1

Direct Stresses and Strains

1.1. GENERAL

External forces applied to a body have the tendency to deform the body which developes an internal resistance against the deforming forces. This resistance increases with the increase in deforming forces but only up to a certain limit, beyond which the deforming forces will cause the failure of that body. The ultimate internal resistance, to the external forces, offered by a body depends upon the type of deformation taking place and the nature of material of which the body is made.

In **strength of materials** the internal effects produced and the deformations of bodies caused by externally applied forces is studied. Whereas in **Engineering mechanics** the study is confined to relations between externally applied forces on rigid bodies, either at rest or in motion.

1.2. STRESS

External forces acting on a rigid body are termed as **loads.** All externally applied loads deform an elastic material. As the material undergoes deformation it sets up internal resistance to the deforming forces. The quantum of internal resisting forces correspondingly increases with the increase in externally applied loads only up to a certain limit beyond which any increase in applied loads will continue the process of deformation to the stage of failure. The deformation is known as **strain** and the resisting forces are called **stresses.** Since within elastic limit (Art. 1.4) the resistance offered by a body is the same as the load applied so the **stress may be defined as load per unit area** and be mathematically expresses as

$$p=\frac{P}{A} \qquad \ldots(1.1)$$

where Stress intensity $=p$

Load applied $=P$

Area of X-section of the loaded section $=A$.

If the load or total force P be expressed in kg and the area A in cm^2 then the unit of stress shall be kg/cm^2.

For the present we plan to study the following three types of stresses—(*i*) Tensile stresses, (*ii*) compressive stresses, and (*iii*) shear stresses.

1.2.1. Tensile stress. Consider a straight bar of uniform *X*-section (Fig. 1.1) subjected to a pair of collinear forces acting in opposite

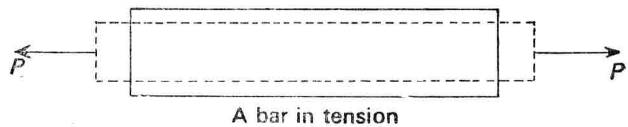


Fig. 1.1.

directions and coinciding with the axis of the bar. If the forces are directed away from the bar then the bar tends to increase in length under the action of applied forces and the stresses developed in the bar are **tensile**. Tensile stresses are generally denoted by p_t.

1.2.2. Compressive stress. In the case discussed above if the forces are directed towards the bar (Fig. 1.2) then the bar tends to

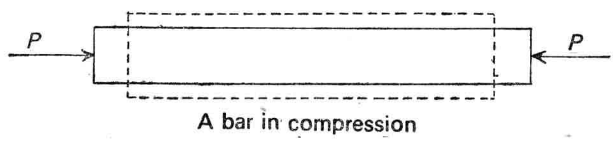


Fig. 1.2.

shorten in length under the action of the applied forces. The stresses developed in the bar are **compressive** and are generally denoted by p_c.

For shear stress refer to Art. 1.8.

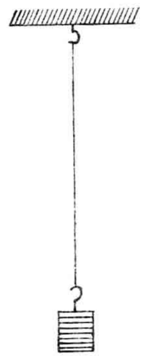
1.3. STRAINS

Strain is a measure of the deformation produced in a member by the load. Direct stresses produce change in length in the direction of the stress. If δl be the change in length l of a member caused by certain stresses then the strain

$$e=\frac{\delta l}{l} \qquad \ldots (1.2)$$

Strain may be defined as change in length per unit length. Tensile stresses increase the lengths whereas compressive stresses decrease the lengths as such tensile strains shall be taken as positive and compressive strains as negative. Since **strain is a ratio of two lengths, it has no units.**

1.4. STRESS-STRAIN DIAGRAM FOR MILD STEEL



Stretching of a steel wire

Fig. 1.3.

Consider a steel wire (Fig. 1.3) held rigidly at its upper end and carrying a weight at its lower end. It is one of the simplest cases of a body under tension. Under the action of the load a small but measurable increase in length of the wire shall be noticed. For small loads extension in length shall be found to be proportional to the applied load. Robert Hooke discovered in 1678 this linear relationship between the applied load and the resulting extension. Materials showing this characteristic are said to obey Hooke's law.

On gradually increasing the loads on the wire a stage is reached where the material ceases to show proportional extensions. The corresponding point *p* on the load extension curve (Fig. 1.4) is known as the **limit of proportionality**. Extension of the wire beyond the limit of proportionality is non-linear. The wire, when loaded up to the limit of proportionality, returns back to its original unstretched position. This property of the material to recover its original position on removal of applied loads is known as **elasticity**. For every material there is definite limiting value of the load up to which the deformation of the material totally disappears on removal of that load. The corresponding stress developed in that material by that limiting load is known as **elastic limit**. The elastic limit is defined as the maximum stress up to which no permanent deformation occurs. The strain that does not disappear on removal of load is known as **permanent set**.

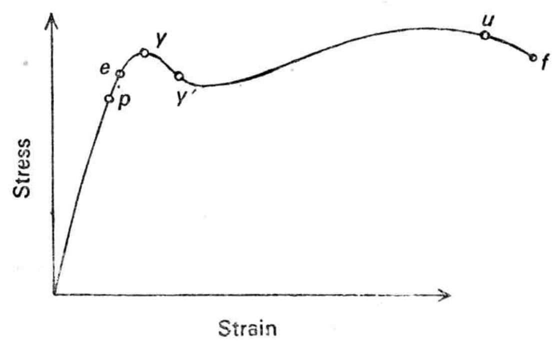


Fig. 1.4.

The stress strain curve is linear up to *p*, thereafter it becomes a little flatter and curved too up to *e*. From *O* to *e* the strain disappears fully on removal of load. Point *e* up to which the material behaves as an elastic material represents the elastic limit. With increase in load the strain goes on increasing along *ey* up to *y*. Immediately beyond the point *y* there is an increase in strain even though there is no appreciable increase in stress. The stress corresponding to the point *y* is called the **yield point** stress. At the yield stress the material begins to flow. At *u* the stress is the maximum and is known as **ultimate strength**. Beyond *u* the bar elongates even with decrease in stress and finally fails at a stage corresponding to point *f*.

The ratio of maximum load, that the specimen is capable of sustaining, to its original area of cross-section is termed as ultimate stress of the material.

After *u* the specimen is greatly reduced in cross-section area. At *f*, the point of failure, the reduced area is the least and this phenomenon is known as *necking*.

1.5. HOOKE'S LAW

It states that for materials loaded within elastic limit the stress is proportional to the strain. Mathematically it can be expressed as:

$$\frac{\text{Stress}}{\text{Strain}}=\text{Constant (called the Young's modulus or the modulus of elasticity and denoted by } E)$$

or

$$\frac{p}{e}=E \qquad \ldots (1.3)$$

Young's modulus E may thus be defined as the stress required to produce unit strain.

Since the unit of stress p is kg/cm^2 and the strain e being only a ratio the unit of E shall be kg/cm^2.

1.6. CHANGE IN LENGTH OF A BODY DUE TO APPLICATION OF LOAD ON IT

Consider the wire in Fig. 1.3 to be subjected to a pull of load P.

Let

l=original length of the wire
A=X-section area of wire
δl=change in length caused by the applied load
e=strain in wire due to applied load
p=stress intensity in the wire due to applied load

From Hooke's law we have:

$$\frac{p}{e}=E \qquad \ldots (\text{Eq. } 1.3)$$

or

$$e=\frac{p}{E} \qquad \ldots (1.4)$$

Substituting for e from Eq. 1.2 and for p from Eq. 1.1 we have:

$$\frac{\delta l}{l}=\frac{P}{AE}$$

or $$\delta l = \frac{Pl}{AE} \qquad \ldots (1.5)$$

1.7. FACTOR OF SAFETY

At stresses below the proportional limit the material is perfectly elastic and beyond this limit a part of the strain usually remains after unloading the member. In order to avoid permanent set in structures it is usual to adopt working stress, p_w well below the limit of proportionality. The usual practice is to adopt working stress as a fraction of the ultimate stress, p_{ult}.

Thus working stress is

$$p_w = \frac{p_{ult}}{n}$$

or $$n = \frac{p_{ult}}{p_w}$$

where n is the **factor of safety and may be defined as the ratio of ultimate stress to the working stress adopted.**

The magnitude of the factor of safety to be adopted depends upon the nature of loading; the homogeneity of materials used and the accuracy with which stresses in members and external forces can be evaluated.

EXAMPLE 1.1. *A steel rod of 20 mm diameter and 500 cm long is subjected to an axial pull of 3000 kg. Determine (i) the intensity of stress, (ii) the strain, (iii) the elongation of rod. Take $E = 2.1 \times 10^6$ kg/cm^2.*

SOLUTION. Diameter of rod = 20 mm = 2.0 cm.

Length of rod = 500 cm.

Load = 3000 kg.

$E_s = 2.1 \times 10^6$ kg/cm^2

X-section area of rod is $$A = \frac{\pi \times 2.0^2}{4} = 3.14 \text{ cm}^2$$

(*i*) Intensity of stress is $$p = \frac{P}{A} = \frac{3000}{3.14} = \mathbf{955.41\ kg/cm^2}$$

(*ii*) Strain $$e = \frac{p}{E} = \frac{955.41}{2.1 \times 10^6}$$

$$= \mathbf{0.000455}$$ (*Strain has no unit*)

(*iii*) Elongation $$\delta l = \frac{Pl}{AE} = \frac{3000 \times 500}{3.14 \times 2.1 \times 10^6}$$

$$= \mathbf{0.2275\ cm.}$$

EXAMPLE 1.2. *A steel rod of 20 mm diameter and 500 cm long elongates by 0.2275 cm when subjected to an axial pull of 3000 kg. Find Young's modulus for steel, the stress and the strain.*

SOLUTION. Diameter of rod = 20 mm = 2.0 cm.

Length of rod is $l=500$ cm
Elongation is $\delta l=0.2275$ cm
Pull is $P=3000$ kg

X-section area of rod is $A=\dfrac{\pi\times 2.0^2}{4}=3.14\ \text{cm}^2$

Using the relation $\delta l=\dfrac{Pl}{AE}$

$\therefore$ $E=\dfrac{Pl}{A\delta l}=\dfrac{3000\times 500}{3.14\times 0.2275}$

$=\mathbf{2.1\times 10^6\ kg/cm^2}$

Stress is $p=\dfrac{P}{A}=\dfrac{3000}{3.14}$

$=\mathbf{955.41\ kg/cm^2}$

Strain is $e=\dfrac{p}{E}=\dfrac{955.41}{2.1\times 10^6}$
$=\mathbf{0.000455.}$

EXAMPLE 1.3. *A short hollow cast iron cylinder of wall thickness 1.0 cm is to carry a compressive load of 60 tonnes. Determine its outside diameter if the ultimate crushing stress for the material is 5400 kg/cm². Use a factor of safety of 6.* (*Banaras Hindu University, 1977*)

SOLUTION. Let outside diameter be D_o. Since wall thickness is 1.0 cm, the inside diameter of the cylinder is (D_o-2) cm (Fig. 1.5).

Area of cross-section

$$=\frac{\pi[D_o^2-(D_o-2)^2]}{4}$$

$$=\frac{\pi(4D_o-4)}{4}$$

$$=\pi(D_o-1)\ \text{cm}^2$$

Crushing load for the column

$$=5400\times\pi(D_o-1)\ \text{kg}$$

$$=5.4\times\pi(D_o-1)\ \text{tonne}$$

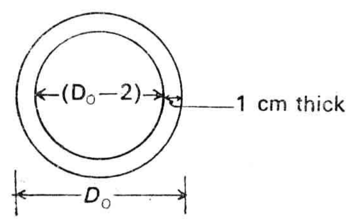


Fig. 1.5.

Factor of safety $=6$

$\therefore$ Safe load $=\dfrac{5.4\times\pi(D_o-1)}{6}=60$

or $D_o=\dfrac{60\times 6}{5.4\times\pi}+1=\mathbf{22.22\ cm.}$

EXAMPLE 1.4. *A 20 cm long steel tube 15 cm internal diameter and 1 cm thick is surrounded closely by a brass tube of same length and thickness. The tubes carry an axial load of 15T. Estimate the load carried by each.* $E_s=2.1\times 10^6\ kg/cm^2$; $E_b=1\times 10^6\ kg/cm^2$.

(*Delhi University; B. Arch., 1974*)

SOLUTION. Outer and inner diameters of steel tube are (15+2) =17 cm and 15 cm respectively and for brass tube they are (17+2) =19 cm and 17 cm respectively.

∴ Cross-sectional area of steel tube is

$$A_s=\frac{\pi(17^2-15^2)}{4}$$

$$=50.265 \text{ cm}^2$$

Cross-sectional area of brass tube is

$$A_b=\frac{\pi(19^2-17^2)}{4}$$

$$=56.549 \text{ cm}^2$$

Young's modulus for steel is

$$E_s=2.1\times 10^6 \text{ kg/cm}^2$$

Young's modulus for brass is

$$E_b=1\times 10^6 \text{ kg/cm}^2$$

Length of each tube is

$$l=20 \text{ cm}$$

Axial load is $P=15000$ kg

Let P_s and P_b be the loads shared by steel and brass tubes respectively.

Thus $P_s+P_b=15000$ . . . (i)

Changes in length for both the tubes are the same.

$$\therefore \quad \frac{P_s l}{A_s E_s}=\frac{P_b l}{A_b E_b}$$

or

$$\frac{P_s}{P_b}=\frac{A_s E_s}{A_b E_b}=\frac{50.265\times 2.1\times 10^6}{56.549\times 1\times 10^6}=1.867$$

$$\therefore \quad P_s=1.867\ P_b$$

From equation (*i*) we have:

$$P_b=\mathbf{5231.95\ kg}$$

and $$P_s=\mathbf{9768.05\ kg.}$$

EXAMPLE 1.5. *A vertical load of 2800 kg is supported by two inclined steel wires AC and BC, each 6.0 metre long as shown in Fig. 1.6. If allowable working stress in wire in tension is 700 kg/cm² determine the required cross-sectional area of each wire and the vertical displacement of the point C. Take E=2.0×10⁶ kg/cm².* (*Osmania University, 1976*)

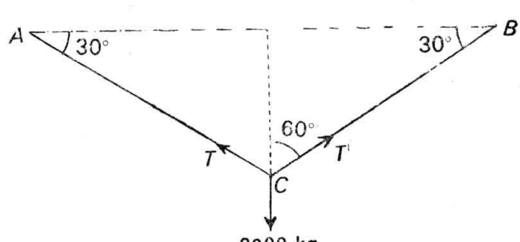


Fig. 1.6.

SOLUTION. Load P will cause tension in both the wires AC and BC and by symmetry the tensions in both the wires shall be equal say T. Balancing the forces in vertical direction at joint C we have by resolving:

$$2T\cos 60 = 2800 = 2T \times \tfrac{1}{2}$$

$$\therefore \quad T = 2800 \text{ kg}$$

Allowed stress in wire is

$$p = 700 \text{ kg/cm}^2.$$

Required X-section area A of each wire

$$= \frac{2800}{700} \qquad \left(\because\ P = p \times A \text{ or } A = \frac{P}{p}\right)$$

$$= \mathbf{4.0\ sq\ cm.}$$

Displacement of point C. Let the point C displace to C' (Fig. 1.7). Since the displacement is small $\angle C'AB$ can still be taken to be 30°.

Draw $CD \perp AC'$.

Now $AC = AD$ and DC' is the elongation of AC.

Pull in the member AC is 2800 kg and its X-section area is 4.0 cm^2.

Elongation DC' is given by equation 1.5

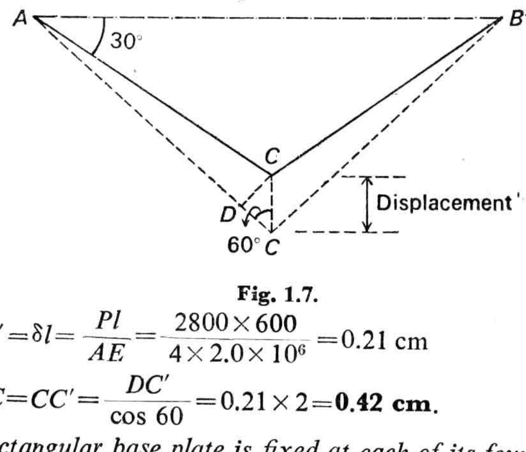


Fig. 1.7.

$$DC' = \delta l = \frac{Pl}{AE} = \frac{2800 \times 600}{4 \times 2.0 \times 10^6} = 0.21 \text{ cm}$$

Displacement of $\quad C = CC' = \dfrac{DC'}{\cos 60} = 0.21 \times 2 = \mathbf{0.42\ cm.}$

EXAMPLE 1.6. *A rectangular base plate is fixed at each of its four corners by 20 mm diameter bolts and nuts as shown in (Fig. 1.8). The plate rests on washers of 22 mm internal diameter and 50 mm external diameter. Upper washers, which are placed between nut and the plate, are of 22 mm internal diameter and 44 mm external diameter. If the base plate carries a load of 12t (including self weight) which is equally distributed at four corners, calculate the stress in the lower washers before the nuts are tightened. What would be the stress in the upper*

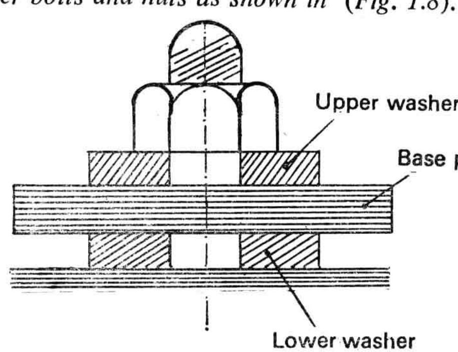


Fig. 1.8.

and lower washers when the nuts are so tightened as to produce the tension of 0.5t in each bolt. *(A.M.I.E., May, 1976)*

SOLUTION. Total load carried by the base plate=12 tonnes

Since the plate is held by 4 bolts, load shared by each bolt

$$=\frac{12}{4}=3 \text{ tonne}=3000 \text{ kg}$$

Area of lower washer $=\frac{\pi}{4}(5^2-2.2^2)=15.83 \text{ cm}^2$.

Stress intensity in the lower washer

$$=\frac{3000}{15.83}=\mathbf{189.51 \ kg/cm^2}.$$

When the nuts are tightened the compressive load in the upper washers equals the tension in each bolt which is $0.5t$ =500 kg.

Area of the upper washer

$$=\frac{\pi}{4}(4.4^2-2.2^2)=11.40 \text{ cm}^2$$

Stress intensity in the upper washer

$$=\frac{500}{11.40}=\mathbf{43.86 \ kg/cm^2}$$

Total compressive load on the lower washer when the nuts have been tightened $=(3000+500)=3500$ kg.

Stress intensity on the lower washer (when the nuts have been tightened)

$$=\frac{3500}{15.83}=\mathbf{221.10 \ kg/cm^2}.$$

1.8. SHEAR STRESS

Consider two plates A and B (Fig. 1.9a) joined together by a rivet C. If the plates be carrying a tensile load P then the rivet may shear along the plane XX (Fig. 1.9b). If d is the diameter of the rivet then the area of X-section of the rivet subjected to shear is

$$A=\frac{\pi d^2}{4}$$

and the shear stress

$$q=\frac{P}{A} \quad \ldots (1.6)$$

or $$q=\frac{4P}{\pi d^2}$$

It should be noted that the applied load is tangential to the resisting area and therefore shear stress is also termed as **tangential**

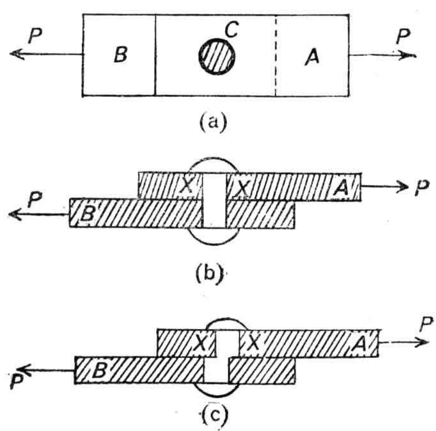


Fig. 1.9.

stress. The tensile and compressive stresses on the other hand are caused by forces acting perpendicular to the areas resisting those forces and as such these stresses are termed as **direct stresses** or **normal stresses.**

1.8.1. Shear strain. Shear force causes relative displacement of the material in the direction of the force. Consider a block *ABCD* fixed at the face *AB* and subjected to a tangential force *P* on the face *DC* (Fig. 1.10). Let *ABC'D'* be the shape to which the block gets distorted under the action of the applied force. The deformation *LL'* is in height *BL*; *MM'* in height *BM* and *CC'* in height *BC*.

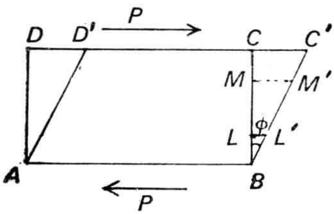


Fig. 1.10.

Shear strain is the deformation caused by the shear force per unit height or length.

$$\text{Thus the shear strain} = \frac{LL'}{BL} = \frac{MM'}{BM} = \frac{CC'}{BC} = \tan\phi = \phi.$$

(*Since* ϕ *is always very small therefore tan* $\phi = \phi$).

Thus ϕ in radians is a measure of the shear strain.

Since intensity of stress is resistance per unit area, its unit will be tonne or kg/cm^2 depending upon the units of force and of area used.

The corresponding shear strain ϕ being in radians is a number.

1.8.2. Complementary shear stress. Consider a rectangular block (Fig. 1.11) of length *l*, width *b* and thickness *t*. Let the faces *AA'B'B* and *DD'C'C* be subjected to a shear stress *q*. Now the shear force on each of the two faces is $(q \times l \times t)$. Under the action of these two forces the block shall be in linear equilibrium but subjected to an anticlockwise couple $M = (q \times l \times t) \times b$. **Only a moment can balance a moment.** Therefore, for the block under consideration to be in equilibrium there must be a couple of equal magnitude but opposite in nature. Shear stress *q* at the two faces *AA'DD'* and *BB'CC'* will thus automatically cause a shear stress **(say)** *q'* on the two faces *AA'D'D* and *BB'C'C* such that they form a couple of magnitude $M = (q \times l \times t) \times b$ and clockwise in nature.

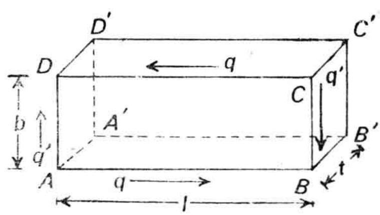


Fig. 1.11.

$$\therefore \qquad (q' \times t \times b) \times l = (q \times l \times t) \times b$$

$$\therefore \qquad q' = q.$$

Hence, it follows that shear stress in one plane on a block in **equilibrium is automatically accompanied by an equal and opposite shear stress known as complementary shear stress in another plane perpendicular to the first plane.**

1.9. NORMAL STRESSES DUE TO SHEAR STRESSES

Figure 1.12 shows a cube of side a subjected to a shear stress q on its faces AB and CD. This gives rise to complementary shear stress of intensity q on faces BC and AD, as discussed in *Art. 1.8.2.* Consider the equilibrium of wedge ABC.

Areas of faces AB and BC are a^2 each. The total forces acting on either face is qa^2.

Resultant R of these two equal forces

$$=\sqrt{(qa^2)^2+(qa^2)^2}$$

$$=qa^2\sqrt{2}$$

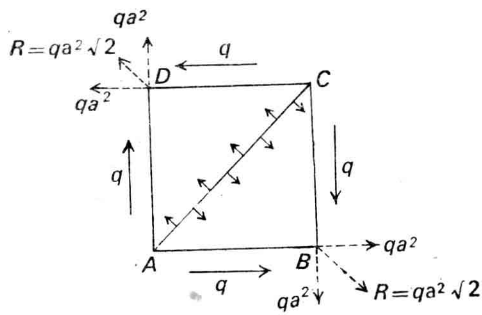


Fig. 1.12.

Since the two forces are equal the resultant shall act normal to the sectional plane at AC i.e. along the diagonal DB. Similarly, by considering the equilibrium of wedge ADC it shall be observed that a force $R=qa^2\sqrt{2}$ shall act along diagonal BD. The diagonal BD will thus be subject to a tensile force of magnitude $qa^2\sqrt{2}$.

The diagonal AC will similarly be found subject to compressive force of magnitude $qa^2\sqrt{2}$.

The length of diagonal BD will thus increase under tension and that of diagonal AC will decrease under compression and the cube shall take up the shape shown in Fig. 1.13.

Length of diagonal $AC=a\sqrt{2}$.

Thus, area of X-section at the plane

$$AC=a\sqrt{2}\times a=a^2\sqrt{2}$$

Thus, tensile stress on the plane

$$AC=\frac{qa^2\sqrt{2}}{a^2\sqrt{2}}=q.$$

Similarly, compressive stress on the plane BD is also q.

Hence, if shear stress q be acting on each one of a pair of two mutually perpendicular plane then it produces a pair of normal stresses of different nature on a pair of mutually perpendicular planes inclined at an angle of 45° to the first set of planes.

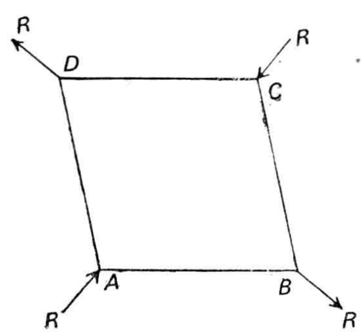


Fig. 1.13.

1.10. MODULUS OF RIGIDITY

For elastic materials it is found that within certain limits shear strain is directly proportional to the shear stress producing it. The ratio $\frac{\text{shear stress}}{\text{shear strain}}$ is called the modulus of rigidity and is generally denoted by G, C, or N.

Thus, $C=\frac{q}{\phi}$ . . . (1.7)

(Units of C are the same as those for q i.e. tonne/cm² or kg/cm²)

If $\phi=1$ then $C=q$

Thus **Modulus of rigidity may be defined as the shear stress needed to produce unit shear strain.**

EXAMPLE 1.7. *An angle bracket transfixed to a vertical column (Fig. 1.14) carries a load of 4000 kg. If the bracket is riveted to the column with two 16 mm diameter rivets find the average shear stress in each of the rivets.*

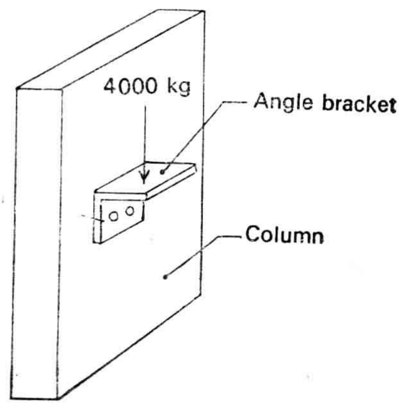


Fig. 1.14.

SOLUTION. Diameter of each rivet=16 mm=1.6 cm

X-section area of each rivet is

$$A=\frac{\pi\times 1.6^2}{4}=2.01 \text{ cm}^2$$

Load of 4000 kg carried by the bracket is shared equally by the two rivets.

∴ Load carried by one rivet is

$$P=\frac{4000}{2}=2000 \text{ kg}$$

Average shear stress in each rivet is

$$q=\frac{2000}{2.01}$$

$$=995.02 \text{ kg/cm}^2$$

$$=\mathbf{9.95 \text{ kg/mm}^2}.$$

EXAMPLE 1.8. *A tie rod made up of two parts as shown in Fig. 1.15 is to carry a load of 9900 kg. Determine a proper diameter for the connecting bolt if the allowable working stress in shear is 700 kg/cm². (Calcutta University, 1976)*

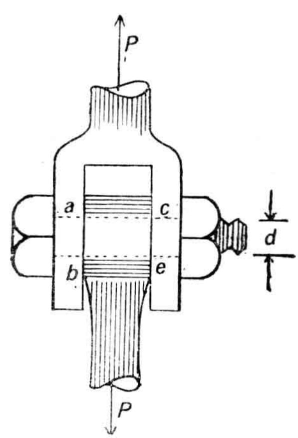


Fig. 1.15.

SOLUTION. Assume that the bolt is free from bending action. For failure of bolt it must shear across two sections *ab* and *cd*. The bolt is thus in double shear. Let the required diameter of bolt be d. Use the relation

$$q=\frac{P}{2A}$$ **(because the bolt is in double shear therefore 2A and not A as in Eq. 1.2)**

$$\therefore \quad q=\frac{P}{2\times\frac{\pi d^2}{4}}=\frac{2P}{\pi d^2}.$$

But it is given that $q=700$ kg/cm² and $P=9900$ kg.

$$\therefore \quad 700=\frac{2\times 9900}{\pi d^2}$$

or $\quad \pi d^2=28.286$

$\therefore \quad d=$ **3.0 cm.**

EXAMPLE 1.9. *It is desired to punch a hole of 20 mm diameter in a plate 20 mm thick. If the shear stress of mild steel is 30 kg/mm² find the force necessary for punching and the stress in the punch.*

SOLUTION. Diameter of hole is $d=20$ mm

Thickness of plate is $t=20$ mm

Area of X-section of punch $=\frac{\pi\times 400}{4}=100\pi$ mm²

Shear stress is $q=30$ kg/cm² *(Given)*

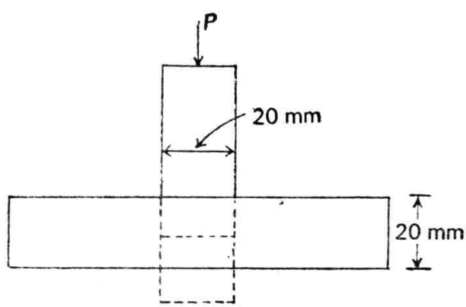


Fig. 1.16.

Punching force is (Fig. 1.16) $P=q\times\pi d\times t=30\times\pi\times 20\times 20$

$=12000\pi$ kg$=$ **37699 kg**

Stress in punch $p=\frac{P}{A}=\frac{12000\pi}{100\pi}=$ **120 kg/mm².**

EXAMPLE 1.10. *With a punch for which the maximum crushing stress is 4 times the maximum shearing stress of the plate, show that the biggest hole that can be punched in the plate is of diameter equal to the plate thickness.* *(Kurukshetra University, 1976)*

SOLUTION. Let

$d=$ Diameter of hole that can be punched in the plate

$t=$ Thickness of plate

p_c = Crushing stress of punch

q = Shear stress of plate

It is given that:

$$p_c = 4q \qquad \ldots (i)$$

Load on punch is

$$P = p_c \times \frac{\pi d^2}{4} \qquad \ldots (ii)$$

Area of plate resisting shear (Fig. 1.17) is

$$= \pi d \times t$$

Force required to punch the hole

$$P = \pi \times d \times t \times q \qquad \ldots (iii)$$

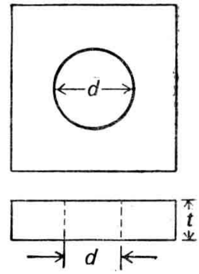


Fig. 1.17.

Equating loads in Eq. (*ii*) and in Eq. (*iii*), we have:

$$p_c \frac{\pi d^2}{4} = \pi\, dtq$$

Now substitute for p_c from equation (*i*) above

$$4q \times \frac{\pi d^2}{4} = \pi\, dtq \quad \text{or} \quad \mathbf{d = t.}$$

∴ Diameter of the hole punched is equal to the plate thickness.

EXAMPLE 1.11. *Two parts of a tie bar of diameter D are connected in such a way that the end of one part fits into the forked end of the other part and a pin of diameter d passes through the two. The pin is in double shear. If p and q be the tensile and shear stresses in the rod and the pin respectively show that for uniform resistance*

$$\frac{d}{D} = \sqrt{\frac{p}{2q}}. \qquad \textit{(London University)}$$

SOLUTION. Let p = tensile stress in the rod

q = shear stress in the pin

∴ Pull in the tie is $P = p \times \dfrac{\pi D^2}{4}$

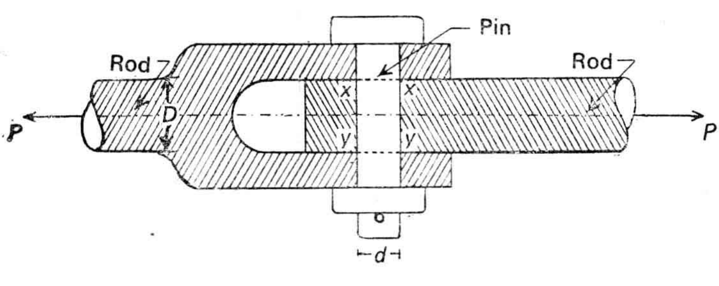


Fig. 1.18.

The pin is in double shear. It shears along two sections XX and YY. (Fig. 1.18)

Strength of pin (in double shear)

$$=2\times q\times \frac{\pi d^2}{4}.$$

Since the resistances offered by the rod and the pin are equal in magnitude

$$\therefore \qquad p\times\frac{\pi D^2}{4}=2\times q\times\frac{\pi d^2}{4}$$

$$\therefore \qquad \frac{d}{D}=\sqrt{\frac{\mathbf{p}}{\mathbf{2q}}}.$$

EXAMPLE 1.12. *A shaft is subjected to a twisting moment which produces a shearing stress of 1200 kg/cm² at its surface in planes perpendicular to its axis. Taking $C=0.86\times10^6$ kg/cm² find changes in the angles of a small square scratched on the surface of the shaft with two of its sides paralled to axis of the shaft.*

SOLUTION. Within limits of proportionality

$$C=\frac{q}{\phi} \text{ or } \phi=\frac{q}{C}$$

$$\therefore \qquad \phi=\frac{1200}{0.86\times10^6}=0.001395 \text{ radian.}$$

But π radian$=180°$

$$\therefore \qquad 0.001395 \text{ radian}=\frac{180}{\pi}\times0.001395=0.0799°$$

$$=4'\ 48''$$

$\therefore$ Change in angle $\phi=4'\ 48''$.

EXERCISE—1.1

1. A steel bar 5 metres long and 2.5 cm in diameter is stretched 2.0 mm by a load of 8 tonnes in pulling it axially. Calculate the stress, strain and the modulus of elasticity of the bar.
   (1629 33 kg/cm²; 0.0004; 4073.33 tonnes/cm²)
2. A circular bar 1 cm diameter and 0.6 m long was tested for Young's modulus of elasticity. It was observed that under a pull of 1360 kg the extension was 0.492 mm. Find the value of Young's modulus of elasticity
   (2.11×10^6 kg/cm²)
3. Determine the load under which a hexagonal bar of 1.5 cm side and 2.5 m length would contract by 2 mm. Take $E=2.15\times10^6$ kg/cm².
   (10070 kg)
4. A short hollow cast iron cylinder of wall thickness 2.0 cm is to carry a load of 40 tonnes. Determine its outside diameter if the working stress for the metal in compression is 800 kg/cm².
   (9.96 cm)
5. Calculate the force required to punch a circular hole 6 cm in diameter in a plate ½ cm thick. Take ultimate shear stress of the plate=3500 kg/cm².
   (33000 kg)
6. A shaft is subjected to a twisting moment which produces a shearing stress of 1200 kg/cm² at its surface in planes perpendicular to its axis. Change in the angles of a square scratched on the surface of the shaft with its two

sides parallel to axis of the shaft was found to be 4′48″. Determine modulus of rigidity for the shaft material.

(0.86×10^6 kg/cm^2)

7. A mild steel round shaft transmitting twisting moment develops shear stress at its surface in a plane at right angles to the axis of shaft. A small square with two sides parallel to the axis of shaft is scratched on the surface of shaft. If 0.0017 radian is the change in angles of square after applcation of stresses calculate the value of q. Take $C = 8.5 \times 10^5$ kg/cm^2

(1445 kg/cm^2)

1.11. ELONGATION OF BARS OF VARYING CROSS-SECTIONS

If a bar is made up of a number of portions of different cross-sections then the total elongation of the bar is the sum of the elongations of each portion constituting the bar length.

Consider a bar (Fig. 1.19) composed of three sections of lengths l_1, l_2 and l_3 having respective areas of cross-sections A_1, A_2 and A_3 subjected to an axial pull P. If δl_1, δl_2, and δl_3 be the respective changes in lengths of the three sections then we have

$$\delta l_1 = \frac{Pl_1}{A_1E} \qquad \ldots (1.5)$$

$$\delta l_2 = \frac{Pl_2}{A_2E}$$

$$\delta l_3 = \frac{Pl_3}{A_3E}$$

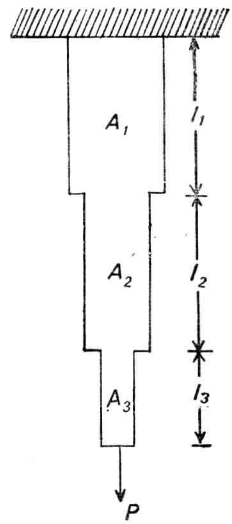


Fig. 1.19.

where E is the modulus of elasticity for the material of which the bar is made.

Now the change in length δl of the entire bar is

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$= \frac{Pl_1}{A_1E} + \frac{Pl_2}{A_2E} + \frac{Pl_3}{A_3E}$$

$$= \frac{P}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right) \qquad \ldots (1.8)$$

EXAMPLE 1.13. *A bar ABCD shown in Fig. 1.20 is subjected to a tensile load of 18000 kg. If stress in the material is limited to 1400*

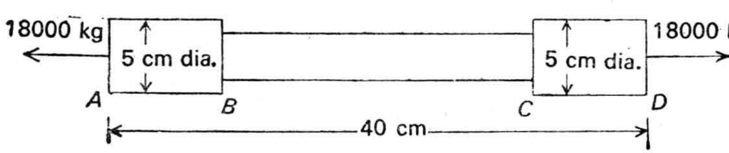


Fig. 1.20.