America)

Use the formula you have found to find your own 10-second pulse count.

Group Activities — Extending the Concept



Using Inequalities Try the following activity. One person picks a point with whole number coordinates on a grid like the one at left without revealing the coordinates. A second person writes the equation of a line passing through the grid region. The first person graphs the line on the grid and indicates whether the secret point lies above, below, or on the line. Continue writing and graphing lines until the second person is able to guess the coordinates of the secret point. Switch roles and try again. What is the fewest number of turns your team required to guess the point?

Group Activities

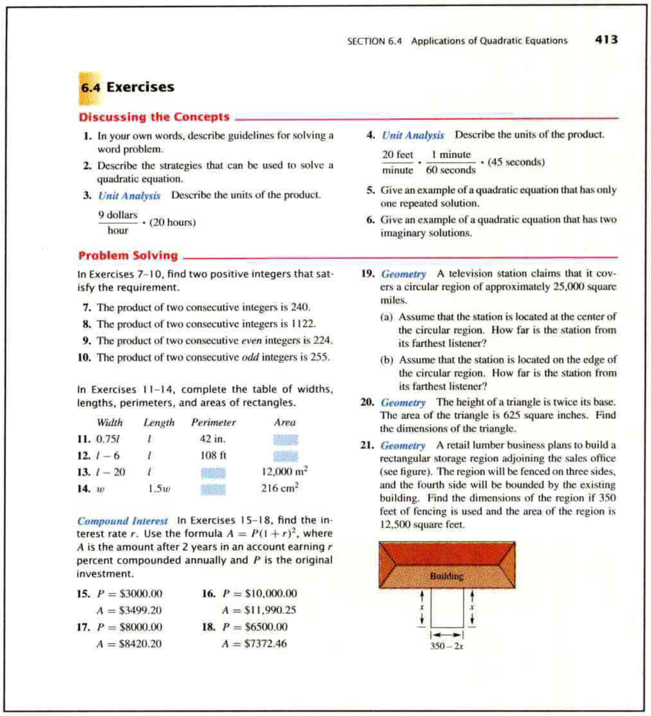
The Group Activities that appear at the end of sections reinforce students' understanding by approaching mathematical concepts in a variety of ways: Communicating Mathematically, You Be the Instructor, Extending the Concept, Problem Solving, and Exploring with Technology. Designed to be completed as group projects in class or as homework assignments, the Group Activities give students opportunities for interactive learning and to think, talk, and write about mathematics.

Group Activities — Problem Solving

Fitting a Quadratic Model The data in the table represents the United States government's annual net receipts y (in billions of dollars) from individual income taxes for the year x from 1990 through 1992, where $x = 0$ corresponds to 1990. (Source: U.S. Department of the Treasury)

x	0	1	2
y	467	468	476

Use a system of three linear equations to find a quadratic model that fits the data. According to your model, what were the annual net receipts from individual income taxes in 1993? The actual annual net receipts for 1993 were $510 billion. How does the value obtained from your quadratic model compare? Suppose you had been involved in planning the 1993 federal budget and had used this model to estimate how much federal income could be expected from 1993 individual income taxes. When you review the actual 1993 tax receipts and see that the model wasn't completely accurate, how do you evaluate the model's prediction performance? Are you satisfied with it? Why or why not?



SECTION 6.4 Applications of Quadratic Equations 413

6.4 Exercises

Discussing the Concepts

1. In your own words, describe guidelines for solving a word problem.
2. Describe the strategies that can be used to solve a quadratic equation.
3. *Unit Analysis* Describe the units of the product.

$\frac{9 \text{ dollars}}{\text{hour}} \cdot (20 \text{ hours})$

4. *Unit Analysis* Describe the units of the product.

$\frac{20 \text{ feet}}{\text{minute}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} \cdot (45 \text{ seconds})$

5. Give an example of a quadratic equation that has only one repeated solution.
6. Give an example of a quadratic equation that has two imaginary solutions.

Problem Solving

In Exercises 7–10, find two positive integers that satisfy the requirement.

7. The product of two consecutive integers is 240.
8. The product of two consecutive integers is 1122.
9. The product of two consecutive *even* integers is 224.
10. The product of two consecutive *odd* integers is 255.

In Exercises 11–14, complete the table of widths, lengths, perimeters, and areas of rectangles.

	Width	Length	Perimeter	Area
11.	$0.75l$	l	42 in.	
12.	$l-6$	l	108 ft	
13.	$l-20$	l		12,000 m^2
14.	w	$1.5w$		216 cm^2

Compound Interest In Exercises 15–18, find the interest rate r. Use the formula $A = P(1+r)^2$, where A is the amount after 2 years in an account earning r percent compounded annually and P is the original investment.

15. $P = \$3000.00$, $A = \$3499.20$
16. $P = \$10{,}000.00$, $A = \$11{,}990.25$
17. $P = \$8000.00$, $A = \$8420.20$
18. $P = \$6500.00$, $A = \$7372.46$

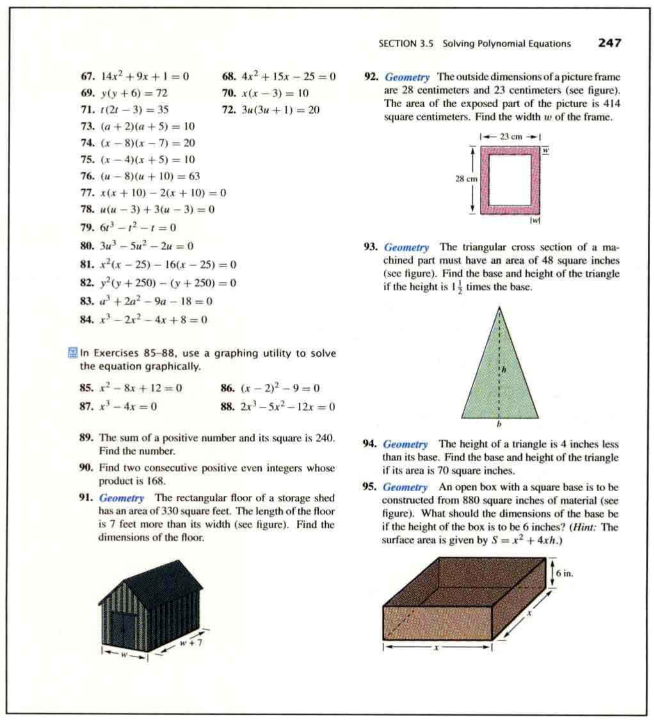
19. *Geometry* A television station claims that it covers a circular region of approximately 25,000 square miles.
    (a) Assume that the station is located at the center of the circular region. How far is the station from its farthest listener?
    (b) Assume that the station is located on the edge of the circular region. How far is the station from its farthest listener?
20. *Geometry* The height of a triangle is twice its base. The area of the triangle is 625 square inches. Find the dimensions of the triangle.
21. *Geometry* A retail lumber business plans to build a rectangular storage region adjoining the sales office (see figure). The region will be fenced on three sides, and the fourth side will be bounded by the existing building. Find the dimensions of the region if 350 feet of fencing is used and the area of the region is 12,500 square feet.

Exercises

In the completely revised and expanded exercise sets of the Second Edition, problems are now grouped into four categories: Discussing the Concepts, Problem Solving, Reviewing the Major Concepts, and Additional Problem Solving. To accommodate a variety of teaching and learning styles, the exercise sets offer numerous computational, conceptual, and applied problems, including multi-part, exploration and discovery, writing, estimation, numeracy, geometry, and challenging exercises, as well as real-life applications, mathematical modeling, graphical comparisons, data interpretation and analysis, fitting a line to data, and exercises that require graphing technology. Applications are labeled for easy reference. Designed to build competence, skill, and understanding, each part of the exercise set is graded in difficulty to allow students to gain confidence as they progress. Detailed solutions to all odd-numbered exercises are given in the Student Solutions Guide, and answers to all odd-numbered exercises appear in the back of the text.

Geometry

Geometric formulas and concepts are reviewed throughout the text. For reference, common formulas are listed inside the back cover of this text.



SECTION 3.5 Solving Polynomial Equations 247

67. $14x^2 + 9x + 1 = 0$
68. $4x^2 + 15x - 25 = 0$
69. $y(y+6) = 72$
70. $x(x-3) = 10$
71. $t(2t-3) = 35$
72. $3u(3u+1) = 20$
73. $(a+2)(a+5) = 10$
74. $(x-8)(x-7) = 20$
75. $(x-4)(x+5) = 10$
76. $(u-8)(u+10) = 63$
77. $x(x+10) - 2(x+10) = 0$
78. $u(u-3) + 3(u-3) = 0$
79. $6t^3 - t^2 - t = 0$
80. $3u^3 - 5u^2 - 2u = 0$
81. $x^2(x-25) - 16(x-25) = 0$
82. $y^2(y+250) - (y+250) = 0$
83. $a^3 + 2a^2 - 9a - 18 = 0$
84. $x^3 - 2x^2 - 4x + 8 = 0$

In Exercises 85–88, use a graphing utility to solve the equation graphically.

85. $x^2 - 8x + 12 = 0$
86. $(x-2)^2 - 9 = 0$
87. $x^3 - 4x = 0$
88. $2x^3 - 5x^2 - 12x = 0$

89. The sum of a positive number and its square is 240. Find the number.
90. Find two consecutive positive even integers whose product is 168.
91. *Geometry* The rectangular floor of a storage shed has an area of 330 square feet. The length of the floor is 7 feet more than its width (see figure). Find the dimensions of the floor.

92. *Geometry* The outside dimensions of a picture frame are 28 centimeters and 23 centimeters (see figure). The area of the exposed part of the picture is 414 square centimeters. Find the width w of the frame.

93. *Geometry* The triangular cross section of a machined part must have an area of 48 square inches (see figure). Find the base and height of the triangle if the height is $1\frac{1}{2}$ times the base.

94. *Geometry* The height of a triangle is 4 inches less than its base. Find the base and height of the triangle if its area is 70 square inches.
95. *Geometry* An open box with a square base is to be constructed from 880 square inches of material (see figure). What should the dimensions of the base be if the height of the box is to be 6 inches? (*Hint:* The surface area is given by $S = x^2 + 4xh$.)

CAREER INTERVIEW

Lisa M. Deitemeyer

Civil Engineer

Johnson-Brittain & Associates, Inc.

Tucson, AZ 85701

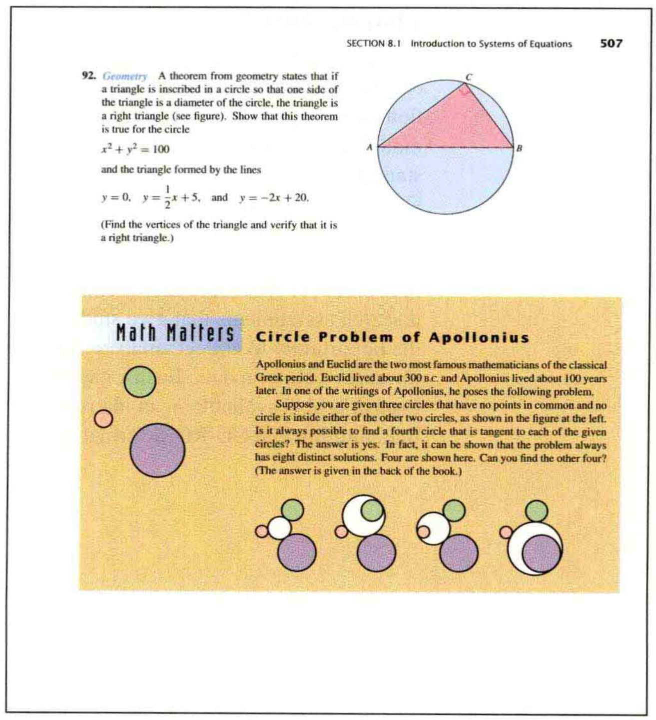
Johnson-Brittain does highway design work, primarily for the Arizona Department of Transportation. I am responsible for drainage design of roadways and intersections. It is important that water properly drain off the road surface to avoid flooding problems. One strategy for removing excess water is to use a pipe drainage system that empties into a retention pond. When designing a pipe system and choosing pipe size, I use the equation $V = Q/A$ to find the velocity V of water moving at flow rate Q (volume per unit time) through a given pipe of cross-sectional area A. Finding the water velocity is very important. If it is too fast, erosion can occur in the retention pond. If it is too slow, sedimentation can clog the pipe. As you can see, algebra is very important to my work. I am always solving for different variables that are needed for drainage design.

Career Interviews

Appearing in each chapter, Career Interviews with people who use algebra in their jobs help students understand that algebra is a modern, problem-solving language.

Math Matters

Each chapter contains a Math Matters feature that engages student interest by discussing an historical note or mathematical problem. For those features that pose a question, the answers appear in the back of the text.



SECTION 8.1 Introduction to Systems of Equations 507

92. *Geometry* A theorem from geometry states that if a triangle is inscribed in a circle so that one side of the triangle is a diameter of the circle, the triangle is a right triangle (see figure). Show that this theorem is true for the circle

$$x^2 + y^2 = 100$$

and the triangle formed by the lines

$$y = 0, \quad y = \frac{1}{2}x + 5, \quad \text{and} \quad y = -2x + 20.$$

(Find the vertices of the triangle and verify that it is a right triangle.)

Math Matters **Circle Problem of Apollonius**

Apollonius and Euclid are the two most famous mathematicians of the classical Greek period. Euclid lived about 300 B.C. and Apollonius lived about 100 years later. In one of the writings of Apollonius, he poses the following problem.

Suppose you are given three circles that have no points in common and no circle is inside either of the other two circles, as shown in the figure at the left. Is it always possible to find a fourth circle that is tangent to each of the given circles? The answer is yes. In fact, it can be shown that the problem always has eight distinct solutions. Four are shown here. Can you find the other four? (The answer is given in the back of the book.)

Mid-Chapter Quizzes

Each chapter contains a Mid-Chapter Quiz with answers in the back of the text. This feature allows the student to perform a self-assessment midway through the chapter.

MID-CHAPTER QUIZ

Take this quiz as you would take a quiz in class. After you are done, check your work against the answers given in the back of the book.

1. Given $f(x) = \left(\frac{4}{3}\right)^x$, find (a) $f(2)$, (b) $f(0)$, (c) $f(-1)$, and (d) $f(1.5)$.

2. Find the domain and range of $g(x) = 2^{-0.5x}$.

In Exercises 3–6, sketch the graph of the function.

3. $y = \frac{1}{2}(4^x)$ **4.** $y = 5(2^{-x})$ **5.** $f(t) = 12e^{-0.4t}$ **6.** $g(x) = 100(1.08)^x$

7. You deposit \$750 at $7\frac{1}{2}\%$ interest, compounded n times per year or continuously. Find the balance A after 20 years.

n	1	4	12	365	Continuous compounding
A					

8. A gallon of milk costs \$2.23 now. If the price increases by 4% each year, what will the price be after 5 years?

9. Given $f(x) = 2x - 3$ and $g(x) = x^3$, find the indicated composition.
(a) $(f \circ g)(-2)$ (b) $(g \circ f)(4)$ (c) $(f \circ g)(x)$ (d) $(g \circ f)(x)$

10. Verify algebraically and graphically that $f(x) = 3 - 5x$ and $g(x) = \frac{1}{5}(3 - x)$ are inverses of each other.

In Exercises 11 and 12, find the inverse of the function.

11. $h(x) = 10x + 3$ **12.** $g(t) = \frac{1}{2}t^3 + 2$

13. Write the logarithmic equation $\log_4\left(\frac{1}{16}\right) = -2$ in exponential form.

14. Write the exponential equation $3^4 = 81$ in logarithmic form.

15. Evaluate $\log_5 125$ without the aid of a calculator.

16. Write a paragraph comparing the graphs of $f(x) = \log_5 x$ and $g(x) = 5^x$.

In Exercises 17 and 18, use a graphing utility to sketch the graph of the function.

17. $f(t) = \frac{1}{2}\ln t$ **18.** $h(x) = 3 - \ln x$

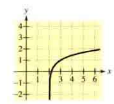


Figure for 19

19. Use the graph of f at the right to determine h and k if $f(x) = \log_5(x - h) + k$.

20. Use a calculator and the change-of-base formula to evaluate $\log_6 450$.

Chapter Project

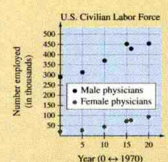
Chapter Projects, referenced in the chapter opener, are engaging applications that use real data, graphs, and modeling to enhance students' understanding of mathematical concepts. Designed as individual or group projects, they offer additional opportunities to think, discuss, and write about mathematics. Many projects include research assignments that give students the opportunity to collect, analyze, and interpret their own data. Each Chapter Project is also available in an interactive, multimedia, CD-ROM format.

CHAPTER PROJECT: Working Men and Women

Data is frequently represented in three ways: *numerically* by a table, *graphically*, or *algebraically* by an equation, formula, or algebraic model. Because each type of representation has its own advantages, the best way to present data is to use a mix of all three representations.

In gathering data, it may be easiest to first organize the data in a table. Finding patterns or trends in the data may be easier to recognize with a graph. And, finally, making future predictions may be easier with an algebraic model.

For instance, the table below shows the numbers of men and women (in thousands) who were practicing physicians from 1970 through 1990. The data is represented graphically by the scatter plot at the left. While investigating the questions below, you will be asked to find algebraic models to represent the data, and then to use the models to interpret the data. (Source: American Medical Association)



Year	1970	1975	1980	1985	1986	1990
Women Physicians	21.4	27.2	44.7	71.9	76.8	93.3
Men Physicians	289.5	313.1	370.2	452.3	429.0	454.0

1. ***Graphical Reasoning*** Use the scatter plot to write a verbal description of the data. Discuss any trend or pattern that is evident from the scatter plot.

2. ***Linear Modeling*** Use graph paper to redraw the scatter plot. Approximate each of the data sets with a line. Then find an equation for each line.

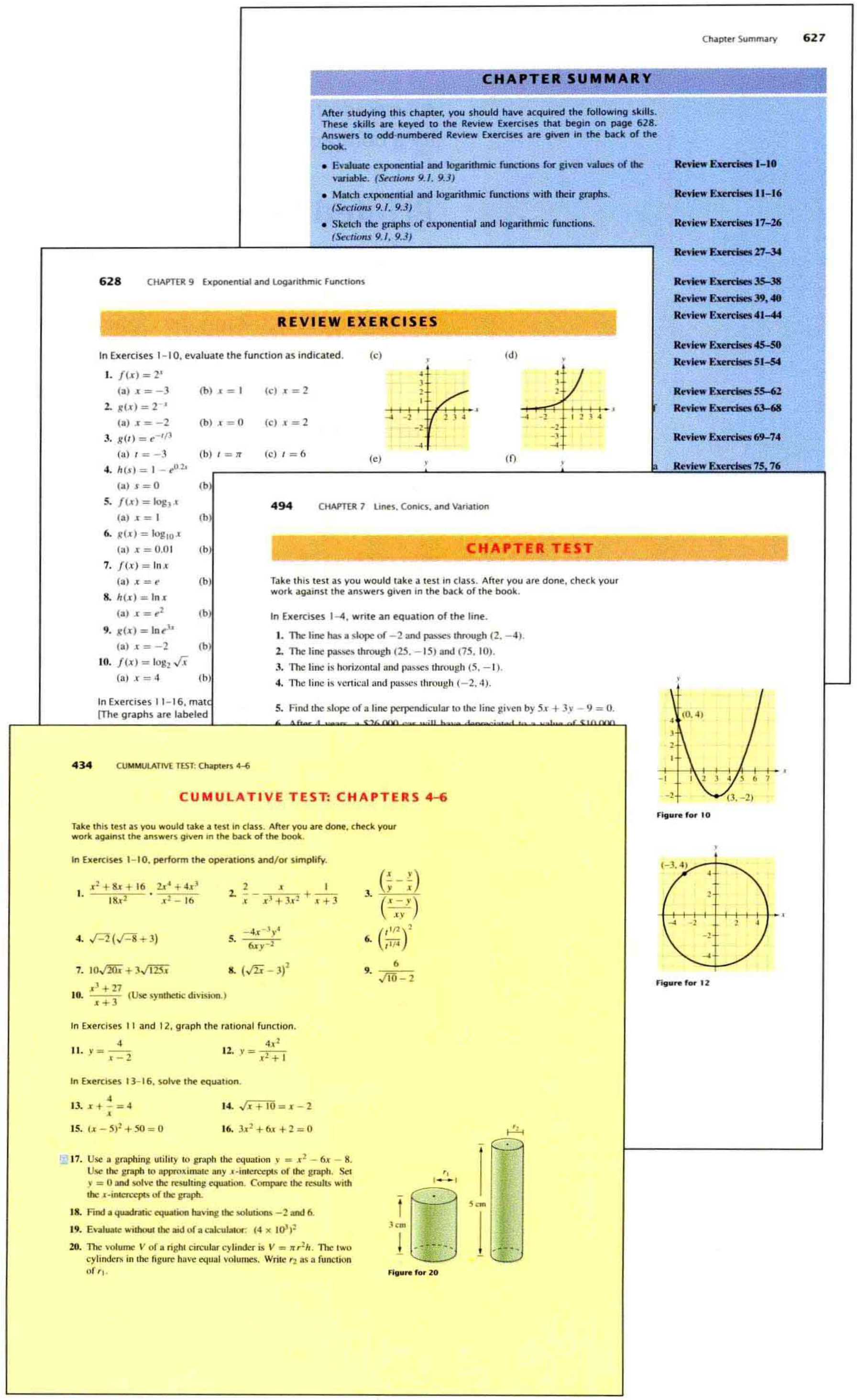
3. ***Linear Modeling*** Use the linear regression program on a graphing utility to find a linear model for each data set. Compare the results with those obtained in Question 2.

4. ***How Well Does It Fit?*** When the regression program in Question 3 is run, it will display a correlation coefficient r that measures how well the linear model fits the data. The closer r is to 1, the better the model fits the data. Which of the two models fits its data better? Does your answer seem reasonable from the graphical point of view?

5. ***Prediction*** Predict the numbers of women and men physicians in 1995. Discuss different ways that you could obtain the prediction. Which method do you prefer? Why?

6. ***Prediction*** Extend the lines you drew in Question 2 until they intersect. What interpretation can you make? Is the interpretation realistic in the context of the data?

7. ***Research Project*** Use your school's library or another reference source to find data for the numbers of men and women in an occupation. Organize the data numerically, graphically, and algebraically. What can you conclude?



Chapter Summary

The Chapter Summary reviews the skills covered in the chapter. Section references for the major topics make this an effective study tool, and correlation to the review exercises offers guided practice.

Review Exercises

The Review Exercises at the end of each chapter offer the student an opportunity for additional practice. Each set of review exercises includes both computational and applied problems covering a wide range of topics.

Chapter Test

Chapter Tests allow students to assess their own level of success.

Cumulative Tests

The Cumulative Tests that appear after Chapters 3, 6, and 9 help students judge their mastery of previously covered material, as well as reinforce the knowledge students have been accumulating throughout the text—preparing them for other exams and for future courses.

Supplements

Intermediate Algebra, Second Edition, by Larson and Hostetler, is accompanied by a comprehensive supplements package. All items are keyed to the text.

Printed Resources

Student Solutions Guide by Gerry C. Fitch, Louisiana State University

- Detailed, step-by-step solutions to all odd-numbered section exercises (except Discussing the Concept) and review exercises
- Detailed, step-by-step solutions to all Mid-Chapter Quiz, Chapter Test, and Cumulative Test questions

Study Guide by Jay Wiestling, Palomar College

- Section summaries
- Additional examples with solutions
- Starter exercises with answers

Graphing Technology Keystroke Guide: Algebra by Benjamin N. Levy

- Keystroke instructions for Texas Instruments, Sharp, Casio, and Hewlett-Packard graphing calculators
- Examples with step-by-step solutions
- Extensive graphics screen output
- Technology tips

Instructor's Annotated Edition

- Includes the entire student edition of the text, with the student answers section
- Instructor's Answers section: answers to all Discussing the Concepts exercises, all remaining even-numbered exercises, and all Discovery Boxes, Technology Boxes, Group Activities, and Chapter Projects
- Specific teaching strategies and suggestions
- Hints for implementing Group Activities
- Common student error annotations
- Additional examples, exercises, class activities, historical notes, and group activities

Test Item File and Instructor's Resource Guide

- Printed test bank with approximately 4000 test items (multiple-choice, open-ended, and writing) coded by level of difficulty
- Technology-required test items coded for easy reference
- Bank of chapter test forms with answer keys
- Two final exams
- Transparency masters
- Notes to the Instructor, which includes information on standardized tests such as the Texas Academic Skills Program (TASP), Florida College Level

Academic Skills Test (CLAST), and the California State University Entry Level Mathematics (ELM) Examination and provides a list of skills covered by the test and the corresponding section(s) in the text where the topic can be found, as well as notes on contemporary instructional strategies such as alternative assessment and cooperative learning

Media Resources

Tutor (IBM, Macintosh)

- Extensive additional practice

Videotapes by Dana Mosely

- Comprehensive coverage keyed to the text by section
- Detailed explanation of important concepts
- Numerous examples and applications, often illustrated via computer-generated animations
- Discussion of study skills
- For media resource centers; by popular demand, also available for student purchase

D.C. Heath Interactive Math Series CD-ROM Projects

- Real-life applications in an interactive, multimedia CD-ROM format
- IBM PC for Windows; Macintosh
- See page xvi for a description.

Computerized Testing

- Test-generating software for both IBM and Macintosh computers
- Approximately 4000 test items
- Also available as a printed test bank

CD-ROM Projects
for Intermediate Algebra, Second Edition

To accommodate a variety of teaching and learning styles, a series of real-life applications is available in a multimedia, interactive CD-ROM format. Suitable for individual or group assignments, these projects reinforce a variety of mathematical concepts. For each text chapter project is a CD-ROM project, allowing students to explore interactively questions that expand upon the topic and goals of the text project. Students have the opportunity to discover the nature of data sets through exploration, using a combination of graphical, numerical, and algebraic approaches in a guided learning environment. Throughout the text, you will notice a CD-ROM icon that reminds you of the availability of this multimedia software in conjunction with the chapter projects.

These multimedia projects broaden the scope of the text's chapter projects by offering additional opportunities for finding patterns and drawing conclusions, covering related topics and concepts, and providing practice with interpreting graphs, charts, and tables. The multimedia format provides access to extensive real data sets and facilitates hands-on data manipulation for practicing data analysis and modeling techniques. In addition, the projects include animations, color photographs, and audio enhancements.

Each multimedia project is presented in four parts: Introduction, Data, Exploration, and Exercises. The Introduction explains the goals of the project and the background of the project topic. The Data section presents all of the data that may be manipulated in the context of the project in a format that is appropriate to the placement in the text; additional history or pertinent facts may often be found in this section. The Exploration section enables students to manipulate data and discover certain facts about or patterns within the data. For example, the Mass Transportation project allows students to use graphs to find patterns and interactively experiment with placing a line on a scatter plot of actual data to approximate a best fitting line. The Exercises section is a set of questions designed to guide the student to the types of discoveries that may be made from exploration of the data. For example, with the Mass Transportation project students are asked to interpret slopes and y-intercepts, consider predictions, and compare various models.

The CD-ROM Projects for *Intermediate Algebra*, Second Edition, are available for use with multimedia Macintosh or IBM with Windows computers. They cover the following topics:

Chapter P	Playing the Stock Market	**Chapter 6**	Gravitation
Chapter 1	Animal Voices and Hearing	**Chapter 7**	Transportation
Chapter 2	Job Comparisons	**Chapter 8**	Retail Sales of Companies
Chapter 3	Volume of a Box	**Chapter 9**	Half-Life and Radioactivity
Chapter 4	Parachutes and Ratios	**Chapter 10**	Mortgages and Finance