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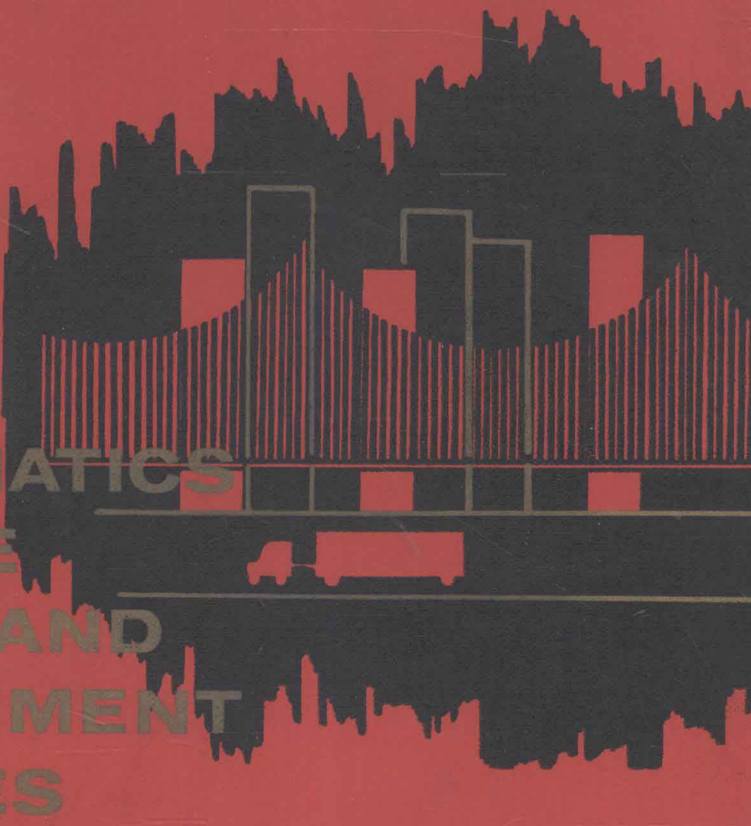

MATHEMATICS FOR THE SOCIAL AND MANAGEMENT SCIENCES

Finite Mathematics

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Mathematics for the Social and Management Sciences: *Finite Mathematics*

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TO MY WIFE

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PREFACE

This book was written to fill a need that has been evident to me for some time. There are, of course, many elementary books on finite mathematics available, some of them written specifically for the social and management sciences; there is none, however, that includes all the subjects in this book. These are subjects which should, in my opinion, be included. There are, in effect, certain topics here that are not found in other books at the level. Now, none of these requires complicated mathematics. Rather, they seem to belong to the "common-sense" school: they are applications to large problems of methods that are automatically used for small problems, and this because they are so obvious.

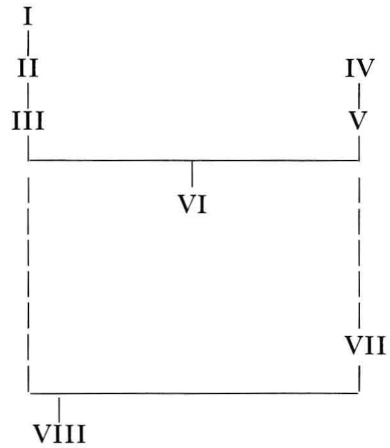
In general, I have tried to introduce the problem first, and have, with this motivation, developed the mathematics. It is my hope that the student will learn more easily in this manner. In some cases I have discussed alternative, but impractical, methods (e.g., enumeration of all extreme points) to show that common sense is not, in itself, sufficient: some practical experience is usually necessary.

Let us look at the book in some slight detail. Chapter I covers topics (systems of linear equations) that have almost certainly been seen before; it takes advantage of them to introduce the more complicated inequalities. Chapter II takes the topics of Chapter I and puts them in the more formal setting of linear algebra. Chapter III uses the results of the first two chapters, applying them to more practical problems.

Chapter IV again deals with subjects that have probably been seen before (sets and logic). These are then used to help in the formalism of Chapter V, which is almost certainly the most difficult one in the book.

Chapter VI, which depends on both III and V, introduces the reader to the important field of game theory. Chapter VII, which is almost independent of the others, develops some new but important methods. Finally, Chapter VIII is of importance in that it introduces the reader to graphical methods. The Appendix includes, for reference, some subjects that have probably been seen by the reader.

The logical connection among the chapters can be seen in the following table:



in which a solid line means that the lower chapter requires reading of the upper chapter, but a broken line means merely that knowledge of the upper chapter is helpful.

It is a pleasure to express my gratitude to those who have helped me in the writing of this book: to my wife, who has been most patient and has encouraged me throughout; to Fordham University, which contributed funds for the preparation of the manuscript; and finally, to Mrs. Adrienne DiFranco, who did some excellent typing and otherwise helped to prepare this manuscript.

G. O.

CONTENTS

ANALYTIC GEOMETRY

1. THE CARTESIAN PLANE

Among the intellectual achievements of the Greeks, their mathematics, and especially their geometry, must be given a place of honor. Thales, Pythagoras, Apollonius, Archimedes—to name but a few—have lent their names to important geometric ideas, while Euclid's *Elements* remains one of the fundamental works of classical antiquity.

Yet for all their excellence in geometry, the Greeks often found themselves baffled by problems that could be solved today by a schoolboy. The trouble lay, not in a lack of ingenuity, but rather in the fact that the ancients lacked one of the fundamental tools of modern mathematics.

It remained to René Descartes (1596-1650) to discover this tool. Descartes is primarily known for his philosophic works—the *Discourse on Method* and the *Meditations on Prime Philosophy.* While no one can deny the extent of his influence on modern philosophy, we would yet venture to say that his mathematical work will prove the more important and enduring.

Descartes' great discovery can best be appreciated if we consider the differences between the two mathematical sciences of geometry and algebra. Geometry deals with the relations between points and lines, while algebra deals with numbers. It is generally easy, when dealing with numbers, to decide what should be done with them; because this is not true of points and lines, algebraic problems have always been easier to solve than geometric problems. Descartes' discovery was, precisely, that it is generally possible to solve geometric problems by algebraic means. He showed that the set of all points on a line has a structure identical to that of the set of real numbers.

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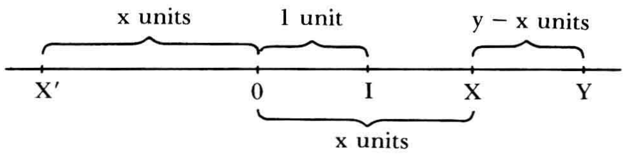


FIGURE I.1.1 The Cartesian line.

Consider, for example, a line (Figure I.1.1). On this line, two points may be taken arbitrarily (the only condition being that they be distinct points), and labeled O and I, respectively. If X is any other point on the line, we associate with X the number

1.1.1 $$x = \frac{\overline{OX}}{\overline{OI}}$$

which is the ratio of the lengths of the two line-segments, $\overline{OX}$ and $\overline{OI}$, with the stipulation that this ratio will be positive if X lies on the same side of O as I (so that $\overline{OX}$ and $\overline{OI}$ have the same direction), while it will be negative if X and I lie on opposite sides of O (so that $\overline{OX}$ and $\overline{OI}$ have opposite directions). The number x is called the *coordinate* of the point X. It is easy to see that the coordinate, x, is merely the distance $\overline{OX}$, measured in units of size $\overline{OI}$, so that the point O, called the *origin,* will have coordinate 0, while the point I will have coordinate 1.

We see, then, that each point on the line can be assigned a number, its coordinate. Conversely, given any positive number x, there are two points X and X' whose distance from the origin is equal to x units. One of these is on the same side of the origin as I and corresponds to x; the other is on the other side of the origin and will correspond to the number $-x$. In this way, each number corresponds to a unique point on the line. We have thus established a one-to-one correspondence between points on the line and real numbers. What is more, the structures of the two systems, in terms of operations that can be performed, are similar. To give an example, the distance $\overline{XY}$ between two points can be expressed in terms of their coordinates by

1.1.2 $$\overline{XY} = y - x$$

so that the geometric relation of distance reduces to the arithmetic operation of subtraction.

Dealing with the geometry of the plane, we find, however, requires a slightly more complicated procedure. It is, in fact, possible to give a one-to-one correspondence between the set of points in the plane and the set of real numbers, but this correspondence is not natural and does not preserve the structure of the system. Instead of assigning a number to each point, then, we assign a pair of numbers.

Consider, in the plane, two straight lines intersecting at a point O. The angle of intersection is not important, but for the sake of convenience, it is best to assume that they intersect at right angles (Figure I.1.2).

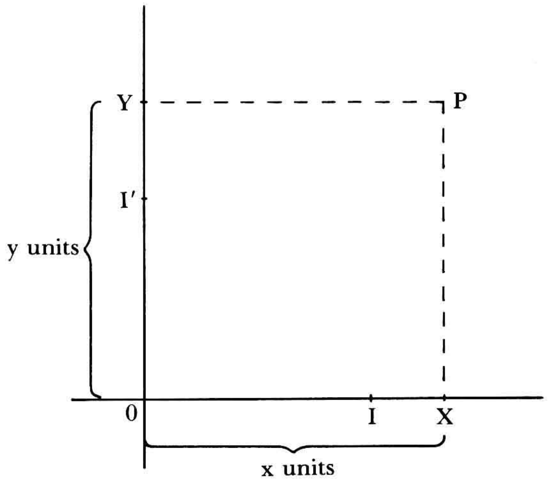


FIGURE I.1.2 The Cartesian plane.

On each of these lines, points I and I' are taken. Although the only need is that they be distinct from O, for convenience they are usually taken to be equidistant from O, and I' is usually obtained from I by a counterclockwise rotation through one right angle.

Consider, now, a point, P, in this plane. Through P, we may draw lines PX and PY, parallel to OI' and OI, respectively. The line PX meets OI at the point X, while PY meets OI' at Y. As in the one-dimensional case, just mentioned, we write

1.1.3 $$x = \frac{\overline{OX}}{\overline{OI}}$$

and

1.1.4 $$y = \frac{\overline{OY}}{\overline{OI'}}$$

where, once again, we agree to let the ratio of two line-segments be positive if they have the same direction, and negative if they have opposite directions.

The two numbers, x and y, given by (1.1.3) and (1.1.4) respectively, are called the *coordinates* of the point P. The number x is generally called the *abscissa*, and y the *ordinate*, of P. The lines OI

and OI' are called the coordinate *axes*; OI is the x-axis, and OI' is the y-axis.

We have thus assigned, to each point in space, a pair of real numbers (x,y), its coordinates. It may be pointed out that, since $OXPY$ is a rectangle, we must have $\overline{OX}=\overline{YP}$, while $\overline{OY}=\overline{XP}$. Thus x and y will be the perpendicular distances of the point P from the y-axis and x-axis, respectively, in terms of the common unit $\overline{OI}$ or $\overline{OI'}$.

2. GRAPHS AND EQUATIONS

We proceed now to study some of the advantages derived from the analytic treatment of geometry. Let us suppose that we are given a relation between two unknowns (or variables), x and y. This relation might be in the form of an equation, say,

1.2.1 $$y = x^2 + 3x$$

or of a word problem,

1.2.2 "x is not smaller than y, but not larger than twice y"

or in many other possible forms. There are necessarily certain pairs of values (x,y) for which the given relation (1.2.1) or (1.2.2) will be true and others for which it will not be true. If we consider the pairs of numbers for which the relation is true, we may plot the position of the corresponding points (i.e., the points having these pairs as their coordinates) on a coordinate plane. The set of these points is called the *graph* of the relation. Figures I.2.1 and I.2.2 show, respectively,

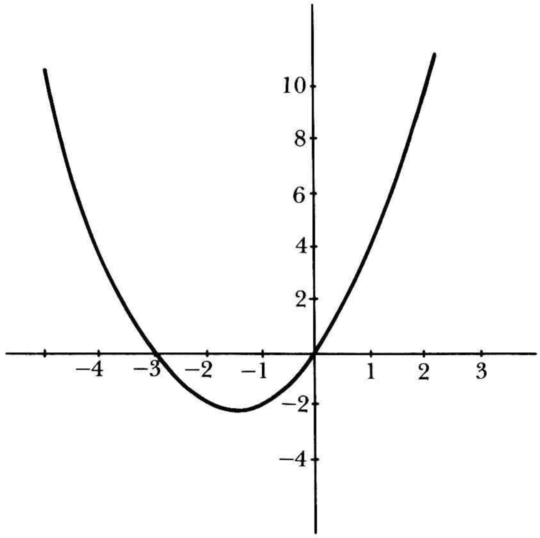
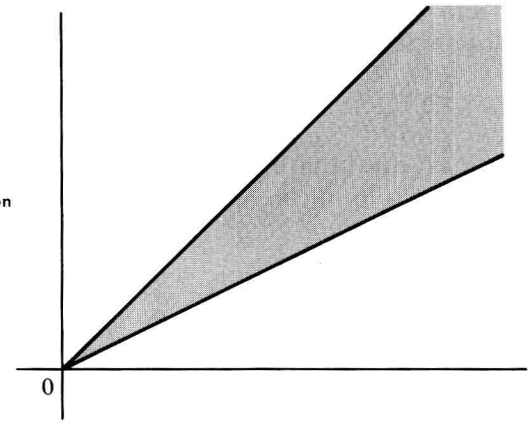


FIGURE I.2.1 Graph of $y = x^2 + 3x$.

FIGURE I.2.2 Graph of the relation 1.2.2.

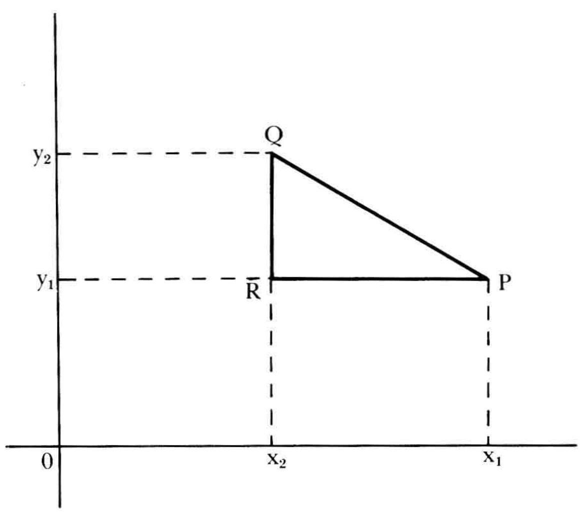


the graphs of the relations (1.2.1) and (1.2.2). For the equation (1.2.1), the graph is a curve (in this case, a *parabola*). As for the relation (1.2.2), we find that its graph consists of all the points inside a wedge with its angle at the origin.

Conversely, if we are given a curve in the plane, it is often possible to find a numerical relation that is satisfied by the coordinates of the points on the curve and by no others. If an equation, this relation is said to be the equation of the curve; (1.2.1) is the equation of the parabola shown in Figure I.2.1.

Relations between points can also be expressed analytically by this means. Consider, for example, two points, P and Q (Figure I.2.3), with coordinates (x_1, y_1) and (x_2, y_2) respectively. If we let R have the same ordinate as P, and the same abscissa as Q, we see that the lines

FIGURE I.2.3



$\overline{PQ}^2 = \overline{RPR}^2 + \overline{RQ}^2 = (x_1 - x_2)^2 + (y_2 - y_1)^2$

PR and *QR*, being parallel to the two coordinate axes, must intersect at right angles, and thus, by Pythagoras' Theorem,

1.2.3 $$\overline{PQ^2} = \overline{PR^2} + \overline{QR^2}$$

We note, now, that $\overline{PR} = \overline{X_1X_2}$ and $\overline{RQ} = \overline{Y_1Y_2}$. Replacing these values in (1.2.3), introducing the coordinates, and taking square roots, we obtain

1.2.4 $$\overline{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

the formula for *distance* between two points in the plane.

Consider, finally, the line *PQ*. It makes an angle, θ, with the x-axis. To find this angle, we note that it is the same as the angle between *PQ* and *PR* (since *PR* is parallel to the x-axis). From elementary trigonometry, we know that

$$\tan \theta = \frac{\overline{RQ}}{\overline{PR}}$$

Now, the tangent of the angle is called the *slope* of *PQ* (see Figure I.2.4). If we let m represent this slope, and introduce coordinates, we have

1.2.5 $$m = \frac{y_2 - y_1}{x_2 - x_1}$$

as the formula for slope. Although the slope is given in terms of the coordinates of *P* and *Q*, it is a property of the line *PQ*, and the formula (1.2.5) will give the same value if we substitute the coordinates of any two points on *PQ*. (Sometimes the slope is defined as a property of the two points, *P* and *Q*; it is then necessary—though easy—to prove that the slope is constant along a line.)

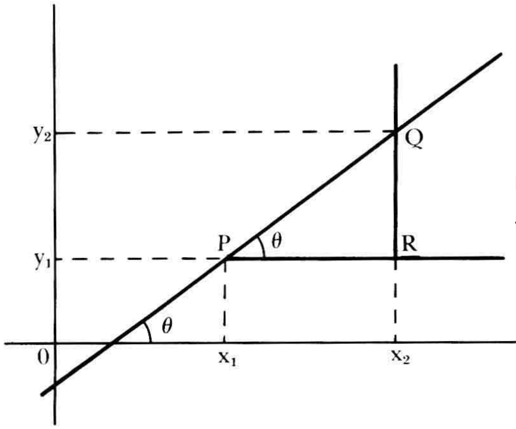


FIGURE I.2.4 The slope of *PQ* is $\tan \theta = \frac{\overline{RQ}}{\overline{PR}}$.

We point out that if a line is horizontal (parallel to the x-axis) its slope must be 0. In fact, points on such a line must have the same ordinate—and the numerator in (1.2.5) vanishes. On the other hand, if two points lie on a vertical line (i.e., parallel to the y-axis), then the denominator in (1.2.5) will vanish. In this case the slope does not exist, although it is sometimes said that the line has infinite slope.

We are now in a position to prove a few theorems concerning lines and their slopes.

I.2.1 Theorem. Two lines are parallel if and only if both have the same slope (or no slope at all).

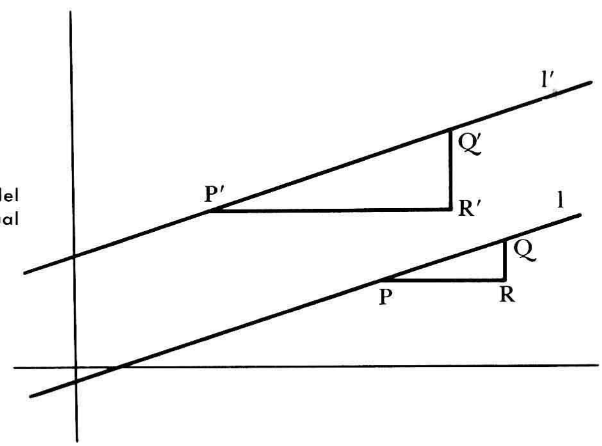


FIGURE I.2.5 The two parallel lines, l and l', have equal slopes.

Proof (see Figure I.2.5). On the lines l and l', take two pairs of points P,Q on l and P', Q' on l'. If l and l' are parallel, the two triangles PQR and $P'Q'R'$ are similar (having corresponding sides parallel) and so

1.2.6
$$\frac{\overline{RQ}}{\overline{PR}} = \frac{\overline{R'Q'}}{\overline{P'R'}}$$

Conversely, if (1.2.6) holds, the two triangles PQR and $P'Q'R'$ are similar. Since PR and $P'R'$ are parallel, this means that PQ and $P'Q'$ are also parallel. This covers the case in which l and l' both have slopes. If the slopes do not exist, then both lines are parallel to the y-axis, and hence to each other.

I.2.2 Corollary. Through a given point P there passes one and only one line with a given slope m.

Proof. This follows from Theorem I.2.1 and the well-known Euclidean fact that through P there passes exactly one line parallel to the line with slope m.

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