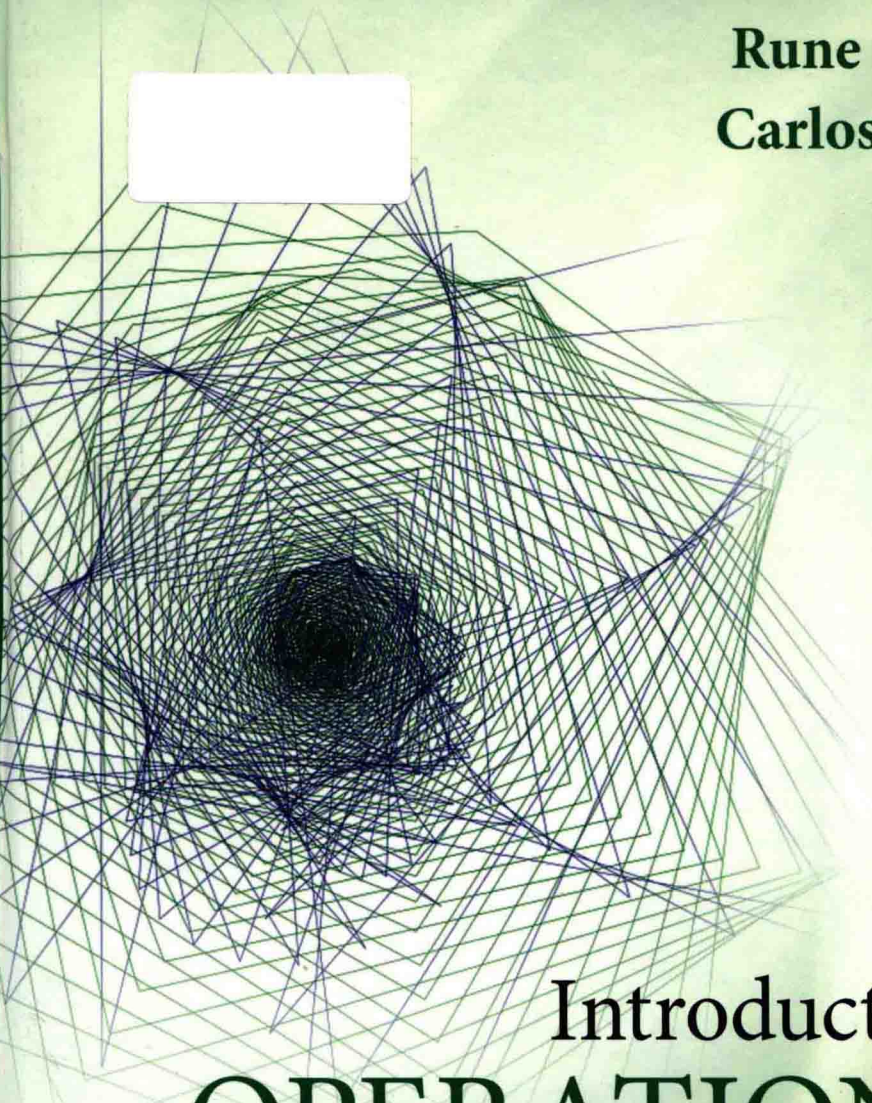


Rune Brincker  
Carlos Ventura



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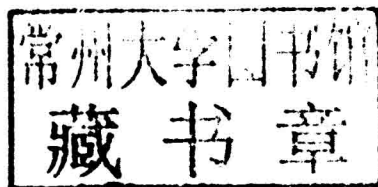
# INTRODUCTION TO OPERATIONAL MODAL ANALYSIS

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**WILEY**

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# **INTRODUCTION TO OPERATIONAL MODAL ANALYSIS**



# Preface

After many years of working on various aspects of operational modal analysis (OMA), conducting vibration tests, analyzing test data from a variety of structures, and giving courses to promote the use and understanding of OMA, we decided to write a book on this topic during a meeting at SVS in Aalborg in the summer 2003. Two years later, in the summer of 2005, we secluded ourselves for one month at the Ventura's family coffee farm in Guatemala where we prepared the first outline of the book and started on the writing process. At that time, we focused our efforts on some key areas, mainly testing, classical dynamics, and signal processing. However, due to a number of circumstances related to work commitments and personal and family affairs, our serious writing process was delayed until the summer of 2011 and continued on until the summer of 2014 when we finished this first edition of the book. It has been a long and demanding effort, but in the end we have prepared a book that reflects our personal views of OMA, both in terms of theory and practice.

It would be preposterous to say that we are the only specialists in the field qualified to write one of the first books on OMA, but both of us have extensive experience with this technology and we recognized that as a team, we were well qualified to write the first book that dealt in a formal manner with the theory behind OMA. We noticed that in the early 1990s people started to pay more attention to OMA and realized the advantages of OMA techniques to determine modal properties of structures. We also noticed that the theory behind OMA was not well understood and that many people were hesitant to use these techniques because of a lack of a clear understanding of why these work so well. So, in 2003 we decided to make an effort to "demystify OMA," and since then the need for such a book has just grown. When we look at our final product, it seems like we made a good decision to work on this book, and we are confident that it will help people working in the field of OMA.

This book is written to be used as a textbook by students, mainly graduate students and PhD students working in research areas where OMA is applied, but it can also be used by scientists and professionals as a reference book for the most important techniques presently being used to analyze vibration data obtained by using OMA testing techniques.

Some people might argue that the classical experimental modal analysis (EMA) and OMA are the same thing, and, therefore, there is no need for a special theory for OMA. But we have compelling reasons to disagree based on our understanding of the fundamental theory of OMA. Our opinion is that actually EMA and OMA are quite different; they have a different history, they use a different technology, they have a different theoretical background, and finally, their applications are different. OMA is indeed a special field that needs to be introduced properly, its mathematical basis and background need to be adequately explained, and good testing practices need to be introduced in order to obtain good data and meaningful results. This is why this book is needed.

The theory in this book is presented heuristically rather than rigorously, thus many mathematical details are omitted for the sake of brevity and conciseness. The aim is not to cover the whole subject in great detail but rather to present a consistent overview of the theories needed to understand the topic and to point out the what these theories have in common and how these theories can be implemented in practice.

This book is rich in mathematical equations that are needed for formulating the theory of OMA, but extensive derivations of equations and formulas are avoided. Our aim was to present each equation or formula in its simplest and clearest formulation. This is also a book rich in simple and clear explanations that will help the reader understand the background for the formulas and how to use them in an effective way in order to perform OMA.

During the writing process, we have been privileged to receive excellent advice from colleagues from around the world who also work on OMA techniques. Without all this advice, we would not have been able to complete the book in a manner that makes us proud of our efforts. We would like to thank all these colleagues for spending their time giving us feedback. We would like to thank Dr. Spilios Fassois and Dr. Nuno Maia for giving us feedback on Chapters 2 and 3, Dr. Anders Brandt and Dr. Henrik Karstoft for their feedback on Chapters 4 and 8, Dr. Manuel Lopez-Aenlle for his feedback on Chapters 5 and 12, Dr. Frede Christensen for his feedback on Chapter 5, Dr. George James and Dr. Lingmi Zhang for their feedback on Chapter 6, Dr. Lingmi Zhang for also giving us feedback on Chapters 9 and 10, Dr. James Brownjohn for his feedback on Chapter 7, and Dr. Bart Peeters for his feedback on Chapter 10. The many useful comments from our PhD students and coworkers, Martin Juul, Anela Bajric, Jannick B. Hansen, Peter Olsen, Anders Skafte, and Mads K. Hovgaard are very much appreciated. We would also like to thank Dr. Palle Andersen of SVS for providing insightful comments on the use of time-domain and frequency-domain techniques as implemented on the ARTeMIS program. The case studies presented in Chapter 11 are based on papers published by the authors and other colleagues. Special thanks are extended to Dr. Alvaro Cunha, Dr. Elsa Caetano, and Dr. Sven-Eric Rosenov for sharing with us the data sets for two of the case studies described in this chapter.

We like to thank our colleagues and friends who have encouraged us to write this book. And last, but not least, we would like to thank our wives, Henriette and Lucrecia for their unconditional support all these years and for having infinite patience with us while we struggled with the preparation of the various drafts of each chapter, and for their willingness to allow us to spend time working on this book rather than attending to family affairs – without their support and unconditional love, this book would have never been a reality.

June 2015  
Rune Brincker  
Carlos E. Ventura

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# 1

## Introduction

*“Torture numbers and they’ll confess to anything”*

– Gregg Easterbrook

The engineering field that studies the modal properties of systems under ambient vibrations or normal operating conditions is called Operational Modal Analysis (OMA) and provides useful methods for modal analysis of many areas of structural engineering. Identification of modal properties of a structural system is the process of correlating the dynamic characteristics of a mathematical model with the physical properties of the system derived from experimental measurements.

It is fair to say that processing of data in OMA is challenging; one can even say that this is close to torturing the data, and it is also fair to say that fiddling around long enough with the data might lead to some strange or erroneous results that might look like reasonable results. One of the aims of this book is to help people who use OMA techniques avoid ending up in this situation, and instead obtain results that are valid and reasonable.

In OMA, measurement data obtained from the operational responses are used to estimate the parameters of models that describe the system behavior. To fully understand this process, one should have knowledge of classical structural mechanics, matrix analysis, random vibration concepts, application-specific simplifying assumptions, and practical aspects related to vibration measurement, data acquisition, and signal processing.

OMA testing techniques have now become quite attractive, due to their relatively low cost and speed of implementation and the recent improvements in recording equipment and computational methods. Table 1.1 provides a quick summary of the typical applications of OMA and how these compare with classical modal testing, also denoted experimental modal analysis (EMA), which is based on controlled input that is measured and used in the identification process.

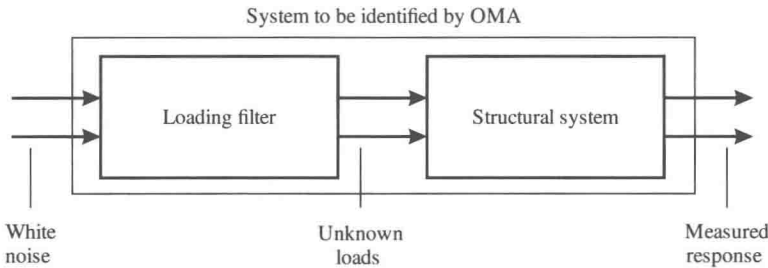
The fundamental idea of OMA testing techniques is that the structure to be tested is being excited by some type of excitation that has approximately white noise characteristics, that is, it has energy distributed over a wide frequency range that covers the frequency range of the modal characteristics of the structure. However, it does not matter much if the actual loads do not have exact white noise characteristics, since what is really important is that all the modes of interest are adequately excited so that their contributions can be captured by the measurements.

Referring to Figure 1.1, the concept of nonwhite, but broadband loading can be explained as follows. The loading is colored, thus does not necessarily have an ideal flat spectrum, but the colored loads can be considered as the output from an imaginary (loading) filter that is loaded by white noise.

**Table 1.1** General characteristics of structural response

	Mechanical engineering	Civil engineering
EMA	<i>Artificial excitation</i> Impact hammer Shakers (hydraulic, electromechanical, etc.) Controlled blasts Well-defined measured input	<i>Artificial excitation</i> Shakers, mainly hydraulic Drop weights Pull back tests Eccentric shakers and exciters Well defined, measured, or unmeasured inputs Controlled blasts
OMA	<i>Artificial excitation</i> Scratching device Air flow Acoustic emissions Unknown signal, random in time and space	<i>Natural excitation</i> Wind Waves Traffic Unknown signal, random in time and space, with some spatial correlation

Source: Adapted from American National Standard: "Vibration of Buildings – guidelines for the measurement of vibrations and their effects on buildings," ANSI S2.47-1990 (ASA 95-1990).



**Figure 1.1** Illustration of the concept of OMA. The nonwhite noise loads are modeled as the output from a filter loaded by a white noise load

It can be proved that the concept of including an additional filter describing the coloring of the loads does not change the physical modes of the system, see Ibrahim et al. [1] and Sections 7.2.7 and 8.3.7. The coloring filter concept shows that in general what we are estimating in OMA is the modal model for "the whole system" including both the structural system and the loading filter.

When interpreting the modal results, this has to be kept in mind, because, some modes might be present due to the loading conditions and some might come from the structural system. We should also note that in practice we often estimate a much larger number of modes than the expected physical number of modes of the considered system.

This means that we need to find ways to justify which modes belong to the structural system, which modes might describe the coloring of the loading, and finally which modes are just noise modes that might not have any physical meaning. These kinds of considerations are important in OMA, and will be further illustrated later in this book.

We can conclude these first remarks by saying that OMA is the process of characterizing the dynamic properties of an elastic structure by identifying its natural modes of vibration from the operating responses. Each mode is associated with a specific natural frequency and damping factor, and these two parameters can be identified from vibration data from practically any point on the structure. In addition, each mode has a characteristic “mode shape,” which defines the spatial distribution of movement over the entire structure.

## 1.1 Why Conduct Vibration Test of Structures?

Vibration measurements are made for a variety of reasons. They could be used to determine the natural frequencies of a structure, to verify analytical models of the structure, to determine its dynamic response under various environmental conditions, or to monitor the condition of a structure under various loading conditions. As structural analysis techniques continually evolve and become increasingly sophisticated, awareness grows of potential shortcomings in their representation of the structural behavior. This is prevalent in the field of structural dynamics. The justification and technology exists for vibration testing and analysis of structures.

Large civil engineering structures are usually too complex for accurate dynamic analysis by hand. It is typical to use matrix algebra based solution methods, using the finite element method of structural modeling and analysis, on digital computers. All linear models have dynamic properties, which can be compared with testing and analysis techniques such as OMA.

## 1.2 Techniques Available for Vibration Testing of Structures

Let us discuss in some detail the two main types of modal testing: the EMA that uses controlled input forces and the OMA that uses the operational forces.

Both forced vibration and in-operation methods have been used in the past and are capable of determining the dynamic characteristics of structures. Forced vibration methods can be significantly more complex than in-operation vibration tests, and are generally more expensive than in-operation vibration tests, especially for large and massive structures. The main advantage of forced vibration over in-operation vibration is that in the former the level of excitation and induced vibration can be carefully controlled, while for the latter one has to rely on the forces of nature and uncontrolled artificial forces (i.e., vehicle traffic in bridges) to excite the structure, sometimes at very low levels of vibration. The sensitivity of sensors used for in-operation vibration measurements is generally much higher than those required for forced vibration tests.

By definition, any source of controlled excitation being applied to any structure in order to induce vibrations constitutes a forced vibration test. In-operation tests that rely on ambient excitation are used to test structures such as bridges, nuclear power plants, offshore platforms, and buildings. While ambient tests do not require traffic shutdowns or interruptions of normal operations, the amount of data collected is significant and it can be a complex task to analyze this data thoroughly.

The techniques for data analysis are different. The theory for forced vibration tests of large structures is well developed and is almost a natural extension of the techniques used in forced vibration tests of mechanical systems. In contrast, the theory for ambient vibration tests still requires further development.



### 1.3 Forced Vibration Testing Methods

Forced vibration tests or EMA methods are generally used to determine the dynamic characteristics of small and medium size structures. In rare occasions, these methods are used on very large structures because of the complexity associated with providing significant levels of excitation to a large, massive structure. In these tests, controlled forces are applied to a structure to induce vibrations. By measuring the structure's response to these known forces, one can determine the structure's dynamic properties. The measured excitation and acceleration response time histories are used to compute frequency response functions (FRFs) between a measured point and the point of input. These FRFs can be used to determine the natural frequencies, mode shapes, and damping values of the structure using well-established methods of analysis. One can apply controlled excitation forces to a structure using several different methods. Forced vibrations encompass any motion in the structure induced artificially above the ambient level. Methods of inducing motion in structures include:

1. Mechanical shakers
  - (a) Electro-magnetic
  - (b) Eccentric mass
  - (c) Hydraulic, including large shaking tables in laboratories
2. Transient loads
  - (a) Pull-back and release, initial displacement
  - (b) Impact, initial velocity
3. Man-excited motions
4. Induced ground motion
  - (a) Underground explosions
  - (b) Blasts with conventional explosives above the ground

The three most popular methods for testing structures are shaker, impact, and pull back or quick-release tests. A brief description of these methods follows:

1. *Shaker tests*: Shakers are used to apply forces to structures in a controlled manner to excite them dynamically. A shaker must produce sufficiently large forces, to effectively excite a large structure in a frequency range of interest. For very large structures, such as long-span bridges or tall buildings, the frequencies of interest are commonly less than 1 Hz. While it is possible to produce considerable forces with relatively small shakers at high frequencies, such as those used to test mechanical systems, it is difficult to produce forces large enough to excite a large structure at low frequencies. Although it is possible to construct massive, low frequency shakers, these are expensive to build, transport, and mount. In such cases, alternative methods to excite the structure are desirable.
2. *Impact tests*: Impact testing is another method of forced vibration testing. Mechanical engineers commonly use impact tests to identify the dynamic characteristics of machine components and small assemblies. The test article is generally instrumented with accelerometers, and struck with a hammer instrumented with a force transducer. While impact testing is commonly used to evaluate small structures, a number of problems may occur when this method is used to test larger structures. To excite lower modes of a large structure sufficiently, the mass of the impact hammer needs to be quite large. Not only is it difficult to build and use large impact hammers with force transducers, but the impact produced by a large hammer could also cause considerable local damage to the test structure.
3. *Pull back tests*: Pull back or quick-release testing has been used in some occasions for testing of large structures. This method generally involves inducing a prescribed temporary deformation to a structure and quickly releasing it, causing the structure to vibrate freely. Hydraulic rams, cables, bulldozers, tugboats, or chain blocks have been used to apply loads that produce a static displacement of the structure. The goal of this technique is to quickly release the load and record the free vibrations of the structure as it tends to return to its position of static equilibrium. The results from quick release tests