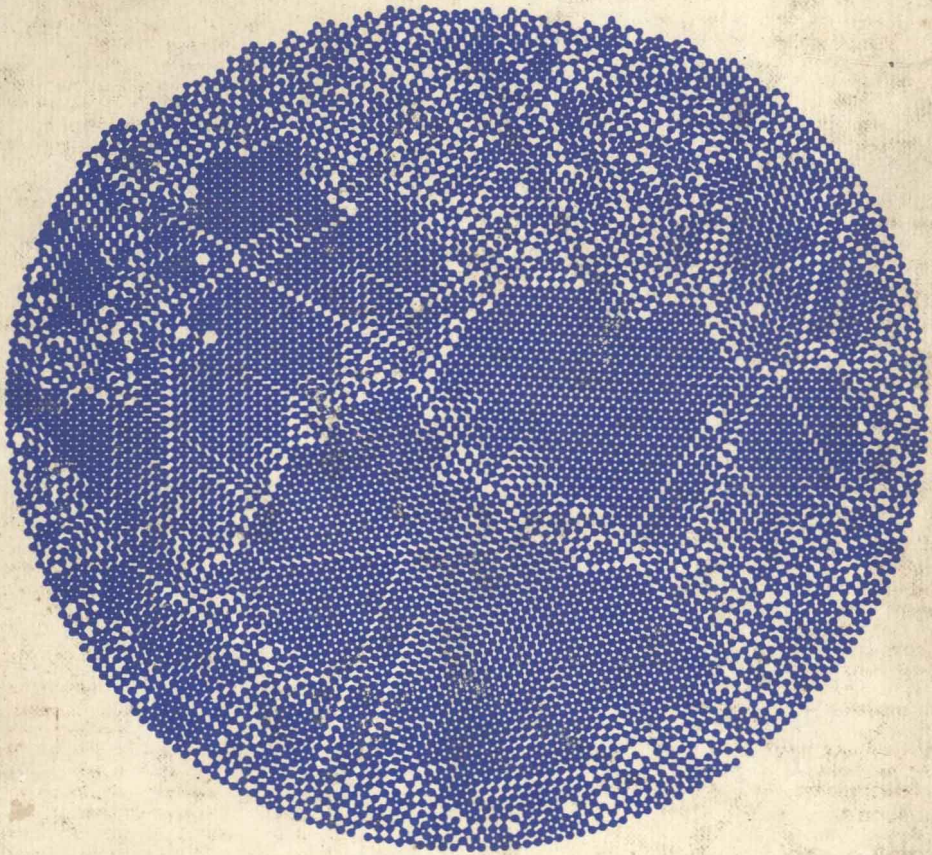


ADVANCES IN THE MECHANICS AND THE FLOW OF GRANULAR MATERIALS

M. SHAHINPOOR

VOLUME I



TRANS TECH PUBLICATIONS

**ADVANCES
IN THE MECHANICS
AND THE FLOW OF
GRANULAR MATERIALS**

VOLUME I

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ADVANCES IN THE MECHANICS
AND THE FLOW OF
GRANULAR MATERIALS

VOLUME I

PREFACE

This two-volume book on the mechanics and the flow of granular materials is a collection of 46 papers contributed by some of the world's leading experts on the subject. It is meant to serve as a survey of recent works on the mechanics, flow and general transport properties of granular materials, i. e., assemblies (aggregates) of distinct (discrete) unit entities (grains, particles, etc.). However, the fundamental purpose of this book is to provide a foundation for bridging the gaps that have existed among seemingly distinct areas of research that nevertheless have been directed towards the same goal, i. e., the understanding of the behavior of granular materials. Examples of diverse fields dealing with all aspects of granular materials are pharmacology, agriculture, ecology, geotechnics, geophysics, stellar physics, nuclear engineering, defense, manufacturing, mathematics-statistics, molecular physics and biomedicine-biometrics. In all of these fields of activities the behavior of granular assemblies is of fundamental and critical importance and poses interesting problems. Problems of immediate interest include the granular jams in capsule and other granular transport, the phenomena of vibratory densification, fluidization, and liquefaction, random packing and critical phenomena, determination of the velocity distribution function for colliding rough inelastic particles, dynamics of planetary granular rings and electromagneto-granular anomalies. Many fundamental problems remain to be solved. For example, can it be shown that the critical density of random close packing of equal sphere is $(2/\pi) \approx 0.6366197$?

The mechanics and flow of granular materials and bulk solids will undoubtedly become much more important in the very near future.

The first volume of this book concentrates on the following areas:

1. Random Packing of Granular Materials
2. General Morphological Characterization and Measurements
3. Statistical Mechanical Considerations
4. Electrical, Thermal and General Transport Properties.

The second volume of this book concentrates on the following main areas:

1. Granular Materials Rapid Flow, General Dynamics and Fluidization
2. Granular Materials and Soil Mechanics.

I hope that the publication of this book will materially contribute to the growth of our understanding of the behavior and the characteristics of granular materials. I would like to thank many friends and colleagues who have generously helped me throughout the preparation of these volumes. Among these I would specifically like to thank Dr. Alexander A. Hayday, Chairman of the Mechanical

and Industrial Engineering Department at Clarkson (where the Granular Materials Research Laboratory is housed), Dr. N. L. Ackermann, Chairman of the Civil and Environmental Engineering Department at Clarkson, and Professors G. Ahmadi and S. P. Lin for their continuous support and encouragement throughout the preparation of these volumes. I wish to thank all of the authors for their cooperation. I would also like to thank a number of research experts who have been most kind in helping me prepare these two volumes, who, however, due to pressure from other commitments, could not make a contribution in time to be included in these two volumes. Among these are Professors N. L. Ackermann, G. K. Batchelor, A. Blinowski, C. Brennen, M. M. Carroll, H. Colijn, S. C. Cowin, P. Cundall, J. Curran, R. Davies, R. A. Davis, K. J. Euler, J. Ghaboussi, R. L. Glicksman, A. S. Goldberg, D. A. Grivas, M. E. Harr, K. Hashiguchi, G. M. Homsy, I. Ishibashi, J. T. Jenkins, D. Jeffrey, G. De Josselin de Jong, J. R. Johanson, B. H. Kaye, R. P. King, J. Konishi, M. M. Mehrabadi, T. P. Meloy, N. Moroto, Z. Mróz, S. Nemat-Nasser, S. Niisek, J. W. Nunziato, M. Oda, R. Oliver, G. D. Parfitt, S. O. Passman, H. B. Poorooshasb, C. H. Rochester, B. Rohani, P. W. Rowe, R. H. Sabsky, M. Satake, S. B. Savage, P. J. Sherrington, A. Shokooh, A. J. M. Spencer, O. L. Strack, F. Tatsuoka, I. G. Vardoulakis, O. T. Walton, K. Yamaguchi and O. C. Zienkiewicz.

I hope to be able to incorporate the contributions from the above experts in future volumes on the mechanics and flow of granular materials.

Special thanks go to my wife Jamileh for her patience during the preparation of this work and to Ms. Rhonda L. Price for typing and preparation of the manuscripts.

Finally I would like to thank Dr.-Ing. Reinhard H. Wöhlbier, Publisher of Trans Tech Publications, and his staff both in the USA and the Federal Republic of Germany for the generous help, encouragement and cooperation extended to me throughout the preparation of this work.

January 1983

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Whatsoever things are true, whatsoever things are honest, whatsoever things are just, whatsoever things are pure, whatsoever things are lovely, whatsoever things are of good report; If there be any virtue, and if there be any praise, think on these things.

Philippians 4:8

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RANDOM PACKING OF GRANULAR MATERIALS

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