BASIC ALGEBRA

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BASIC ALGEBRA

To Anna
and
to the memory of my mother

PREFACE

This text is written for university and junior college students with little or no background in the subject taking a first course in basic algebra. Its main objective is to help students develop the algebraic skills they will need in subsequent courses. The text is designed for either a one-semester or a one-quarter course. Instructors may want to create a syllabus, set their own pace, and establish their own areas of emphasis.

This book is traditional in content. Special emphasis is placed on informal presentation and on the relevance of algebra to real-world situations. Each section begins with a statement of objectives and concludes with exercises graduated in difficulty. There are close to 3000 exercises and worked-out examples, designed to enhance student comprehension. Each chapter ends with a review summarizing the main ideas discussed in the chapter and a sample test. Answers to all the odd-numbered exercises and review problems and tests are given at the back of the book.

In addition, students are asked to respond to numerous "Self-Tests." The answers to these questions are provided in the margin for instant feedback. This feature prepares students for the exercises. Finally, historical developments of some concepts and biographical sketches of some famous mathematicians who contributed to the development of mathematics are presented intermittently throughout.

A workbook containing section summaries, solved problems, answers to the even exercises, and additional sample tests is available to instructors. It also discusses the use of electronic calculators so that instructors who wish to teach the course with the aid of calculators may do so.

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September 1980

M.N.M.

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SOME BASIC OPERATIONS

1.1 THE INTEGERS: ADDITION AND SUBTRACTION

OBJECTIVES

1. **Define numeral, digit, constant, variable, natural number, integers, and number line.**
2. **Use numerals to write number names.**
3. **State the commutative and associative rules for addition.**
4. **Add and subtract integers.**
5. **Use number lines to illustrate addition and subtraction of integers.**

The word *algebra* is derived from the word *Al-jabr* in the title of the book *Ilm Al-jabr Wal Muqabalah* written in 830 A.D. by the Arab mathematician Mohammed ibn Musa al-Khowarizmi. One of the earliest writings on algebra dates back to an ancient book written on papyrus about 2000 B.C. by the Egyptian priest Ahmes. Clay tablets dating from 1800 B.C. indicate that the Babylonians dealt with algebraic equations, progressions, and the algebra of right triangles. Although Babylonian algebra was highly developed, apparently the Babylonians used no algebraic symbolism in their system of writing.

Our system of writing numbers, called the Hindu-Arabic system, originated in India around 200 B.C. and appeared in Europe only by 500 A.D. The logical foundation of this system was not formulated until the late nineteenth century.

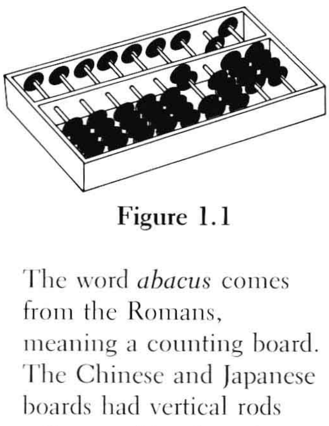


Figure 1.1

The word *abacus* comes from the Romans, meaning a counting board. The Chinese and Japanese boards had vertical rods with round beads and wedge-shaped beads, respectively. The Armenian *tchoreb* and the Russian *s'choty*, like that of the Romans, had horizontal rods. The abacus has been in use for over 2000 years.

The notion of counting is as ancient as man. One of the earliest counting methods was *tallying*. A tally (a mark or a scratch) was made into cave walls or on wooden or ivory sticks to represent a single object. As societies developed, so also did the counting system. One of the earliest counting devices, used worldwide, is the human hand. In North America, for example, the Shoshone tribe used finger counting and assigned names to numbers, such as "sople" for 1, "vuy" for 2, "pa" for 3, and so on. This was a first step in developing a counting system that could be used for calculating. Thus, for 6 the Shoshone used "kuan-sople," which meant five and one, a primitive form of addition. Having five fingers on each hand, man found it natural to count in groups of fives and tens. From this has come our present decimal (from the Latin *decimus* meaning "ten") system. In some societies groups of twenties were used. The French still use numbers such as "quatre vingts," which means "four 20s." This suggests that in some parts of France the French counted in groups of twenty. Counting methods developed from finger counting to the abacus to the present-day electronic calculators.

Because writing out number names was not convenient, symbols called *numerals* were introduced to represent numbers. In our number system the *natural* (or *counting*) *numbers* are named one, two, three, four, and so on,

and the numerals used to represent these numbers are

1, 2, 3, 4, and so on

In this system we make use of only ten symbols called *digits*. These digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. By means of these ten digits we can express every number, no matter how large or how small. Thus, when we write the number 634 we mean 6 (hundreds), 3 (tens), and 4 (ones, or units). The number is read "six hundred thirty-four." Larger numbers consisting of any number of digits can easily be read by separating the numbers into blocks of three digits, each beginning at the right, as illustrated in the following example.

Note. In the United States the tenth place from the right represents billions. That is, one billion is one thousand millions. However, in certain other countries, including England, one billion means one million million.

	hundred billions	ten billions	billions	hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones (or units)
	9	5	7	2	4	8	3	0	1	6	8	4
block name →	billions			millions			thousands					

The number 957,248,301,684 is read "nine hundred fifty-seven billion, two hundred forty-eight million, three hundred one thousand, six hundred eighty-four."

Answers

1. (a) 2,004 (b) 2,040 (c) 2,044 (d) 2,000,404
2. (a) Ninety thousand, ten (b) Ninety thousand, one hundred one. (c) Nine thousand one

SELF-TEST

1. Write each word description in Hindu-Arabic notation.
   (a) Two thousand four ______
   (b) Two thousand forty ______
   (c) Two thousand forty-four ______
   (d) Two million, four hundred four ______
2. Write each number in words.
   (a) 90,010 ______
   (b) 90,101 ______
   (c) 9,001 ______

Algebra, the theory of arithmetic, is a study of numbers and their properties. It provides a tool for solving many of our daily practical problems in such diverse fields as business, engineering, biology, ecology, medicine, archeology, politics, meteorology, and many others.

In algebra we use *letters* such as a, b, c, x, y, and z as the names of numbers. We refer to a numeral or a letter as a *constant* if it is the name of exactly one number. Commonly, the first few letters of the alphabet, such as a, b, and c, are used as constants. If a letter names more than one number or it is a symbol whose value is unknown, such a letter is called a *variable*. The letters x, y, and z are commonly used as variables.

One of the basic operations involving numbers is *addition*, which is denoted by +. Two numbers may be added and the result, which is always a number, is called the *sum*. That is, if a and b are natural numbers, then the sum $a + b$ is a natural number. This is called the *closure rule for addition*. Thus, in $5 + 8 = 13$, the number 13, which is a natural number, is called the sum and 5 and 8 are called the terms of the sum. We are all familiar with the equality symbol = read "is equal to," or "is," or "equals." This symbol is used whenever the quantities are exactly the same. If the quantities are not the same, we use the symbol ≠, read "is not equal to." For example, $13 = 13$, whereas $13 \neq 10$.

THE CLOSURE RULE FOR ADDITION

The notion of addition arises naturally in our everyday experiences. For example, if Carlos buys two adjacent lots, one with an area of 35 acres and the other with an area of 27 acres, the total area of land bought by Carlos is the sum of 35 and 27. We write $35 + 27 = 62$ acres. Note that the order of addition is not important. That is, $35 + 27 = 27 + 35 = 62$.

In general, if a and b are natural numbers, then the sum of a and b is equal to the sum of b and a. We write

Note. This rule is not restricted to the natural numbers.

$$a + b = b + a \tag{1}$$

This is called the *commutative rule for addition*.

THE COMMUTATIVE RULE FOR ADDITION

Let us return to Carlos in the above example. Suppose Carlos has a wife, Yvonne, and two children. In his will Carlos leaves 20 acres to his wife and the remaining acreage to his children. How many acres are left for the children? The operation involved in computing the acreage left to the children is called *subtraction*. In this case, the acreage left to the children is the difference when 20 is subtracted from 62. We write

$$62 - 20 = 42$$

If a is a natural number, then $-a$ is the additive inverse of a and is called the *negative* of the natural number a. In general, if a and b are numbers, the difference when b is subtracted from a is denoted by

$$a - b = a + (-b). \tag{2}$$

Subtraction is indicated by the *minus* symbol − used in arithmetic. The expression $a - b$ is read "a minus b" and the result of the operation is called the *difference*.

EXAMPLE 1. If you buy a \$30,000 house and you pay for it with a standard mortgage, you will pay back \$75,000. The difference represents the interest charged. How much have you paid in interest?

SOLUTION. The interest paid is the difference between the amount paid and the cost of the house. Thus we have

$$75{,}000 - 30{,}000 = 45{,}000$$

That is, the interest paid is \$45,000.

Note. The interest paid is more than the cost of the house.

SELF-TEST. A father's age is 42 and his son's is 16. The difference between their ages is ______________.

Answer

$42 - 16 = 26$.

In using the commutative rule for addition care should be exercised when negative numbers are involved. For example,

$$20 + (-5) = (-5) + 20 = 15$$

However, $20 - 5 \neq 5 - 20$. We note that

$$20 - 5 = 15$$

whereas

$$5 - 20 = -15$$

Negative numbers were invented about 1000 A.D. and were generally accepted around the end of the sixteenth century. Negative numbers arise naturally in our everyday life. Consider the following opposites.

A fall in temperature is the opposite of a rise in temperature.
A loss of \$5 is the opposite of a gain of \$5.
A 6-yard (yd) loss is the opposite of a 6-yd gain.
A loss in the stock market is the opposite of a gain in the stock market.

We use the symbols + and − with numbers to express such events. For example,

A 4° rise in temperature	$+4$
A loss of \$560	-560
A building 30 meters (m) high	$+30$

SELF-TEST. Use natural numbers and negative natural numbers to express each of the following:

1. Twenty degrees below zero ______________
2. A gain of seven kilograms ______________
3. One hundred twenty-three meters below sea level ______________

Answers

1. -20
2. $+7$
3. -123

Note. The number zero, denoted by 0, was introduced by the Hindus about 700 A.D.

The set consisting of the natural numbers, the negative of the natural numbers, and zero is called the set of *integers*. The natural numbers are called *positive integers*, and the negatives of the natural numbers are called *negative integers*. The number 0, which is neither positive nor negative, obeys the following rules:

$$c + 0 = 0 + c = c \tag{3}$$

and

$$c + (-c) = c - c = 0 \tag{4}$$

where c is any number. 0 is called the *additive identity* element. For example,

$$7 + 0 = 7, \qquad (-3) + 0 = -3$$

and

$$5 + (-5) = 5 - 5 = 0, \qquad 26 + (-26) = 26 - 26 = 0$$

We also have

$$-(-c) = c. \tag{5}$$

For example, $-(-11) = 11$.

The integers are ordered; that is, it is always possible to determine whether one integer is greater than, equal to, or less than another. Because of this property we can represent the integers as points on a line. We do this as follows.

1. Draw a straight line horizontally (see Figure 1.2).
2. Select a point 0 and associate it with the number zero. This point is called the *origin*.
3. Select a convenient point to the right of zero and let it represent the number 1.
4. Select the *positive* direction as the direction traveled when going from left to right. Select the opposite direction, called the *negative* direction, when going from right to left.

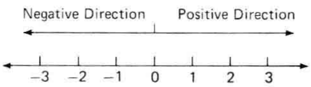


Figure 1.2

The line segment from 0 to 1 is called the *unit* segment, and its length is chosen as the unit length. Next, we reproduce the unit length successively on both sides of 0 and 1 to obtain the graphical representation of the set of integers—the positive integers in ascending order to the right of 0 and the negative integers in descending order to the left of 0. The line is then called the *number line* (also called *line graph*, *real line*, or *directed line*).

Consider a football game in progress. We may chart the gains and losses in yardage as positive and negative integers. If in two plays 6 and 3 yd are gained, respectively, the net gain may be represented by

$$(+6) + (+3) = +9$$

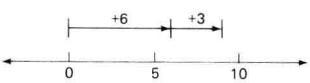


Figure 1.3

As a visual aid we use a representation as in Figure 1.3.

EXAMPLE 2. On a number line illustrate each of the following gains and losses of yardage.

1. Gain of 5 and loss of 3
2. Loss of 7 and gain of 11
3. Loss of 3 and loss of 2.

SOLUTION. We illustrate the gains and losses on a number line as follows.

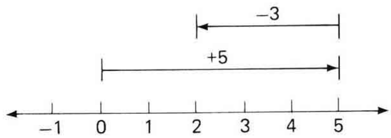
1.


Figure 1.4

We have

$$(+5) + (-3) = 5 - 3 = +2$$

which represents a net gain of 2 yd (+2).

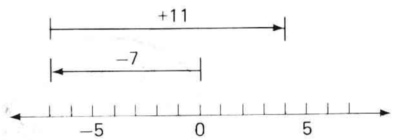
2.


Figure 1.5

We have

$$(-7) + (+11) = 11 + (-7) = 11 - 7 = +4$$

which represents a net gain of 4 yd (+4).

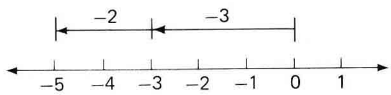
3.


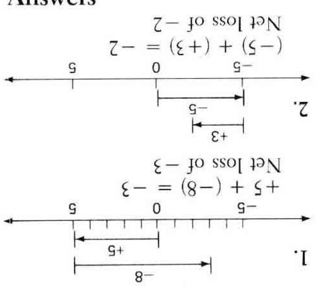
Figure 1.6

We have

$$(-3) + (-2) = -5$$

which represents a net loss of 5 yd (−5).

Answers



SELF-TEST. On a number line illustrate each of the following.

1. Gain of 5 and loss of 8.

   Net (gain/loss) of ______________________.

2. Loss of 5 and gain of 3.

   Net (gain/loss) of ______________________.