CALCULUS

with Applications

SECOND EDITION

Coughlin • Zitarelli

Calculus with Applications

Second Edition

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Saunders College Publishing
Harcourt Brace Jovanovich College Publishers
Fort Worth Philadelphia San Diego New York
Orlando Austin San Antonio Toronto
Montreal London Sydney Tokyo

Text Typeface: Times Roman
Compositor: General Graphic Services
Acquisitions Editor: Robert Stern
Developmental Editor: Richard Koreto
Managing Editor: Carol Field
Project Editor: Laura Maier
Copy Editor: Merry Post
Manager of Art and Design: Carol Bleistine
Associate Art Director: Doris Bruey
Text Designer: William Boehme
Cover Designer: Lawrence R. Didona
Text Artwork: Vantage Art, Inc.
Director of EDP: Tim Frelick
Production Manager: Robert Butler
Marketing Manager: Monica Wilson

Cover Credit: Cover illustration produced by James T. Hoffman of the Center for Geometry Analysis, Numerics, and Graphics, University of Massachusetts at Amherst.

Printed in the United States of America

CALCULUS WITH APPLICATIONS, Second edition

0-03-055757-7

Library of Congress Catalog Card Number: 92-050299

2345 049 987654321

THIS BOOK IS PRINTED ON **ACID-FREE, RECYCLED** PAPER

Preface

In the summer of 1991 a committee of the National Research Council, which is part of the National Academy of Sciences, issued a report called *Moving Beyond Myths*.* The report describes many serious problems in undergraduate mathematics education and provides an action plan for attacking them. One of the myths that this committee identified (see page 12) is

Myth: *Only scientists and engineers need to study mathematics.*

Reality: Mathematics is a science of patterns that is useful in many areas. Indeed, the most rapid areas of growth in applications of mathematics have been in the social, biological, and behavioral sciences. Financial analysts, legal scholars, political pollsters, and sales managers all rely on sophisticated mathematical models to analyze data and make projections. Even artists and musicians use mathematically based computer programs to aid in their work. No longer just a tool for the physical sciences, mathematics is a language for all disciplines.

The goal of this book is to reverse this myth. Although the table of contents is standard, the means of achieving the goal differs from other texts in two notable ways. First, in every section the Referenced Exercises refer to areas outside mathematics and are accompanied by footnotes so the student can pursue them in greater depth, or the instructor can assign them as part of special projects. Because such applications deal with real-life situations, calculators are frequently necessary to carry out the computations.

We hope to show students through these exercises that mathematics is everywhere. No matter what major a student chooses, mathematics can play a key role in solving interesting problems in that discipline. Here is a representative list of selected Referenced Exercises, along with the section where each can be found.

Application	Source	Section
stocks and bonds	*Forbes Magazine*	1–3
income tax	*Wall Street Journal*	2–3
telecommunications	*Forecasting Public Utilities*	3–2
regeneration of trees	*Ecology*	4–5
learning curves	*Decision Sciences*	5–2

*William E. Kirwan (Editor), *Moving Beyond Myths: Revitalizing Undergraduate Mathematics*, Washington D.C., National Academy Press, 1991.

Application	Source	Section
pollution control	*American Journal of Agricultural Economics*	6–4
gifted students	*Journal of Mathematical Psychology*	7–1
cancer treatment	*Physics in Medicine and Biology*	8–2
new technologies	*Management Science*	9–2
sound transmission	(text) *Musical Acoustics: Piano and Wind Instruments*	10–4
swimming records	*The American Scientist*	11–3
insurance	*Journal of Risk and Insurance*	12–2

In addition, almost every chapter has at least one Case Study that treats a particular application in greater detail than the Referenced Exercises. The topics have been chosen not only to illustrate areas where mathematics has played a crucial role in solving an important problem, but to highlight areas of particular human interest.

Even if the instructor does not have time to cover all the Referenced Exercises or Case Studies, it is hoped that their relevance and wealth (in both diversity and number) will make a lasting impression.

Changes in the Second Edition

This second edition of the book differs from the first edition in several ways.

1. The exercises are divided into three sets. The first set consists of the standard assortment of problems, usually numbering between 50 and 70. The first 20 or so generally reflect the examples in the text and are routine, while the later problems are a bit more challenging. The second set, titled "Referenced Exercises," contains problems from areas outside mathematics; these exercises are accompanied by complete references to the literature. The third set, "Cumulative Exercises," contains problems whose solutions call on material from the preceding sections in that chapter. These problems often require different skills than do the usual problems. The authors have found this feature very useful in their own classes as a way to continually help students review old material as they study the current section. In total, there are over 4000 exercises in this book, about 20% of them new to this edition. Although the book continues to contain application exercises associated with many subjects, many simple drill problems have been added as well to help students master concepts before applying them.
2. There are many new and updated Referenced Exercises.
3. There is a new Case Study, "Volumes: Old and New."
4. The normal exercise sets and the cumulative exercise sets include two types of problems that several users of the first edition requested: those that are stated in words and require the translation from ordinary language into mathematical terms and those that require a geometric interpretation.
5. The new, inexpensive graphics calculators promise to revolutionize the way we learn mathematics. They are especially helpful when learning calculus because of their amazing capability to sketch the graph of any function

almost instantaneously. We have included two ways to help students incorporate graphics calculators into the study of calculus. One is an appendix for beginners. The other consists of Graphics Calculator Explorations, which appear at the end of every chapter and center on some geometric ideas that lend themselves to this new technology. These Explorations are open-ended in the sense that the student can extend the ideas presented in the text. Found in the Explorations are instructions on the use of Casio and Texas Instruments graphing calculators.

6. Many instructors who used the first edition suggested that two important topics should have their own sections. Section 2-4 is now exclusively devoted to continuous and differentiable functions and Section 8-6 is devoted to volume.

Organization

The text can be divided into three parts: differential calculus (Chapters 1 through 5), integral calculus (Chapters 6 and 7), and special topics (Chapters 8 through 12).

Differential Calculus

Chapter 1 covers the algebraic techniques needed for the subsequent material. Functions are defined and studied, and various properties of polynomial functions and rational functions are illustrated. Chapter 2 explains the limit of a function by discussing velocity, rate of change of a moving object, and the tangent to a graph. The definition of the derivative is used to calculate the derivatives of several functions.

Chapter 3 presents various techniques for computing the derivatives of functions. Chapter 4 applies these techniques in a wide variety of problems.

Chapter 5 introduces exponential and logarithmic functions. Their derivatives are obtained by calculator experiments aimed at suggesting general rules. Those rules are then proved rigorously.

Integral Calculus

Chapter 6 introduces integration via antidifferentiation. After indefinite integrals have been defined, the Fundamental Theorem of Calculus relates the derivative to the integral. Then the integral is used to compute areas bounded by curves.

Chapter 7 presents several techniques of integration, including the use of tables of integrals and numerical integration.

Special Topics

Chapter 8 extends the definition of the derivative and the definition of the integral to functions of more than one variable. Partial derivatives are defined and applied to the sketching of surfaces, while double integrals are evaluated for functions of two variables.

An equation involving properties about an unknown function and its derivatives is called a differential equation. Chapter 9 discusses some of the basic concepts of differential equations.

The trigonometric functions, studied in Chapter 10, allow us to create models in such diverse disciplines as astronomy, ecology, navigation, and music when we must consider "periodic" functions, which are functions that repeat their functional values periodically.

The two most widely used methods of finding a functional value of functions such as e^x, $\ln x$, and $\sin x$, is to look up the value in a table or to use a calculator. In Chapter 11 we see what steps the calculator uses to compute this number. It approximates the number $e^{1.1}$ by substituting 1.1 into a particular polynomial.

The theory of probability, studied in Chapter 12, is the branch of mathematics that helps find patterns in seemingly haphazard occurrences. Some problems in probability study experiments that have a finite number of possible outcomes. When the outcomes in an experiment can be any number in an interval, calculus is used to discover the properties of the experiment.

Format

We have tried to keep the length of each section to what can be covered in a typical 50-minute class. Sometimes, however, the topic has dictated more extensive coverage. Each numbered section has been partitioned into subsections to help the instructor prepare lectures and to help the student organize the material.

There are two kinds of examples. One explains a new skill that is being encountered for the first time. It is labeled simply EXAMPLE. The other illustrates a skill that was explained beforehand. It uses the following format:

EXAMPLE

Problem

Solution

The student should be able to make a good attempt at solving the problem independently before reading its solution.

Each chapter ends with a review of the terms, notation, and formulas that were introduced in the chapter. It also contains review problems that can be used to review for tests.

Supplements

A number of supplements are available for use with this text.

The **Instructor's Manual**, by Richard Shores, Lynchburg College, contains the partially worked-out solutions to all exercises in the text. (Answers to the odd-numbered exercises are also in the back of the text.) Also contained are additional teaching hints and aids for the instructor.

A **Student Solutions Manual and Study Guide**, also by Richard Shores, contains fully worked-out solutions to every other odd-numbered problem. This

manual is designed to help students with their problem-solving skills: These solutions can be used as models in solving similar problems.

The software package **Graph 2D/3D** by George Bergeman, Northern Virginia Community College, is available free to users of this text. It supplies graphical and computational support for many of the important topics in each chapter. The software requires an IBM or IBM-compatible computer with at least 512K.

The test bank by Robert Kurtz and Pao-sheng Hsu of the University of Maine at Orono, contains about 1000 questions, all new to this edition. "Writing Across the Curriculum" problems, which apply mathematics to different fields, make this manual unique. Even instructors who use their own test banks will find this one useful.

The test bank is also available in a computerized format for Macintosh, IBM, and IBM-compatible computers. This computerized version allows an instructor to custom-design tests and to sort the questions by several different categories. The IBM version requires 256K and two disk drives or 384K and a hard drive, graphics card and monitor, and a printer capable of handling graphs. Full instructions are included.

Acknowledgments It is a pleasure to acknowledge the help we received from many people. Our biggest support has continued to come from our wives, Anita and Judy, and children, Paul and Nicole, and Virginia, Christina, and Sara.

Several reviewers deserve special thanks for their constructive criticism of the entire manuscript.

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We would like to thank again the very helpful reviewers who read the manuscript of the first edition at various stages of development.

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Daniel Symancyk, Anne Arundel Junior College

A very important part of the reviewing process is the checking of errors. For their aid in this process, we would like to thank Bob Martin, Tarrant County Junior College, and Paul Allen, University of Alabama, who performed the arduous task of checking galleys and page proofs for mathematical accuracy.

We extend special gratitude to our friend Vincent Damiano, who is a Senior Project Engineer for Chevron USA, Inc. He read the entire manuscript and suggested several realistic linear programming problems.

The staff at Saunders has been a pleasure to work with. We especially thank our editors, Robert Stern and Richard Koreto, for the interest, encouragement, guidance, and persistence they have bestowed upon this project. We also thank the project editor, Laura Maier, and the art directors, Carol Bleistine and Doris Bruey, not only for their professional expertise but for their patience and humor as well. Our appreciation also goes to Merry Post for her excellent copyediting.

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Philadelphia, PA
December 1992

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