



# Intermediate Algebra

## A FUNCTIONAL APPROACH

SHOKO AOGAICHI BRANT  
EDWARD A. ZEIDMAN

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 HarperCollins *College Publishers*

This book is dedicated to our parents,  
Tadashi and Toshiye Aogaichi  
and  
Paul and Elsie Zeidman.

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*Intermediate Algebra*

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## THE AUDIENCE

*Intermediate Algebra: A Functional Approach* is an algebra text for students who have completed a course in beginning algebra. (Appendices provide a quick review of topics from beginning algebra.) Upon a successful completion of a course using this text, students should be prepared to take courses in college algebra, statistics, and trigonometry.

## THE APPROACH

Our textbook reflects the National Council of Teachers of Mathematics (NCTM) and American Mathematical Association of Two-Year Colleges (AMATYC) standards by presenting algebra in a functional framework. Even though traditional topics are covered, they are not covered in a traditional order; rather, topics are covered in context, as they are needed in the study of functions. For example, factoring quadratic expressions is covered just before solving quadratic equations and determining zeros of quadratic functions.

Graphics calculators are integrated throughout for connecting graphical, numerical, and symbolic representations of functions.

In addition to asking students to interpret formulas, we ask students to write formulas that represent real-life data. Our ultimate aim is to further the students' ability to use functions in modeling and analyzing numerical information.

As the exploration of functions proceeds, we introduce the appropriate language of algebra as a concise method of describing and generalizing events. Students are asked to write in precise mathematical terms to summarize accurately what they observe.

## THE DISTINGUISHING FEATURES

### Order of topics

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Functions serve as the organizational framework of this text, with algebra of expressions and equation solving discussed within this framework. We begin by leading the student to explore linear functions completely, then quadratic functions and their inverses, square root functions, as well as sequences and series. The subsequent study of each function—cubic, polynomial, rational, exponential, and logarithmic—follows the pattern established with the linear and quadratic functions: algebra of expressions, solving equations, graphing functions, determining the inverse, data fitting, and other applications.

## 2

## USES OF LINEAR FUNCTIONS

Linear functions have many uses, especially in describing situations that change at a constant rate. Let's look at one such example:

The spawning of salmon declines when industrial waste is introduced into the river. Two tons of waste causes the fish population to decrease to 2,500. Twelve tons of waste causes the population to decrease to 1,500.

Assuming the salmon population is related to the amount of industrial waste introduced into the river, and is declining at a constant rate, the linear function that represents the population is

$$P(x) = -100x + 2700,$$

where  $x$  is the amount of industrial waste in tons. The graph of  $P$  is shown in Figure 1. Using either the function or its graph, we can determine that when the amount of industrial waste introduced into the river reaches 27 tons, there will be no salmon left.

The idea of *function* has been written about for more than two thousand years. For example, Ptolemy (c. 178–100 B.C.) wrote formulas to predict planetary positions. The word *function* seems to have been introduced by Leibniz in 1694. Euler in 1748 began his text on precalculus with the following definition of a function: a function is an equation or formula involving variables and constants. He was credited as the first to use the notation  $f(x)$  that we will discuss in Section 2.3. This notation is an example of how mathematicians generalize and extend their concepts. Another hundred years later, Dirichlet gave the definition of the function that we use now.



**FIGURE 1**  
Graph of salmon population  
function  $P(x) = -100x + 2700$

### 2.1 SOLUTIONS OF LINEAR EQUATIONS

- ▲ Find  $x$ -intercepts of lines.
- Use intercepts to draw comprehensive graphs of lines.
- Approximate solutions of linear equations.

In this section, we explore the *x-intercept point*, where a line crosses the  $x$ -axis. The  $x$ -intercept of the line or graph is the *root* of an equation.

▲ Find  $x$ -intercepts of lines.

We know from previous work what the *y-intercept* is. It is the  $y$ -coordinate of the point where a line crosses the  $y$ -axis. What about the point where a line crosses the  $x$ -axis?

### CHAPTER OPENER

Examples or historical background begin each chapter, providing students with insight into the topics to be covered.

### SECTION OPENER

A list of objectives outlines the topics covered.



## TITLED EXAMPLES

Many of the title examples present students with forums for investigation and discovery. We then generalize concepts from these examples. Data-fitting examples allow students to use previously learned algebraic methods to develop formulas.

Checking (including estimating and predicting) the solution is a consistent and integral part of each example.

## TECHNICAL NOTES

The Technical Notes provide an interface between algebra and graphics calculators. Although keystrokes are not included, the notes include methods for appropriate use of calculators, as well as explanations of how calculators work.

### EXAMPLE 3 Finding the root of a linear equation

The population of some species of birds is declining. In 1990, the estimated population of redheaded woodpeckers in one region was 4,000. By 1995, it was 3,000.

(i) Assuming that the population declines at the same constant rate, find a linear function that describes the redheaded woodpecker population. (Use  $x = 0$  for the year 1990.)

(ii) In what year can we expect the redheaded woodpecker population to be 0 in this particular region?

**Solution** (i) To determine the linear equation relating year and population, write the equation of a line passing through the two points  $(0, 4000)$  and  $(5, 3000)$ . The slope is

$$m = \frac{3000 - 4000}{5 - 0} = -200.$$

Using the point  $(0, 4000)$  and the slope-intercept form of the equation, we have  $y = -200x + 4000$ .

(ii) To find the year in which the population will be 0, find the root of the equation  $y = -200x + 4000$ .

$$\begin{aligned} -200x + 4000 &= 0 \\ -200x &= -4000 \\ x &= 20 \end{aligned}$$

Thus, in the year 2010  $(1990 + 20)$ , if the current rate continues, we can expect the redheaded woodpeckers to have vanished from this particular region. ◀

#### Use intercepts to draw comprehensive graphs of lines.

The  $x$ - and  $y$ -intercept points are important in getting a comprehensive picture of the graph of a linear function. The following technical note illustrates their usefulness.

#### TECHNICAL NOTE Good Window Size

In previous examples, we suggest the window sizes to use. How do we determine a **good window size**? Figure 5 shows the graph of the function in the last

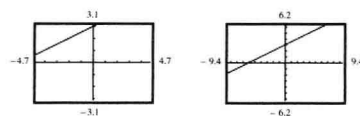


FIGURE 5  
Two views of the graph of  $y = \frac{1}{2}x + 3$

#### Graph step functions.

This idea of a function being *connected* or having a *break* is an important part of the study of calculus. A function can of course have more than one break and have infinitely many breaks.

A **step function** is a piecewise-defined function made up of constant functions. A step function is so named because its graph looks like stair steps. Consider a step function with one step:

$$f(x) = \begin{cases} -1, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

Its graph is shown in Figure 43. As before, an open circle indicates that  $(0, -1)$  is not part of the first stair (because  $y = -1$  when  $x < 0$ ). A solid circle indicates that  $(0, 1)$  is part of the second stair (because  $y = 1$  when  $x \geq 0$ ). At  $x = 0$ , the graph of this function steps up, from  $y = -1$  to  $y = 1$ . The domain of  $f$  is the set of real numbers, but the range of  $f$  is the set containing two integers,  $\{-1, 1\}$ .

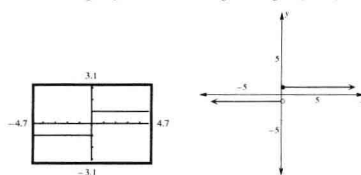


FIGURE 43  
Graph and sketch of  $f(x) = \begin{cases} -1, & x < 0 \\ 1, & x \geq 0 \end{cases}$

A function with infinitely many steps is the **greatest integer function**,  $f(x) = \lfloor x \rfloor$ .

#### DEFINITION Greatest integer

The **greatest integer** of  $x$ ,  $\lfloor x \rfloor$ , is defined as the *greatest integer less than or equal to  $x$* .

If we position a real number on a number line, the number's greatest integer is the first integer to its left. For example, in Figure 44,  $\lfloor -1.3 \rfloor$  is  $-2$  and  $\lfloor 1.5 \rfloor$  is  $1$ . The greatest integer of an integer is itself, so  $\lfloor 3 \rfloor$  is  $3$  and  $\lfloor 0 \rfloor$  is  $0$ .

**CAUTION:** Notice that the greatest integer of a number is always the integer to its left on a number line, *not* the number rounded to an integer. In Figure 44,  $\lfloor -1.3 \rfloor$  is  $-2$ , not  $-1$ .

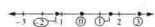
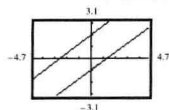


FIGURE 44  
Some real numbers and their greatest integers, plotted on the number line

## DEFINITIONS, RULES, SUMMARIES, AND CAUTIONS

For easy reference, important definitions, rules, and summaries are titled and boxed. Cautions are provided for steps that are ripe for errors, emphasizing what should be done to avoid mistakes.

**EXAMPLE 5** ▶ Recognizing parallel lines

**FIGURE 63**  
Graphs of  $y_1 = \frac{3}{4}x - 1$  and  $y_2 = \frac{3}{4}x + 2$

**Solution**

The lines are graphed in Figure 63. Are they parallel? Be careful—looks are deceiving. Because  $m_1 = \frac{3}{4} = 0.75$  and  $m_2 = \frac{3}{4} = 0.75$ ,  $m_1 = m_2$ . Therefore, the lines are **not** parallel. ◀

**EXAMPLE 6** ▶ Recognizing perpendicular lines

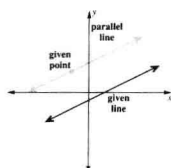
**FIGURE 64**  
Graphs of  $y_1 = -\frac{3}{4}x - 1$  and  $y_2 = \frac{3}{4}x + 2$

**Solution**

Graph the lines using a square window, as shown in Figure 64. Are they perpendicular? Does  $m_1 m_2 = -1$ ? Because  $m_1 = -\frac{3}{4}$  and  $m_2 = \frac{3}{4}$ , we have  $m_1 m_2 = -\frac{3}{4} \cdot \frac{3}{4} = -\frac{9}{16}$ , not  $-1$ . Therefore, the lines are **not** perpendicular. ◀

**B Find equations of parallel lines.**

We know from geometry that there is one and only one line through a given point and **parallel** to a given line. See Figure 65. Given a line and a point not on the line, we can write an equation of the line through the point and **parallel** to it. Let's see how to find the equation of a line parallel to a given line.



**FIGURE 65**  
Line through a given point and parallel to a given line

**FIGURES AND CALCULATOR SCREEN DISPLAYS**

The text contains almost 1,500 figures and graphs. Since the use of graphics calculators is assumed with this book, the figures include TI-82 graphics calculator screen displays where appropriate, recognizing that in most situations other graphics calculators have similar displays.

**EXERCISE SETS**

The exercises are graded, paired, and coded to each section objective. Writing exercises are scattered throughout each exercise set and often follow a group of exercises that develop a pattern or display a concept. Application problems (including statistical and geometric) are also included within many sets. Each section ends with comprehensive exercises that synthesize and extend the material.

$$\begin{cases} z = 3 \\ w + x + y + z = 5 \\ 8w + 4x + 2y + z = 2 \\ 27w + 9x + 3y + z = 0 \end{cases}$$

$$48. \begin{cases} -8w + 4x - 2y + z = 3 \\ -w + x - y + z = -1 \\ z = -4 \\ w + x + y + z = 5 \end{cases}$$

(Leave answer as fractions, or as decimals rounded to one decimal place.)

49. The Richmond Baking Company produces three types of breads: white, whole wheat, and sourdough. The company has three types of machines for producing these breads: mixing, kneading, and baking. The weekly production schedule showing the number of minutes a loaf of each bread needs on each machine is given in the following table:

(Time in Minutes per Loaf)	White Bread	Whole Wheat Bread	Sourdough Bread	Total Time Available
Mixing Machine	2	3	1	1,221
Kneading Machine	5	7	6	3,671
Baking Machine	12	15	11	7,757

The total time available is the number of minutes each machine is available for use. The remaining time is needed for preparation and maintenance.

- (I) Find the number of loaves of each bread that can be made if the company uses all the available time.
- (II) Suppose the Richmond Baking Company must make 150 loaves of white bread and 150 loaves of wheat. Find the maximum number of loaves of sourdough bread the company can make, without using more than the total time available on each machine.
50. The Anika Computer Company produces three types of computers, the slow Fx-088, medium speed Gx-426, and a faster model Hx-486. The Fx-088 is built with five slow chips, two medium speed chips, and three fast chips; the Gx-426 is built with two slow chips, four medium speed chips, and seven fast chips; the Hx-486 is built with one slow chip, six medium speed chips, and nine fast chips. The company has in its inventory 6,200 slow chips, 12,500 medium speed chips, and 19,700 fast chips.
- (I) Construct a production table with the type of chip listed in the rows, the type of computer listed in the columns, and total number of chips available in the last column.
- (II) If the company wants to use all its chips in inventory, find the number of Fx-088, Gx-426, and Hx-486 computers that can be built.
- (III) Suppose the company wants to make only 600 Fx-088 computers and 900 Gx-426 computers. Find the maximum number of Hx-486 computers they can make, using as many of the computer chips as available in inventory.
51. A bottle of soda containing 1.6 ounces of syrup, 7.5 ounces of juice, and 20.9 ounces of carbonated water costs \$1.85. A second bottle of soda containing 2.3 ounces of syrup, 8.3 ounces of juice, and 19.4 ounces of carbonated water costs \$2.25. A third bottle of soda containing 1.2 ounces of syrup, 6.9 ounces of juice, and 21.9 ounces of carbonated water costs \$1.61. Assuming that these prices are for the syrup, juice, and carbonated water only and not for the packaging, find to the nearest penny the cost per ounce for the syrup, the juice, and the carbonated water.
52. Alena is on a special diet consisting of three types of foods: P, Q, and R. The diet is to consist of at least 4 milligrams of Vitamin B-1, 5 milligrams of Vitamin B-2, and 8 milligrams of Vitamin B-6. The table below gives the amounts of these vitamins on contained per unit of each type food.

	Food P	Food Q	Food R
Vitamin B-1 (mg)	0.08	0.05	0.06
Vitamin B-2 (mg)	0.05	0.08	0.08
Vitamin B-6 (mg)	0.1	0.1	0.2

If she is to meet the minimum vitamin requirements, how many units of each type of food should she include in her diet? (Round your answer to the nearest whole unit.)

## QUICK REVIEW

Each chapter ends with a summary, review exercises, group projects, and a chapter quiz. Coded to the chapter sections, the review exercises include writing exercises that seek conceptual understanding and synthesis. The group projects are designed to provide and encourage cooperative and collaborative learning in small groups. The exercises in the chapter quizzes are scrambled.

[2.4]

14. Graph the absolute value function of  $f(x) = |x + 5|$ .

EXERCISES 15–16. Graph the function, and determine its domain and range. Is the graph connected at  $x = 3$ ?

$$15. f(x) = \begin{cases} 1 - 2x, & x \leq 3 \\ 2x - 1, & x > 3 \end{cases}$$

$$16. f(x) = \begin{cases} 1 - 2x, & x \leq 3 \\ -5, & x > 3 \end{cases}$$

[2.5]

17. For the sequence  $u_n = \{5, 9, 13, 17, 21, \dots\}$ :

(i) Plot the sequence.

(ii) Write a formula for the  $n^{\text{th}}$  term.

18. Describe any similarities and differences between the sequence

$$u_n = 2n + 5$$

and the function

$$f(x) = 2x + 5.$$

19. For the parametric functions

$$d_1(t) = \begin{cases} 100 - 2t \\ 2 \end{cases} \quad \text{and} \quad d_2(t) = \begin{cases} 5(t - 30) \\ 3 \end{cases} \quad \text{with} \quad 0 \leq t \leq 50.$$

(i) Graph the parametric equations simultaneously. Use a  $[0, 100]$  by  $[0, 5]$  window.

(ii) Give a comparative description of the speed and direction of the motion simulated by  $d_1$  and  $d_2$ .

20. On a foggy day, a speed boat is traveling due south at 10 knots; its position relative to a lighthouse is 20 nautical miles east and 40 nautical miles north. A fishing boat is traveling due east at 3.5 knots; its position relative to the same lighthouse is 10 nautical miles west and 20 nautical miles south. Model the situation. Will they collide? Verify algebraically.

### C Group Project

1. For the U.S. population data displayed in the table below:

Year	1950	1960	1970	1980	1990
Pop.*	151.3	179.3	203.3	226.5	248.7

\* in millions

(i) Find linear equations for  $P$ , where  $P$  is the U.S. population in millions and  $t$  is the year.

- Group 1. Use points (1950, 151.3) and (1960, 179.3).
- Group 2. Use points (1950, 151.3) and (1970, 203.3).
- Group 3. Use points (1950, 151.3) and (1980, 226.5).
- Group 4. Use points (1950, 151.3) and (1990, 248.7).
- Group 5. Use points (1960, 179.3) and (1970, 203.3).
- Group 6. Use points (1960, 179.3) and (1980, 226.5).
- Group 7. Use points (1960, 179.3) and (1990, 248.7).
- Group 8. Use points (1970, 203.3) and (1980, 226.5).
- Group 9. Use points (1970, 203.3) and (1990, 248.7).
- Group 10. Use points (1980, 226.5) and (1990, 248.7).
- Group 11. Use the linear regression command on a graphics calculator.

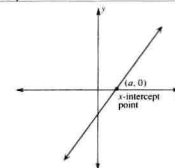
## CHAPTER 2 REVIEW

### A Summary

#### 2.1 SOLUTIONS OF LINEAR EQUATIONS

The  $x$ -intercept point  $(a, 0)$  is the point where a non-horizontal line crosses the  $x$ -axis.

#### Examples

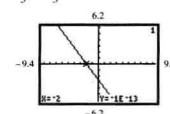


The  $x$ -intercept of the graph of  $y = mx + b$  is the root (solution) of the equation  $mx + b = 0$ .

The  $x$ -intercept of the graph of  $y = -\frac{4}{3}x - \frac{8}{3}$  is  $-2$  because the root of the equation  $-\frac{4}{3}x - \frac{8}{3} = 0$  is  $x = -2$ .

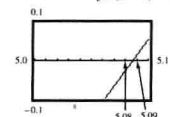
A comprehensive graph of a linear function shows both the  $x$ - and  $y$ -intercept points of the line.

A comprehensive graph of  $y = -\frac{4}{3}x - \frac{8}{3}$  is:



Approximate the solution of a linear equation graphically by approximating the  $x$ -intercept.

An approximate solution of  $1.08x - 2.87(3 - 4x) = 8.4(x + 2) - 4.25$  is the approximate  $x$ -intercept of the graph of  $y = [1.08x - 2.87(3 - 4x)] - [8.4(x + 2) - 4.25]$ .



The approximate solution is  $5.09$ , with an error  $\leq 0.01$ .

## GROUP PROJECT



## CONTENT HIGHLIGHTS

The text begins with graphing and interpreting linear functions. In **Chapter 1**, we explore the four representations of a linear function and the effects and applications of the y-intercept and slope.

In **Chapter 2**, we present the connections between symbolic algebra (solving a linear equation) and graphs (x-intercept of the graph of a linear function). We also introduce mathematical modeling for problem solving (including determining the general term of a sequence), function notation, and ideas of domain and range.

**Chapter 3** extends the study of linear functions to systems of linear equations. We solve systems graphically and algebraically (substitution and elimination methods), and also include solving systems by matrices, because graphics calculators make matrices accessible to intermediate algebra students.

In **Chapters 4 and 5** we proceed to study quadratic functions. Although Chapter 4 covers solving quadratic equations using traditional methods, they are constantly reinforced by graphical and numerical techniques. Related topics include the square root functions, simulations with parametric equations, and sequences and series.

In **Chapter 6**, the ideas of linear and quadratic functions are extended to the more general polynomial functions. Beginning with cubic functions, we repeat the pattern of development established for quadratics and introduce the Fundamental Theorem of Algebra and the Binomial Theorem.

From the beginning of our study of rationals in **Chapter 7**, we pay careful attention to the restriction on the values of the variables of a rational expression. The restrictions, as well as algebraic operations, are supported by the visualization obtained with graphics calculators.

Using an example that differentiates between exponential and quadratic growth, we introduce exponential functions in **Chapter 8**. Following the pattern set with quadratic and square root functions, we also introduce logarithmic functions as inverse functions of exponential functions. We fit exponential and logarithmic functions to data by solving exponential and logarithmic equations.

After their definitions and graphs, conic sections are presented in **Chapter 9** from a functional point of view as a combination of two functions. In addition, exercise sets include solving nonlinear systems of equations.

Finally, **Appendices 1–6** give a quick review of beginning algebra concepts.

## THE SUPPLEMENT PACKAGE FOR INTERMEDIATE ALGEBRA: A FUNCTIONAL APPROACH

The text is supported with an extensive package of supplements for both the instructor and the student.

For the instructor

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***Instructor's Annotated Exercises*** With this volume, instructors have immediate access to all the answers in the text. Each answer is printed in bold type next to or below the pertinent exercise.

The ***Instructor's Solution Manual*** provides worked solutions to all even exercises in the text.

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#### **For the student**

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A ***Student Solution Manual*** contains worked solutions to all of the odd-numbered exercises in the text. To order use ISBN 0-673-99537-2.

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***Videos*** tie specific graphics calculator keystrokes to examples in the text.

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Shoko Brant  
Ed Zeidman

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