

Mathematics Monograph Series **30**

The Elements of Financial Econometrics

Jianqing Fan Qiwei Yao (范剑青 姚琦伟)

(计量金融精要)



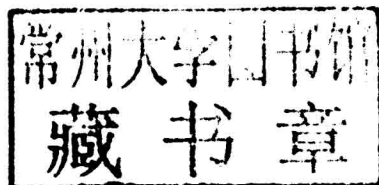
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Preface to Mathematics Monograph Series

Science Press asked me to write a preface for their series of books called “Mathematics Monograph Series”. They told me that the Press had published nearly 30 mathematical monographs in this series since 2006. This reminded me that, also in 2006, I received an email message from the Editor in Chief of “Sugaku Tushin” (“Mathematical Communications”, the membership magazine of the Mathematical Society of Japan). The Editor in Chief told me that they were planning to have a special section on “Recent development in Chinese mathematical community”, and invited me to write an article for the special section. As a result, I published in their magazine an article called “Some Aspects of Mathematical Community in China”. Among other things, in the article I demonstrated that, with the favorable environment, the Chinese mathematical community had made great progress since the late 1970s (when China started to implement “Reform and Opening-up Policy”): A large number of publications (including articles and monographs) have been written by Chinese mathematicians, there have been always Chinese mathematicians presenting their speeches at various international academic conferences or workshops, many Chinese mathematicians have served as editors of international academic journals, or as members in various academic organizations. All these reveal that the Chinese mathematical community which has been growing rapidly has exerted more and more influence in the world.

Indeed, the series of mathematical monographs published by Science Press reflects partly more and more influence of Chinese mathematical community in the world. Chinese people are good at mathematics. In the past, Science Press published many high level mathematical monographs and textbooks. Among them some were written in Chinese and some were written in English. Some monographs which appeared originally in Chinese have been purchased by international publishers who then re-published them abroad in English, and this has gained influence in corresponding areas of the international mathematical community.

In recent years most Chinese mathematicians have mastered good English. In accordance with this situation, Science Press has decided to publish “Mathematics Monograph Series” — a series in which high level mathematical monographs and textbooks are written directly in English. The goal of this series is to provide further good service for Chinese mathematicians and to enhance further the influence of the mathematics study in China in the international mathematical community.

I would like to conclude this short preface with the following wish which I expressed also at the end of the afore mentioned article “Some Aspects of Mathematical Community in China”:

The Chinese mathematical community will continuously make its effort to work hard, and to strengthen its international exchanges and collaborations, so as to make more contributions to the study and development of mathematics in the world.

Zhi-Ming Ma
March 15, 2015

Preface

This is an introductory textbook for financial econometrics at the master's level. Readers are assumed to have some background in calculus, linear algebra, statistics, and probability, all at undergraduate level. Knowledge in economics and finance is beneficial but not essential.

This book grew out of the lecture notes for the “Financial Econometrics” course taught by Jianqing Fan for Master in Finance students at Princeton University since 2003 and for Master in Financial Engineering students at Fudan University since 2011. The audiences are always very broad with diverse background in mathematics, physics, computer science, economics, or finance. Those talented students wish to learn fundamentals of financial econometrics in order to work in financial industry or to pursue a Ph.D. degree in related fields. The challenges are to give all of them sufficient financial econometrics knowledge upon the completion of the class. This text book is written with the aim to achieve such an ambitious goal.

We trust that the book will be of interest to those coming to the area for the first time, to readers who already have economics and finance background but would like to further sharpen their quantitative skills, and also to those who have strong quantitative skills and would like to learn how to apply them to finance. Application-oriented analysts will also find this book useful, as it focuses on methodology and includes numerous case studies with real data sets. We purposely keep the level of mathematics moderate, as the power of the quantitative methods can be understood without sophisticated technical details. We also avoid the cook book style of writing, as good understanding of statistical and econometric principles in finance enable readers to apply the knowledge far beyond the problems stated in the book. Numerical illustration with real financial data is throughout the book. We also indicate, whenever possible, where to find the relevant R codes to implement the various methods. Due to the nature of the subject, it is inevitable that we occasionally step into more sophisticated techniques which rely on more advanced mathematics; such sections are marked with “*” and can be ignored for beginners. Most technical arguments are collected in a “Complements” section at the end of some chapters,

but key ideas are left within the main body of the text.

What is financial econometrics? Broadly speaking, it is an interdisciplinary subject that uses statistical methods and economic theory to address a variety of quantitative problems in finance. These include building financial models, testing financial economics theory, simulating financial systems, volatility estimation, risk management, capital asset pricing, derivative pricing, portfolio allocation, proprietary trading, portfolio and derivative hedging, among others. Financial econometrics is an active field of integration of finance, economics, probability, statistics, and applied mathematics. Financial activities generate many new problems and products, economics provides useful theoretical foundation and guidance, and quantitative methods such as statistics, probability and applied mathematics are essential tools to solve quantitative problems in finance. Professionals in finance now routinely use sophisticated statistical techniques and modern computation power in portfolio management, proprietary trading, derivative pricing, financial consulting, securities regulation, and risk management.

When the class was first taught in 2003, there were very few books on financial econometrics. The books that have strong impact on our preparation of lecture notes are Campbell et al. (1997) and Fan and Yao (2003). With one semester of teaching, we can only cover the important elements of financial econometrics. We use this name as the title of the book, as it also reflects the modulus aspect of the book. This allows us to expand this textbook to cover other fundamental materials in the future. For example, “Simulation methods in finance” and “Econometrics of continuous time finance” are taught at Princeton, but are not in this book nor in Fudan’s financial engineering class due to time constraints. Another important topic that we wish to cover is the analysis of high-frequency financial data.

The book consists of two integrated parts: The first four chapters are on time series aspects of financial econometrics while the last five chapters on cross-sectional aspects. The introduction in Chapter 1 sets the scene for the book: using two financial price time series we illustrate the stylized features in financial returns. The efficient markets hypothesis is deliberated together with statistical tests for random walks and white noise. A compact view of linear time series models is given in Chapter 2, including ARMA models, random walks, and inference with trends. We also include a brief introduction on the exponential smoothing based forecasting techniques for trends and momentum, which are widely used in financial industry. Chapter 3 introduces various heteroscedastic volatility models. A compact introduction to state space models including the techniques such as Kalman filter and particle filters is included as an appendix. Chapter 4 contains some selective topics in multivariate time series analysis. Within the context of vector autoregressive models, we also introduce the topics such as Granger causality, impulse response

functions, and cointegration, as they play important roles in economics and finance.

The second begins with Chapter 5, which introduces portfolio theory and derives the celebrated capital asset model. We also introduce statistical techniques to test such a celebrated model and provide extensive empirical studies. Chapter 6 extends the capital asset pricing model to multi-factor pricing model. The applications of the factor models and econometrics tests on the validity of such pricing models are introduced. In addition, principal component analysis and factor analysis are briefly discussed. Chapter 7 touches several practical aspects of portfolio allocation and risk management. The highlights of this chapter include risk assessments of large portfolios, portfolio allocation under gross-exposure constraints, and large volatility matrix estimation using factor models and covariance regularization. Chapter 8 derives the capital asset pricing model from consumption, investment, and saving point of view. This gives students different perspective on where the financial prices come from and a chance to appreciate its differences from pricing financial derivatives. Chapter 9 calculates the prices implied by the models of returns. It gives us an idea of what the fundamental price of a stock is and how the prices are related to the dividend payments and short-term interest rates.

Many people have been of great help to our work on this book. Early drafts of this book has been taught to about over five hundred students and there are our enthusiastic readers. In particular, we are grateful to Yingying Fan, Yue Niu, Jingjing Zhang, Feng Yang, Weijie Gu, Xin Tong, Wei Dai, Jiawei Yao, Xiaofeng Shi, and Weichen Wang for their gracious assistance of teaching Financial Econometrics class at Princeton. Réne Carmona provides us his course outlines on Financial Econometrics that helps the selection of the topics of the course. Many treatments of ARIMA models are inspired by the lecture notes of George Tiao. Alex Furger and Michael Lachans spend a great amount of their precious time to proof-read the final version of the book. We are very grateful to their contributions and generosity. We would like to thank to Yacine Ait-Sahalia and Yazhen Wang for stimulating discussions on the topic and Shaojun Guo for formatting the references of the book. We are indebted to Jiaan Yan for his encouragement and support to publish this book in China and to Xiongwen Lu for inviting us to teach the course in Fudan University. We would also like to thank to Yuzuo Chen for providing various editorial assistance.

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December 2014

Jianqing Fan, Princeton
Qiwei Yao, London

Contents

Preface to Mathematics Monograph Series

Preface

Chapter 1	Asset Returns	1
1.1	Returns	1
1.1.1	One-period simple returns and gross returns	1
1.1.2	Multiperiod returns	2
1.1.3	Log returns and continuously compounding	2
1.1.4	Adjustment for dividends	4
1.1.5	Bond yields and prices	5
1.1.6	Excess returns	6
1.2	Behavior of financial return data	7
1.2.1	Stylized features of financial returns	12
1.3	Efficient markets hypothesis and statistical models for returns	16
1.4	Tests related to efficient markets hypothesis	20
1.4.1	Tests for white noise	20
1.4.2	Remarks on the Ljung-Box test*	22
1.4.3	Tests for random walks	23
1.4.4	Ljung-Box test and Dickey-Fuller test	26
1.5	Appendix: Q-Q plot and Jarque-Bera test	26
1.5.1	Q-Q plot	26
1.5.2	Jarque-Bera test	27
1.6	Further reading and software implementation	28
1.7	Exercises	29
Chapter 2	Linear Time Series Models	31
2.1	Stationarity	31
2.2	Stationary ARMA models	33
2.2.1	Moving average processes	34
2.2.2	Autoregressive processes	38
2.2.3	Autoregressive and moving average processes	45

2.3	Nonstationary and long memory ARMA processes	50
2.3.1	Random walks	50
2.3.2	ARIMA model and exponential smoothing	52
2.3.3	FARIMA model and long memory processes*	53
2.3.4	Summary of time series models	54
2.4	Model selection using ACF, PACF and EACF*	55
2.5	Fitting ARMA models: MLE and LSE	59
2.5.1	Least squares estimation	59
2.5.2	Gaussian maximum likelihood estimation	61
2.5.3	Illustration with gold prices	63
2.5.4	A snapshot of maximum likelihood methods*	67
2.6	Model diagnostics: residual analysis	69
2.6.1	Residual plots	69
2.6.2	Goodness-of-fit tests for residuals	72
2.7	Model identification based on information criteria	73
2.8	Stochastic and deterministic trends	75
2.8.1	Trend removal	76
2.8.2	Augmented Dickey-Fuller test	77
2.8.3	An illustration	79
2.8.4	Seasonality	82
2.9	Forecasting	84
2.9.1	Forecasting ARMA processes	84
2.9.2	Forecasting trends and momentum of financial markets	89
2.10	Appendix: Time series analysis in R	97
2.10.1	Start up with R	97
2.10.2	R-functions for time series analysis	98
2.10.3	TSA – an add-on package	99
2.11	Exercises	100
Chapter 3	Heteroscedastic Volatility Models	104
3.1	ARCH and GARCH models	105
3.1.1	ARCH models	105
3.1.2	GARCH models	110
3.1.3	Stationarity of GARCH models	113
3.1.4	Fourth moments	115
3.1.5	Forecasting volatility	118
3.2	Estimation for GARCH models	120
3.2.1	Conditional maximum likelihood estimation	120
3.2.2	Model diagnostics	122

3.2.3	Applications of GARCH modeling	124
3.2.4	Asymptotic properties*	131
3.2.5	Least absolute deviations estimation*	132
3.3	ARMA-GARCH models	136
3.4	Extended GARCH models	137
3.4.1	EGARCH models	138
3.4.2	Asymmetric power GARCH	143
3.4.3	Excess returns and GARCH-in-Mean	146
3.4.4	Integrated GARCH model	147
3.5	Stochastic volatility models	148
3.5.1	Probabilistic properties	149
3.5.2	Parameter estimation	149
3.5.3	Leverage effects	152
3.6	Appendix: State space models*	153
3.6.1	Linear models	153
3.6.2	Kalman recursions for Gaussian models	153
3.6.3	Nonlinear models	156
3.6.4	Particle filters	158
3.7	Exercises	160
Chapter 4	Multivariate Time Series Analysis	163
4.1	Stationarity and auto-correlation matrices	163
4.1.1	Stationary vector processes	163
4.1.2	Sample cross-covariance/correlation matrices	165
4.2	Vector autoregressive models	168
4.2.1	Stationarity	169
4.2.2	Parameter estimation	170
4.2.3	Model selection and diagnostics	173
4.2.4	Illustration with real data	175
4.2.5	Granger causality	179
4.2.6	Impulse response functions	182
4.3	Cointegration	185
4.3.1	Unit roots and cointegration	186
4.3.2	Engle-Granger method and error correction models	187
4.3.3	Johansen's likelihood method*	191
4.3.4	Illustration with real data	195
4.4	Exercises	199
Chapter 5	Efficient Portfolios and Capital Asset Pricing Model	201
5.1	Efficient portfolios	201

5.1.1	Returns and risks of portfolios	201
5.1.2	Portfolio optimization	202
5.1.3	Efficient portfolios and Sharpe ratios	205
5.1.4	Efficient frontiers	206
5.1.5	Challenges of implementation	207
5.2	Optimizing expected utility function	208
5.3	Capital asset pricing model	210
5.3.1	Market portfolio	210
5.3.2	Capital asset pricing model	212
5.3.3	Market β and its applications	214
5.4	Validating CAPM	215
5.4.1	Econometric formulation	215
5.4.2	Maximum likelihood estimation	216
5.4.3	Testing statistics	218
5.5	Empirical studies	223
5.5.1	An overview	223
5.5.2	Fama-French portfolios	224
5.5.3	Further remarks	227
5.6	Cross-sectional regression	227
5.7	Portfolio optimization without a risk-free asset	228
5.8	CAPM with unknowing risk free rate	236
5.8.1	Validating the Black version of CAPM	237
5.8.2	Testing statistics	237
5.9	Complements	240
5.9.1	Proof of (5.43)	240
5.9.2	Proof of (5.48)	240
5.10	Exercises	241
Chapter 6	Factor Pricing Models	245
6.1	Multifactor pricing models	245
6.1.1	Multifactor models	245
6.1.2	Factor pricing models	249
6.2	Applications of multifactor models	250
6.3	Model validation with tradable factors	251
6.3.1	Existence of a risk-free asset	252
6.3.2	Estimation of risk premia	252
6.3.3	Testing statistics	253
6.3.4	An empirical study using Fama-French portfolios	256
6.3.5	Absence of a risk-free asset*	258

6.4	Macroeconomic variables as factors*	260
6.5	Selection of factors	261
6.5.1	Principal component analysis	262
6.5.2	Factor analysis*	267
6.6	Exercises	269
Chapter 7	Portfolio Allocation and Risk Assessment	272
7.1	Risk assessment of large portfolios	272
7.1.1	Stability of a portfolio	273
7.1.2	Stability and risk approximations	274
7.1.3	Errors in risk assessments	279
7.1.4	Representative portfolios with a given exposure	281
7.2	Estimation of a large volatility matrix	282
7.2.1	Exponential smoothing	282
7.2.2	Regularization by thresholding	284
7.2.3	Projections onto semi-positive and positive definite matrix spaces	287
7.2.4	Regularization by penalized likelihood*	288
7.2.5	Factor model with observable factors	291
7.2.6	Approximate factor models with observable factors	295
7.2.7	Approximate factor models with unobservable factors	298
7.3	Portfolio allocation with gross-exposure constraints	301
7.3.1	Portfolio selection with gross-exposure constraint	302
7.3.2	Relation with covariance regularization*	305
7.4	Portfolio selection and tracking	306
7.4.1	Relation with regression	306
7.4.2	Portfolio selection and tracking	307
7.5	Empirical applications	308
7.5.1	Fama-French 100 portfolios	309
7.5.2	Russell 3000 stocks	311
7.6	Complements	312
7.6.1	Proof of Theorem 7.2	312
7.6.2	Proof of Theorem 7.3	313
7.6.3	Proof of (7.48)	313
7.7	Exercises	314
Chapter 8	Consumption based CAPM	316
8.1	Utility optimization	316
8.2	Consumption-based CAPM	319
8.2.1	CCAPM	319
8.2.2	Power utility	321

8.3	Mean-variance frontier*	325
8.4	Exercises	327
Chapter 9	Present-value Models	328
9.1	Fundamental price	328
9.2	Rational bubbles	330
9.3	Time-varying expected returns	332
9.4	Empirical evidence	336
9.5	Linear regression under dependence	344
9.6	Exercises	346
References		348
Author Index		358
Subject Index		362

List of Figures

1.1	Plots of log returns against simple returns of the Apple Inc.....	3
1.2	Yield spreads and returns of bonds.....	6
1.3	Daily, weakly and monthly returns of S&P 500 index.....	8
1.4	Daily, weakly and monthly returns of Apple stock.....	9
1.5	Histograms and Q-Q plots of log returns of S&P 500.....	9
1.6	Histograms and Q-Q plots of log returns of Apple stock.....	10
1.7	ACF of log-, squared and absolute returns of S&P 500.....	11
1.8	ACF of log-, squared and absolute returns of Apple stock.....	12
1.9	Tail distributions of S&P 500 index.....	14
1.10	VIX and S&P 500 index.....	16
1.11	Relationship among different processes.....	18
1.12	Ljung-Box and Dickey-Fuller tests.....	26
2.1	Time series plot and sample ACF plot of four moving-average processes of different orders.....	36
2.2	Time series plot and sample PACF plot of 4 autoregressive processes.....	44
2.3	Five stationary time series with the same marginal distribution.....	47
2.4	Sample ACF and PACF for five stationary time series plotted in Figure 2.3.....	48
2.5	Relationship among different processes.....	50
2.6	Time series plot and ACF of a random walk.....	52
2.7	The yields of a basket HY bonds and their differences.....	53
2.8	A schematic overview of time series models.....	55
2.9	Plots for the daily gold prices and their differences.....	64
2.10	Daily gold prices and their one-step-ahead predicted prices.....	66
2.11	Good and bad residual patterns.....	70
2.12	Diagnostic plots for daily gold prices.....	71
2.13	Diagnostic plots for daily gold prices.....	72
2.14	Q-Q plot of the residuals from fitted ARIMA models for the daily gold prices.....	72

2.15	A random walk plus white noise with a linear trend	76
2.16	log daily S&P 500 index prices and their sample ACF	79
2.17	Time series plot of the residuals	81
2.18	Quarterly earnings of IBM and Johnson and Johnson	82
2.19	ACFs for the earnings of Johnson and Johnson	83
2.20	Illustration of the best predictor	85
2.21	MACD technical indicators	93
2.22	RSI technical indicators	96
3.1	Figures of an ARCH(1) model	108
3.2	Features of a simulated ARCH(1) model	110
3.3	Relationship among different white noise processes	112
3.4	Features of GARCH(1,1) model	118
3.5	Features of GARCH(1,1) model	119
3.6	Diagnostic plot of GARCH(1,1) fit	124
3.7	Daily prices of the S&P 500 index, Goldman Sachs and Ford	125
3.8	Daily returns of the S&P 500 index, Goldman Sachs and Ford and their predictive intervals	126
3.9	ACFs for squared and the absolute residuals GARCH(1,1) fit	128
3.10	Daily returns and their predicted VaRs	130
3.11	Absolute errors of Gaussian MLE and LADE	134
3.12	Fitted volatility using EGARCH for S&P 500, Goldman Sachs and Ford.	142
3.13	Q-Q plots of standardized residuals	142
3.14	Fitted volatility based on APGARCH(1,1) fit	144
3.15	VaRs based on APGARCH(1,1) fit	145
3.16	Weights in exponential smoothing and its equivalent window size	147
3.17	Predicted VaR from stochastic volatility models	152
4.1	Daily log close prices of FTSE 100 index, FTSE MidCap index, and FTSE SmallCap index	167
4.2	Sample cross-correlations of the log returns	168
4.3	Sample cross-correlations of the residuals	178
4.4	Daily returns and their predictions	179
4.5	Daily log prices of S&P 500 index and JP Morgan	185
4.6	Impulse response functions of the fitted AR(1) model	185
4.7	A random walk path of a drunk and an associated path of his dog	190
4.8	U.S. Treasury real yield curve with different maturities	196
4.9	Time series and ACF plots for the five candidate cointegrated variables	198