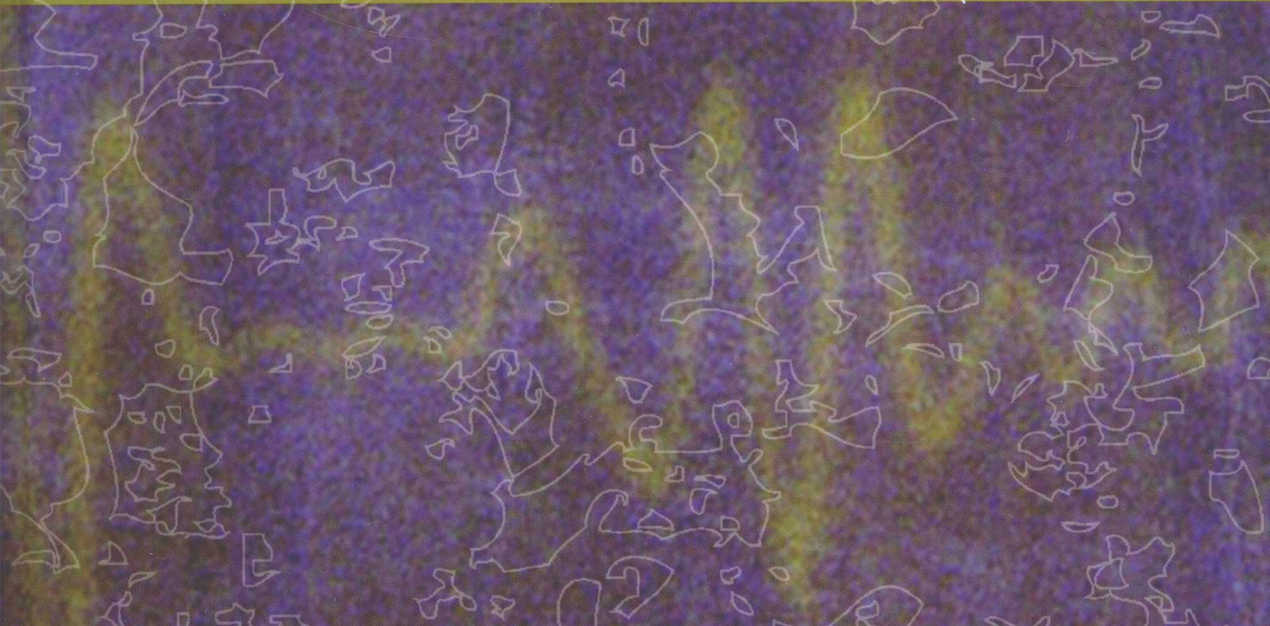
Second Edition

The Rock Physics Handbook

Tools for Seismic Analysis of Porous Media



Gary Mavko
Tapan Mukerji
Jack Dvorkin

CAMBRIDGE

The Rock Physics Handbook, Second Edition

Tools for Seismic Analysis of Porous Media

Gary Mavko
Stanford University, USA

Tapan Mukerji
Stanford University, USA

Jack Dvorkin
Stanford University, USA

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521861366

First published 2009

Printed in the United Kingdom at the University Press, Cambridge

A catalog record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication Data

Mavko, Gary, 1949–
The rock physics handbook : tools for seismic analysis of porous media / Gary Mavko, Tapan Mukerji, Jack Dvorkin. – 2nd ed.
p. cm.
Includes bibliographical references and index.
ISBN 978-0-521-86136-6 (hardback)
1. Rocks. 2. Geophysics. I. Mukerji, Tapan, 1965– II. Dvorkin, Jack, 1953– III. Title.
QE431.6.P5M38 2009
552'.06–dc22

2008049235

ISBN 978-0-521-86136-6 hardback

The Rock Physics Handbook, Second Edition

Tools for Seismic Analysis of Porous Media

The science of rock physics addresses the relationships between geophysical observations and the underlying physical properties of rocks, such as composition, porosity, and pore fluid content. The *Rock Physics Handbook* distills a vast quantity of background theory and laboratory results into a series of concise, self-contained chapters, which can be quickly accessed by those seeking practical solutions to problems in geophysical data interpretation.

In addition to the wide range of topics presented in the First Edition (including wave propagation, effective media, elasticity, electrical properties, and pore fluid flow and diffusion), this Second Edition also presents major new chapters on granular material and velocity–porosity–clay models for clastic sediments. Other new and expanded topics include anisotropic seismic signatures, nonlinear elasticity, wave propagation in thin layers, borehole waves, models for fractured media, poroelastic models, attenuation models, and cross-property relations between seismic and electrical parameters. This new edition also provides an enhanced set of appendices with key empirical results, data tables, and an atlas of reservoir rock properties expanded to include carbonates, clays, and gas hydrates.

Supported by a website hosting MATLAB routines for implementing the various rock physics formulas presented in the book, the Second Edition of *The Rock Physics Handbook* is a vital resource for advanced students and university faculty, as well as in-house geophysicists and engineers working in the petroleum industry. It will also be of interest to practitioners of environmental geophysics, geomechanics, and energy resources engineering interested in quantitative subsurface characterization and modeling of sediment properties.

Gary Mavko received his Ph.D. in Geophysics from Stanford University in 1977 where he is now Professor (Research) of Geophysics. Professor Mavko co-directs the Stanford Rock Physics and Borehole Geophysics Project (SRB), a group of approximately 25 researchers working on problems related to wave propagation in earth materials. Professor Mavko is also a co-author of *Quantitative Seismic Interpretation* (Cambridge University Press, 2005), and has been an invited instructor for numerous industry courses on rock physics for seismic reservoir characterization. He received the Honorary Membership award from the Society of Exploration Geophysicists (SEG) in 2001, and was the SEG Distinguished Lecturer in 2006.

Tapan Mukerji received his Ph.D. in Geophysics from Stanford University in 1995 and is now an Associate Professor (Research) in Energy Resources Engineering and a member of the Stanford Rock Physics Project at Stanford University. Professor Mukerji co-directs the Stanford Center for Reservoir Forecasting (SCRF) focusing on problems related to uncertainty and data integration for reservoir modeling. His research interests include wave propagation and statistical rock physics, and he specializes in applied rock physics and geostatistical methods for seismic reservoir characterization, fracture detection, time-lapse monitoring, and shallow subsurface environmental applications. Professor Mukerji is also a co-author of *Quantitative Seismic Interpretation*, and has taught numerous industry courses. He received the Karcher award from the Society of Exploration Geophysicists in 2000.

Jack Dvorkin received his Ph.D. in Continuum Mechanics in 1980 from Moscow University in the USSR. He has worked in the Petroleum Industry in the USSR and USA, and is currently a Senior Research Scientist with the Stanford Rock Physics Project at Stanford University. Dr Dvorkin has been an invited instructor for numerous industry courses throughout the world, on rock physics and quantitative seismic interpretation. He is a member of American Geophysical Union, Society of Exploration Geophysicists, American Association of Petroleum Geologists, and the Society of Petroleum Engineers.

Preface to the Second Edition

In the decade since publication of the *Rock Physics Handbook*, research and use of rock physics has thrived. We hope that the First Edition has played a useful role in this era by making the scattered and eclectic mass of rock physics knowledge more accessible to experts and nonexperts, alike.

While preparing this Second Edition, our objective was still to summarize in a convenient form many of the commonly needed theoretical and empirical relations of rock physics. Our approach was to present *results*, with a few of the key assumptions and limitations, and almost never any derivations. Our intention was to create a quick reference and not a textbook. Hence, we chose to encapsulate a broad range of topics rather than to give in-depth coverage of a few. Even so, there are many topics that we have not addressed. While we have summarized the assumptions and limitations of each result, we hope that the brevity of our discussions does not give the impression that application of any rock physics result to real rocks is free of pitfalls. We assume that the reader will be generally aware of the various topics, and, if not, we provide a few references to the more complete descriptions in books and journals.

The handbook contains 101 sections on basic mathematical tools, elasticity theory, wave propagation, effective media, elasticity and poroelasticity, granular media, and pore-fluid flow and diffusion, plus overviews of dispersion mechanisms, fluid substitution, and V_P–V_S relations. The book also presents empirical results derived from reservoir rocks, sediments, and granular media, as well as tables of mineral data and an atlas of reservoir rock properties. The emphasis still focuses on elastic and seismic topics, though the discussion of electrical and cross seismic-electrical relations has grown. An associated website (http://srb.stanford.edu/books) offers MATLAB codes for many of the models and results described in the Second Edition.

In this Second Edition, Chapter 2 has been expanded to include new discussions on elastic anisotropy including the Kelvin notation and eigenvalues for stiffnesses, effective stress behavior of rocks, and stress-induced elasticity anisotropy. Chapter 3 includes new material on anisotropic normal moveout (NMO) and reflectivity, amplitude variation with offset (AVO) relations, plus a new section on elastic impedance (including anisotropic forms), and updates on wave propagation in stratified media, and borehole waves. Chapter 4 includes updates of inclusion-based effective media models, thinly layered media, and fractured rocks. Chapter 5 contains extensive new sections on granular media, including packing, particle size, sorting, sand–clay mixture models, and elastic effective medium models for granular materials. Chapter 6 expands the discussion of fluid effects on elastic properties, including fluid substitution in laminated media, and models for fluid-related velocity dispersion in heterogeneous poroelastic media. Chapter 7 contains new sections on empirical velocity–porosity–mineralogy relations, V_P–V_S relations, pore-pressure relations, static and dynamic moduli, and velocity–strength relations. Chapter 8 has new discussions on capillary effects, irreducible water saturation, permeability, and flow in fractures. Chapter 9 includes new relations between electrical and seismic properties. The Appendices has new tables of physical constants and properties for common gases, ice, and methane hydrate.

This *Handbook* is complementary to a number of other excellent books. For in-depth discussions of specific rock physics topics, we recommend *Fundamentals of Rock Mechanics, 4th Edition,* by Jaeger, Cook, and Zimmerman; *Compressibility of Sandstones,* by Zimmerman; *Physical Properties of Rocks: Fundamentals and Principles of Petrophysics*, by Schon; *Acoustics of Porous Media*, by Bourbié, Coussy, and Zinszner; *Introduction to the Physics of Rocks*, by Guéguen and Palciauskas; *A Geoscientist's Guide to Petrophysics*, by Zinszner and Pellerin; *Theory of Linear Poroelasticity*, by Wang; *Underground Sound*, by White; *Mechanics of Composite Materials*, by Christensen; *The Theory of Composites*, by Milton; *Random Heterogeneous Materials*, by Torquato; *Rock Physics and Phase Relations*, edited by Ahrens; and *Offset Dependent Reflectivity – Theory and Practice of AVO Analysis*, edited by Castagna and Backus. For excellent collections and discussions of classic rock physics papers we recommend *Seismic and Acoustic Velocities in Reservoir Rocks*, Volumes 1, 2 and 3, edited by Wang and Nur; *Elastic Properties and Equations of State*, edited by Shankland and Bass; *Seismic Wave Attenuation*, by Toksöz and Johnston; and *Classics of Elastic Wave Theory*, edited by Pelissier *et al.*

We wish to thank the students, scientific staff, and industrial affiliates of the Stanford Rock Physics and Borehole Geophysics (SRB) project for many valuable comments and insights. While preparing the Second Edition we found discussions with Tiziana Vanorio, Kaushik Bandyopadhyay, Ezequiel Gonzalez, Youngseuk Keehm, Robert Zimmermann, Boris Gurevich, Juan-Mauricio Florez, Anyela Marcote-Rios, Mike Payne, Mike Batzle, Jim Berryman, Pratap Sahay, and Tor Arne Johansen, to be extremely helpful. Li Teng contributed to the chapter on anisotropic AVOZ, and Ran Bachrach contributed to the chapter on dielectric properties. Dawn Burgess helped tremendously with editing, graphics, and content. We also wish to thank the readers of the First Edition who helped us to track down and fix errata.

And as always, we are indebted to Amos Nur, whose work, past and present, has helped to make the field of rock physics what it is today.

Gary Mavko, Tapan Mukerji, and Jack Dvorkin.

Contents

1 Basic tools

1.1 The Fourier transform

Synopsis

The **Fourier transform** of $f(x)$ is defined as

$$F(s) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi xs}\, dx$$

The inverse Fourier transform is given by

$$f(x) = \int_{-\infty}^{\infty} F(s) e^{+i2\pi xs}\, ds$$

Evenness and oddness

A function $E(x)$ is *even* if $E(x) = E(-x)$. A function $O(x)$ is *odd* if $O(x) = -O(-x)$.

The Fourier transform has the following properties for even and odd functions:

- *Even functions.* The Fourier transform of an even function is even. A *real even* function transforms to a *real even* function. An *imaginary even* function transforms to an *imaginary even* function.
- *Odd functions.* The Fourier transform of an odd function is odd. A *real odd* function transforms to an *imaginary odd* function. An *imaginary odd* function transforms to a *real odd* function (i.e., the "realness" flips when the Fourier transform of an odd function is taken).

$$\begin{aligned} \text{real even (RE)} &\rightarrow \text{real even (RE)} \\ \text{imaginary even (IE)} &\rightarrow \text{imaginary even (IE)} \\ \text{real odd (RO)} &\rightarrow \text{imaginary odd (IO)} \\ \text{imaginary odd (IO)} &\rightarrow \text{real odd (RO)} \end{aligned}$$

Any function can be expressed in terms of its even and odd parts:

$$f(x) = E(x) + O(x)$$

where

$$E(x) = \tfrac{1}{2}[f(x) + f(-x)]$$

$$O(x) = \tfrac{1}{2}[f(x) - f(-x)]$$

Then, for an arbitrary complex function we can summarize these relations as (Bracewell, 1965)

$$f(x) = \mathrm{re}(x) + i\,\mathrm{ie}(x) + \mathrm{ro}(x) + i\,\mathrm{io}(x)$$

$$F(x) = \mathrm{RE}(s) + i\,\mathrm{IE}(s) + \mathrm{RO}(s) + i\,\mathrm{IO}(s)$$

As a consequence, a real function $f(x)$ has a Fourier transform that is *hermitian,* $F(s) = F^*(-s)$, where $*$ refers to the complex conjugate.

For a more general complex function, $f(x)$, we can tabulate some additional properties (Bracewell, 1965):

$$\begin{aligned}
f(x) &\Leftrightarrow F(s)\\
f^*(x) &\Leftrightarrow F^*(-s)\\
f^*(-x) &\Leftrightarrow F^*(s)\\
f(-x) &\Leftrightarrow F(-s)\\
2\,\mathrm{Re}f(x) &\Leftrightarrow F(s) + F^*(-s)\\
2\,\mathrm{Im}f(x) &\Leftrightarrow F(s) - F^*(-s)\\
f(x) + f^*(-x) &\Leftrightarrow 2\,\mathrm{Re}F(s)\\
f(x) - f^*(-x) &\Leftrightarrow 2\,\mathrm{Im}F(s)
\end{aligned}$$

The **convolution** of two functions $f(x)$ and $g(x)$ is

$$f(x) * g(x) = \int_{-\infty}^{+\infty} f(z)\,g(x-z)\,dz = \int_{-\infty}^{+\infty} f(x-z)\,g(z)\,dz$$

Convolution theorem

If $f(x)$ has the Fourier transform $F(s)$, and $g(x)$ has the Fourier transform $G(s)$, then the Fourier transform of the convolution $f(x) * g(x)$ is the product $F(s)\,G(s)$.

The **cross-correlation** of two functions $f(x)$ and $g(x)$ is

$$f^*(x) \star g(x) = \int_{-\infty}^{+\infty} f^*(z-x)\,g(z)\,dz = \int_{-\infty}^{+\infty} f^*(z)\,g(z+x)\,dz$$

where f^* refers to the complex conjugate of f. When the two functions are the same, $f^*(x) \bigstar f(x)$ is called the **autocorrelation** of $f(x)$.

Energy spectrum

The modulus squared of the Fourier transform $|F(s)|^2 = F(s)\,F^*(s)$ is sometimes called the **energy spectrum** or simply the **spectrum**.

If $f(x)$ has the Fourier transform $F(s)$, then the autocorrelation of $f(x)$ has the Fourier transform $|F(s)|^2$.

Phase spectrum

The Fourier transform $F(s)$ is most generally a complex function, which can be written as

$$F(s) = |F|e^{i\varphi} = \mathrm{Re}\,F(s) + i\,\mathrm{Im}\,F(s)$$

where $|F|$ is the modulus and φ is the **phase**, given by

$$\varphi = \tan^{-1}[\mathrm{Im}\,F(s)/\mathrm{Re}\,F(s)]$$

The function $\varphi(s)$ is sometimes also called the **phase spectrum**.

Obviously, both the modulus and phase must be known to completely specify the Fourier transform $F(s)$ or its transform pair in the other domain, $f(x)$. Consequently, an infinite number of functions $f(x) \Leftrightarrow F(s)$ are consistent with a given spectrum $|F(s)|^2$.

The **zero-phase** equivalent function (or zero-phase equivalent wavelet) corresponding to a given spectrum is

$$F(s) = |F(s)|$$
$$f(x) = \int_{-\infty}^{\infty} |F(s)|\, e^{+i2\pi xs}\, ds$$

which implies that $F(s)$ is real and $f(x)$ is hermitian. In the case of zero-phase *real* wavelets, then, both $F(s)$ and $f(x)$ are real even functions.

The **minimum-phase** equivalent function or wavelet corresponding to a spectrum is the unique one that is both *causal* and *invertible*. A simple way to compute the minimum-phase equivalent of a spectrum $|F(s)|^2$ is to perform the following steps (Claerbout, 1992):

(1) Take the logarithm, $B(s) = \ln\,|F(s)|$.
(2) Take the Fourier transform, $B(s) \Rightarrow b(x)$.
(3) Multiply $b(x)$ by zero for $x < 0$ and by 2 for $x > 0$. If done numerically, leave the values of b at zero and the Nyquist frequency unchanged.
(4) Transform back, giving $B(s) + i\varphi(s)$, where φ is the desired phase spectrum.
(5) Take the complex exponential to yield the minimum-phase function: $F_{\mathrm{mp}}(s) = \exp[B(s) + i\varphi(s)] = |F(s)|e^{i\varphi(s)}$.
(6) The causal minimum-phase wavelet is the Fourier transform of $F_{\mathrm{mp}}(s) \Rightarrow f_{\mathrm{mp}}(x)$.

Another way of saying this is that the phase spectrum of the minimum-phase equivalent function is the Hilbert transform (see Section 1.2 on the Hilbert transform) of the log of the energy spectrum.

Sampling theorem

A function $f(x)$ is said to be *band limited* if its Fourier transform is nonzero only within a finite range of frequencies, $|s| < s_c$, where s_c is sometimes called the *cut-off frequency*. The function $f(x)$ is fully specified if sampled at equal spacing not exceeding $\Delta x = 1/(2s_c)$. Equivalently, a time series sampled at interval Δt adequately describes the frequency components out to the *Nyquist frequency* $f_N = 1/(2\Delta t)$.

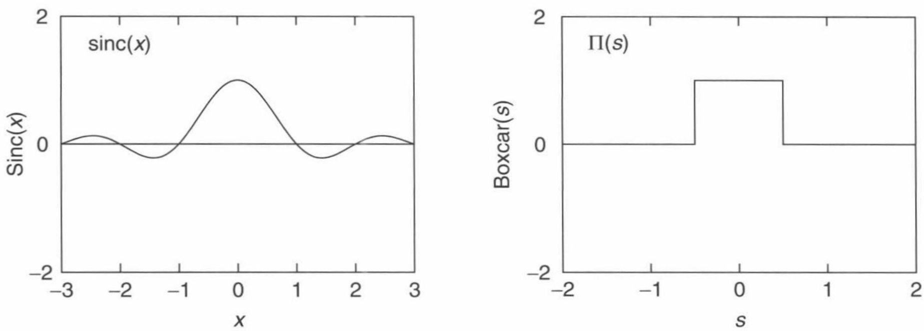


Figure 1.1.1 Plots of the function sinc(x) and its Fourier transform $\Pi(s)$.

The numerical process to recover the intermediate points between samples is to convolve with the *sinc function*:

$$2s_c\mathrm{sinc}(2s_c x) = 2s_c\sin(\pi 2s_c x)/\pi 2s_c x$$

where

$$\mathrm{sinc}(x) \equiv \frac{\sin(\pi x)}{\pi x}$$

which has the properties:

$$\left.\begin{array}{l}\mathrm{sinc}(0) = 1\\ \mathrm{sinc}(n) = 0\end{array}\right\} \; n = \text{nonzero integer}$$

The Fourier transform of sinc(x) is the boxcar function $\Pi(s)$:

$$\Pi(s) = \begin{cases} 0 & |s| > \frac{1}{2} \\ 1/2 & |s| = \frac{1}{2} \\ 1 & |s| < \frac{1}{2} \end{cases}$$

Plots of the function sinc(x) and its Fourier transform $\Pi(s)$ are shown in Figure 1.1.1.

One can see from the convolution and similarity theorems below that convolving with $2s_c\,\mathrm{sinc}(2s_c x)$ is equivalent to multiplying by $\Pi(s/2s_c)$ in the frequency domain (i.e., zeroing out all frequencies $|s| > s_c$ and passing all frequencies $|s| < s_c$.

Numerical details

Consider a band-limited function $g(t)$ sampled at N points at equal intervals: $g(0)$, $g(\Delta t)$, $g(2\Delta t), \ldots,$ $g((N-1)\Delta t)$. A typical fast Fourier transform (FFT) routine will yield N equally spaced values of the Fourier transform, $G(f)$, often arranged as

1	2	3	$\cdots$	$\left(\frac{N}{2}+1\right)$	$\left(\frac{N}{2}+2\right)$	$\cdots$	$(N-1)$	N
$G(0)$	$G(\Delta f)$	$G(2\Delta f)$	$\cdots$	$G(\pm f_N)$	$G(-f_N+\Delta f)$	$\cdots$	$G(-2\Delta f)$	$G(-\Delta f)$

time domain sample rate Δt
Nyquist frequency $f_N = 1/(2\Delta t)$
frequency domain sample rate $\Delta f = 1/(N\Delta t)$

Note that, because of "wraparound," the sample at $(N/2 + 1)$ represents both $\pm f_N$.

Spectral estimation and windowing

It is often desirable in rock physics and seismic analysis to estimate the spectrum of a wavelet or seismic trace. The most common, easiest, and, in some ways, the worst way is simply to chop out a piece of the data, take the Fourier transform, and find its magnitude. The problem is related to sample length. If the true data function is $f(t)$, a small sample of the data can be thought of as

$$f_{\mathrm{sample}}(t) = \begin{cases} f(t), & a \le t \le b \\ 0, & \text{elsewhere} \end{cases}$$

or

$$f_{\mathrm{sample}}(t) = f(t)\,\Pi\left(\frac{t - \frac{1}{2}(a+b)}{b-a}\right)$$

where $\Pi(t)$ is the boxcar function discussed above. Taking the Fourier transform of the data sample gives

$$F_{\mathrm{sample}}(s) = F(s) * \left[|b-a|\,\mathrm{sinc}((b-a)s)e^{-i\pi(a+b)s}\right]$$

More generally, we can "window" the sample with some other function $\omega(t)$:

$$f_{\mathrm{sample}}(t) = f(t)\,\omega(t)$$

yielding

$$F_{\mathrm{sample}}(s) = F(s) * W(s)$$

Thus, the estimated spectrum can be highly contaminated by the Fourier transform of the window, often with the effect of smoothing and distorting the spectrum due to the convolution with the window spectrum $W(s)$. This can be particularly severe in the analysis of ultrasonic waveforms in the laboratory, where often only the first 1 to $1\frac{1}{2}$ cycles are included in the window. The solution to the problem is not easy, and there is an extensive literature (e.g., Jenkins and Watts, 1968; Marple, 1987) on spectral estimation. Our advice is to be aware of the artifacts of windowing and to experiment to determine the sensitivity of the results, such as the spectral ratio or the phase velocity, to the choice of window size and shape.

Fourier transform theorems

Tables 1.1.1 and 1.1.2 summarize some useful theorems (Bracewell, 1965). If $f(x)$ has the Fourier transform $F(s)$, and $g(x)$ has the Fourier transform $G(s)$, then the Fourier