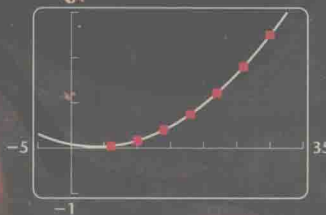


INTERMEDIATE ALGEBRA

GRAPHS & MODELS



BITTINGER • ELLENBOGEN • JOHNSON

Intermediate Algebra

GRAPHS AND MODELS

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Preface

Appropriate for a one-term course in intermediate algebra, *Intermediate Algebra: Graphs and Models* is intended for those students who have completed a first course in algebra. This text is more interactive than most other intermediate algebra texts. Our goal is to enhance the learning process through the use of technology and to provide as much support and help for students as possible in their study of algebra.

Content Features

- **Integrated Technology** The technology of the graphing calculator is completely integrated throughout the text to provide a visual means of increasing understanding. In this text, we use the term “grapher” to refer to all graphing calculator technology. The use of the grapher is woven throughout the exposition, the exercise sets, and the testing program without sacrificing algebraic skills. We use the grapher technology to enhance, not to replace, the students’ mathematical skills and to alleviate the tedium associated with certain procedures. It is assumed that each student is required to have a grapher (or at least access to one) while enrolled in this course.
- **Learning to Use the Technology** To minimize the need to spend valuable class time teaching students how to use a grapher, we have included an introduction to the graphing calculator as well as explanations of the grapher features used throughout the text. The features are introduced on an “as-needed” basis. Specific keystrokes for various graphers are given in the *Graphing Calculator Manual*, which is bundled free with every new copy of this text.
- **Interactive Discoveries** The grapher provides an exciting teaching opportunity in which a student can discover and further investigate mathematical concepts. This unique Interactive Discovery feature is used

to introduce new topics and provides a vehicle for students to “see” a concept quickly. This feature reinforces the idea that grapher technology is an integral part of the course as well as an important learning tool. It invites the student to develop analytic and reasoning skills while taking an active role in the learning process. (See pp. 120, 244, and 294.)

- **Function Emphasis** The use of technology with its immediate visualization of a concept encourages the early presentation of functions. Functions are introduced in the first section of Chapter 2. The study of the family of functions (linear, quadratic, higher-degree polynomial, rational, exponential, and logarithmic) has been enhanced and streamlined with the inclusion of the grapher. Applications with graphs are incorporated throughout to amplify and add relevance to the study of functions. (See pp. 108, 273, and 632.)

- **Variety in Approaches to Solutions** Skill in solving mathematical problems is expanded when a student is exposed to a variety of approaches to finding a solution. We have carefully incorporated three solution approaches throughout the text: algebraic, graphical, and numerical. Chapter openers illustrate an application with a concurrent grapher presentation of both a table and a graph (see pp. 67 and 487). The TABLE feature on a grapher provides a numerical display or check of the solution (see pp. 299 and 553).

To highlight both the algebraic- and graphical-solution approaches in solving equations, we have used a two-column solution format in numerous examples (see pp. 170, 308, and 623). In the algebraic/graphical side-by-side features, both methods are presented together; each method provides a complete solution. This feature emphasizes that there is more than one way to obtain a result and illustrates the comparative efficiency and accuracy of the two methods.

- **Real-Data Applications** Throughout the writing process, we conducted an energetic search for real-data applications. The result of that effort is a variety of examples and exercises that connect the mathematical content with the real world. Source lines appear with most real-data applications and charts and graphs are frequently included. Many applications are drawn from the fields of health, business and economics, life and physical sciences, social science, and areas of general interest such as sports and daily life. We encourage students to “see” and interpret the mathematics that appears around them every day. (See pp. 52, 510, and 585.)

- **Regression** Using regression or curve fitting to model data is introduced in Chapter 2 with linear functions. This visual theme is continued with quadratic, cubic, quartic, exponential, and logarithmic functions. Although the theoretic aspects of curve fitting cannot be developed in this course, the power of the grapher is very apparent in this area as the technique is applied to real data. Students can quickly make the “what is this used for?” connection between real data and the extrapolated results of the curve fitting, thus giving them a better conceptual understanding of the material. (See pp. 130–131 and 336.)

- **Verifying Identities** Identities can be partially verified with a grapher using both the GRAPH and TABLE features (see pp. 276, 284, and 446). This content feature allows a visual answer to such frequent questions as “Why isn’t $(x + 2)^2$ equal to $x^2 + 4$?” This approach also provides a unique lead-in to the development of the properties of exponents and logarithms.

Pedagogical Features


- **Use of Color** The text uses full color in an extremely functional way, as seen in the design elements and artwork on nearly every page. The choice of color has been carried out in a methodical and precise manner so that its use carries a consistent meaning, which enhances the readability of the text for both student and instructor. (See pp. 73 and 299.)
- **Art Package** The text contains nearly 1000 art pieces. The exceptional situational art and statistical graphs throughout the text highlight the abundance of real-world applications while helping students visualize the mathematics (see pp. 55, 303, and 562). The design and use of color with the grapher windows exemplifies the impact that technology has in today’s mathematical curriculum (see pp. 75, 84, and 234).
- **Annotated Examples** Over 470 examples fully prepare the student for the exercise sets. Learning is carefully guided with numerous color-coded art pieces and step-by-step annotations, with substitutions and annotations highlighted in red (see pp. 162 and 590). The basis for problem solving is a five-step process established early in the text to aid the student in strategically approaching and solving applications (see pp. 91 and 389–390).
- **Collaborative Corners** In today’s professional world, teamwork is essential. We have included optional Collaborative Corner features throughout the text to allow students to work in groups to solve problems. There is an average of three Collaborative Corner activities per chapter, each one appearing after the appropriate section’s exercise set (see pp. 101, 341–342, and 588). Additional Collaborative Corner activities and suggestions for directing collaborative learning appear in the *Printed Test Bank/Instructor’s Resource Guide*.
- **Variety of Exercises** There are over 6400 exercises in this text. The exercise sets are enhanced not only by the inclusion of real-data applications with source lines, detailed art pieces, and technology windows that include both tables and graphs, but also by the following features.


Technology Exercises Since use of the grapher is totally integrated in this text, exercise sets include both grapher and nongrapher exercises. In some cases, detailed instruction lines indicate the approach the

student is expected to use. In others, the student is left to choose the approach that seems best, thereby encouraging critical thinking. (See pp. 47 and 328.)

Skill Maintenance The exercises in this section have been specifically selected to review concepts previously taught in the text that are foundations for the material presented in the following section. They are often chosen to prepare the student for the new concept(s) that will be covered next. (See p. 543.)

Synthesis Exercises These exercises, which appear at the end of each exercise set, encourage critical thinking by requiring students to synthesize concepts from several sections or to take a concept a step further than in the regular exercises. (See pp. 60–61.)

Thinking and Writing Exercises for thinking and writing, at the beginning of the synthesis exercises, are denoted with a maze icon . They encourage students to both consider and write about key mathematical ideas in the chapter. Many of these exercises are open-ended, making them particularly suitable for use in class discussions or as collaborative activities. (See p. 124.)

Stop and Think Exercises Throughout the text certain problems are marked with a magnifying glass icon —these problems can usually be solved without lengthy computation or use of a grapher. They are intended to remind the student to look at the problem carefully to help develop the habit of thinking about every problem before attempting to solve it. (See p. 20 and 132.)

- **Chapter Openers** Each chapter opens with an application illustrated with both technology windows and situational art. The openers also include a table of contents listing section titles. (See pp. 151 and 579.)
- **Highlighted Information** Important definitions, properties, and rules are displayed in screened boxes. Summaries and procedures are listed in color-outlined boxes. Both of these design features present and organize the material for efficient learning and review. (See pp. 104 and 387.)
- **Summary and Review** The Summary and Review at the end of each chapter provides an extensive set of review exercises along with a list of important properties and formulas covered in that chapter. This feature provides an excellent preparation for chapter tests and the final examination. Answers to all review exercises appear in the text along with section references that direct students to material to reexamine if they have difficulty with a particular exercise. (See pp. 145 and 342.)
- **Cumulative Review** After every three chapters, and at the end of the text, we have included a Cumulative Review, which reviews skills and concepts from all preceding chapters of the text. (See pp. 216, 414, 651, and 695.)

Supplements for the Instructor

Instructor's Solutions Manual

The *Instructor's Solutions Manual* by Judith A. Penna contains worked-out solutions to all exercises in the exercise sets, including the thinking and writing exercises. It also includes a sample test with answers for each chapter and answers to the exercises in the appendixes. The sample tests are also included in the *Student's Solutions Manual*.

Printed Test Bank/Instructor's Manual

This supplement contains the following:

- Extra practice problems.
- Black-line masters of grids and number lines for transparency masters or test preparation.
- A videotape index and section cross references to the available tutorial software packages.
- Additional collaborative learning activities and suggestions.

The test bank portion contains the following:

- Six free-response test forms for each chapter, following the format of and with the same level of difficulty as the tests in the *Student's Solutions Manual*.
- Two multiple-choice test forms for each chapter.
- Eight alternative forms of the final examination, three with questions organized by type, three with questions organized by chapter, and two with multiple-choice questions.

Testgen-EQ CD-ROM

Testgen-EQ is a computerized test generator that allows instructors to select test questions manually or randomly from selected topics or to use a ready-made test for each chapter. The test questions are algorithm-driven so that regenerated number values maintain problem types and provide a large number of test items in both multiple-choice and open-ended formats for one or more test forms. Test items can be viewed on screen, and the built-in question editor lets instructors modify existing questions or add new ones that include pictures, graphs, accurate math symbols, and variable text and numbers.

Additional features in the new Testgen-EQ CD-ROM allow the instructor to customize both the look and content of test banks and tests. Test questions are easily transferred from the test bank to a test and can be sorted, searched, and displayed in various ways. Testgen-EQ is available on a dual platform (Macintosh/Windows) CD-ROM.

Course Management and Testing System

InterAct Math Plus for Windows and Macintosh (available from Addison Wesley Longman) combines course management and on-line testing with

the features of the basic tutorial software (see “Supplements for the Student”) to create an invaluable teaching resource. Consult your local Addison Wesley Longman sales consultant for details.

Supplements for the Student

Graphing Calculator Manual

The *Graphing Calculator Manual* by Judith A. Penna, with the assistance of Daphne Bell, contains keystroke level instruction for the Texas Instruments TI-83/83+, TI-86, and TI-89.

Bundled free with every copy of the text, the *Graphing Calculator Manual* uses actual examples and exercises from *Intermediate Algebra: Graphs and Models* to help teach students to use their graphing calculator. The order of topics in the *Graphing Calculator Manual* mirrors that of the text, providing a just-in-time mode of instruction. Keystroke manuals for other calculators are available upon request.

Student's Solutions Manual

The *Student's Solutions Manual* by Judith A. Penna contains completely worked-out solutions with step-by-step annotations for all the odd-numbered exercises in the exercise sets in the text, with the exception of the thinking and writing exercises. It also includes a self-test with answers for each chapter and a final examination.

The *Student's Solutions Manual* can be purchased by your students from Addison Wesley Longman.

InterAct Math Tutorial Software CD-ROM

InterAct Math Tutorial Software CD-ROM has been developed and designed by professional software engineers working closely with a team of experienced math educators.

InterAct Math Tutorial Software includes exercises that are linked with every objective in the textbook and require the same computational and problem-solving skills as their companion exercises in the text. Each exercise has an example and an interactive guided solution that are designed to involve students in the solution process and to help them identify precisely where they are having trouble. In addition, the software recognizes common student errors and provides students with appropriate customized feedback.

With its sophisticated answer recognition capabilities, *InterAct Math Tutorial Software* recognizes appropriate forms of the same answer for any kind of input. It also tracks student activity and scores for each section, which can then be printed out.

InterAct Math Tutorial Software is available in CD-ROM for both Windows and Macintosh computers.

World Wide Web Supplement

<http://www.GraphsModels.com>

This specially developed Web site provides additional practice and learning resources. For each chapter, students can find additional practice exercises, Web links for further exploration, and expanded Summary and Review pages that reinforce the concepts and skills learned throughout the chapter. Students can also download a plug-in for Addison Wesley Longman's *InterAct Math Tutorial Software* that allows students to access tutorial problems directly through their Web browser.

Videotapes

Developed and produced especially for this text, these videotapes feature an engaging team of instructors presenting material and concepts from every section of the text in a student-interactive format. The lecturer's presentations include examples and problems from the text and support an approach that emphasizes the use of technology, visualization, and problem solving.

Math Tutor Center

The Math Tutor Center is a service provided by Addison Wesley Longman. Staffed by qualified mathematics instructors, this service offers live tutoring for students via telephone, fax, and/or e-mail, five days a week, seven hours a day. Students can receive tutoring on examples, exercises, and problems contained in their text. This service is free to any student who purchases a new Bittinger/Ellenbogen/Johnson text bundled with a valid registration number. A valid registration number can be purchased separately for those students with used texts.

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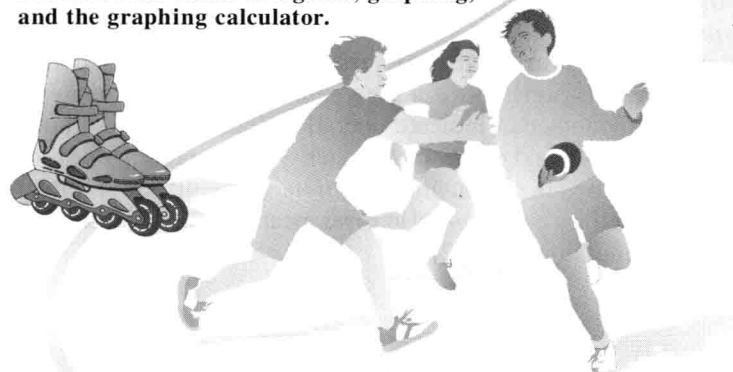
*Answers**A-1**Index**I-1**Index of Applications**I-11*

Basics of Algebra and Graphing

1



The theme of this text is problem solving in algebra. Both symbolic algebra and graphs are used to develop models and solve applications. Models are used extensively in many fields, ranging from sociology to medicine to meteorology. In this chapter, we see what is meant by a mathematical model, and we introduce the basics of algebra, graphing, and the graphing calculator.



APPLICATION

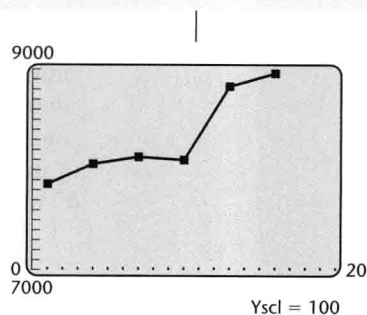
CHILD EXPENDITURE. The following table shows the annual expenditures on a child in 1996 by families with an average income of \$46,100. Use the data to draw a line graph.

AGE OF CHILD	ANNUAL EXPENDITURE
1	\$7860
4	8060
7	8130
10	8100
13	8830
16	8960

Source: *The Wall Street Journal Almanac*, 1998

This situation can be modeled by graphing the data.

This problem appears as Exercise 49 in Section 1.6.



- 1.1 Some Basics of Algebra
- 1.2 Operations with Real Numbers
- 1.3 Equivalent Algebraic Expressions
- 1.4 Exponential and Scientific Notation
- 1.5 Graphs
- 1.6 Mathematical Models
- SUMMARY AND REVIEW
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