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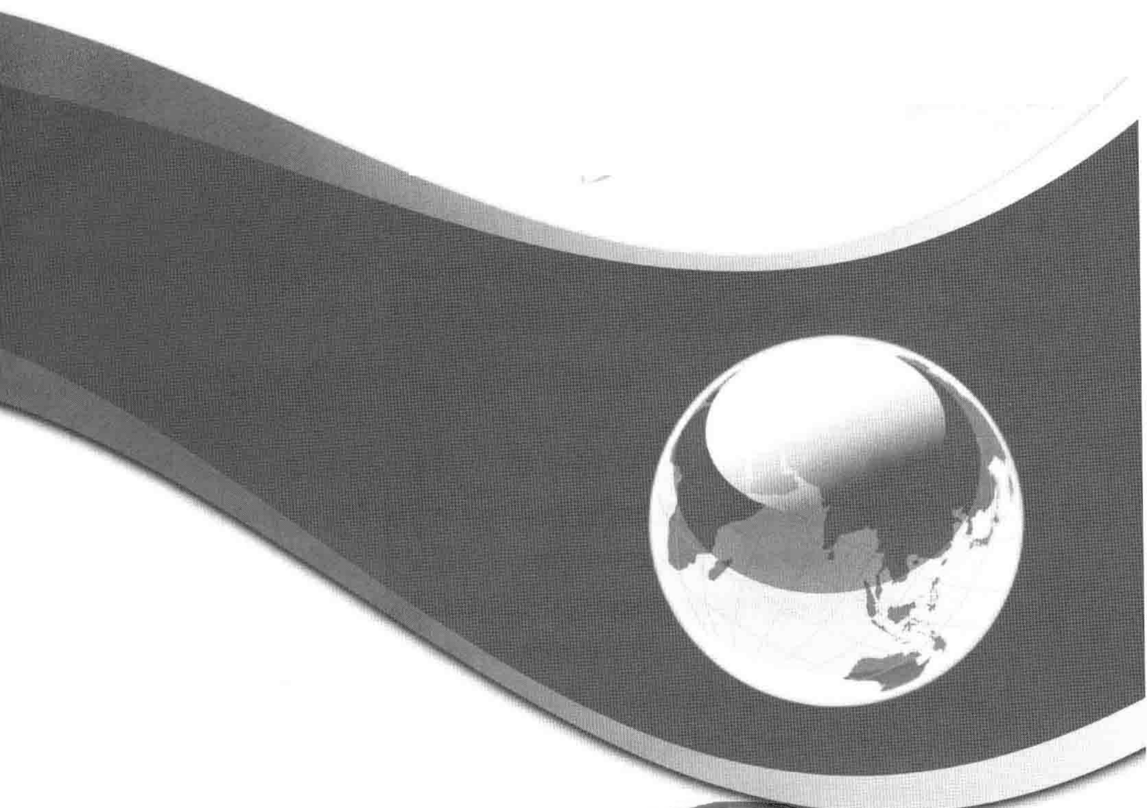

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Preface

The research described herein was conducted during my postdoctoral stay in the Department of Geomatics at the University of New Brunswick in Canada and later continued during my lecturing in the National School of Surveying at the University of Otago in New Zealand and in the School of Geodesy and Geomatics at the Wuhan University in China. Theoretical definitions of the rigorous orthometric height and the geoid-to-quasigeoid correction were discussed with Prof. Petr Vaníček (University of New Brunswick), Prof. Will E. Featherstone (Curtin University), Prof. Lars E. Sjöberg (Royal Institute of Technology), Prof. Pavel Novák (University of West Bohemia), Dr. Christian Hirt (Curtin University), and Dr. Sten Claessens (Curtin University). The presented numerical results were compiled with the help of Dr. Nadim Dayoub (University of Newcastle upon Tyne), Dr. Robert Čunderlík (Slovak Technical University), Prof. Viliam Vatrt (Brno University of Technology) and my former PhD student Dr. Ahmed Abdalla (University of Otago). The digital density model of New Zealand was complied with the help of Dr. Pascal Sirguey (University of Otago) and the advice of Dr. Mark Rattenbury (GNS Science). The gravity database was kindly provided by the GNS Science New Zealand, and the levelling and GPS data and the NZGeoid2009 official quasigeoid model of New Zealand by the Land Information New Zealand.

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Summary

The definition and practical realization of the World Height System (WHS) requires the unification of several continental, national and local geodetic vertical controls currently established over the world. This can be done partially (on a continental scale) by a joint adjustment of levelling networks, while a global realization requires finding their relation with respect to the geoidal geopotential value W_0. Another major issue, associated with the vertical datum unification, is a choice of a height system. Either Helmert's orthometric heights or Molodensky's normal heights are practically used in countries where the leveiling networks were realized through geodetic spirit levelling and gravity measurements along levelling lines. In countries, where these gravity measurements are absent, the normal gravity values were used to approximate the actual gravity. The vertical datum is in this case defined in the system of normal-orthometric heights. The conversion between different types of heights is thus indispensible for the unification of geodetic vertical datums. The rigorous relation between the orthometric and normal heights is utilized in definitions of the geoid-to-quasigeoid correction in the spatial and spectral domains presented in this work. However, the practical application of these expressions in computing the geoid-to-quasigeoid correction is not simple, because it requires the knowledge of the terrain geometry, topographic density distribution and importantly also the facilitation of advanced numerical techniques. After reviewing the numerical models of computing the geoid-to-quasigeoid correction in the spatial and spectral domains, some of these practical aspects are discussed in the context of the (experimental) vertical datum unification in New Zealand. The height reference system in New Zealand was realized by several local vertical datums (LVDs), which were established throughout the country based on precise levelling from tide gauges or connecting to existing levelling networks. Moreover, the LVDs were defined in the system of the normal-orthometric heights, because of the absence of measured gravity values along levelling lines. Asserting that all geodetic data available should be incorporated in the vertical datum realization, the unification of LVDs at the North and South Islands of New Zealand was realized in several processing steps, which comprised the levelling network adjustment, the gravimetric geoid and quasigeoid determination, the height conversion, the compilation of digital terrain and density models, the analysis of the mean dynamic topography offshore, the conversion between permanent tidal systems, and the estimations of LVD offsets. These procedures are summarized here and possible methods of improving the accuracy of the height conversion are also discussed.

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1. Introduction

For a practical realization of the geodetic vertical datum, Helmert's (1884, 1890) orthometric heights are preferably used. The reason is a simple computation of the mean gravity using Poincaré-Prey's gravity reduction while assuming a uniform topographic density distribution, and the acceptable accuracy for most of the regions where the levelling networks are established. To determine the orthometric heights in the mountainous, polar and geologically complex regions with the accuracy of a few centimetres or even better, Helmert's definition is not sufficient. In this case, more accurate methods for the evaluation of mean gravity have to be applied.

A more accurate method was introduced by Niethammer (1932, 1939). He took the terrain effect into consideration while assuming a uniform topographic density. According to his method, the mean value of the planar terrain correction is evaluated as a simple average of values computed at the finite number of points along the plumbline within the topography; see also Baeschlin (1948) who summarized his work. Mader (1954) estimated the difference between Helmert and Niethammer's methods of ~6 cm for Hochtor (2,504 m) in the European Alps, see also Heiskanen and Moritz (1967, Chapters 4-6). Mader (1954) and Ledersteger (1968) also presupposed that the terrain correction varies linearly with depth. Based on this assumption, the mean terrain correction is averaged from two values computed for points at the topographic surface and the geoid. Flury and Rummel (2009), however, demonstrated that the non-linear changes of the terrain correction could not be disregarded. Hence, the mean terrain correction should be computed according to Niethammer (1939) or Flury and Rummel (2009). Wirth (1990) modified Niethammer's method by means of computing the topographic gravity potential (instead of the terrain correction) at points at the topographic surface and the geoid.

It is a well-known fact that the mean gravity within the topography depends also on the actual topographic density distribution. The variation of topographic density can cause changes in orthometric height up to several decimetres (e.g. Vaníček et al., 1995). The correction to Helmert's orthometric height due to the lateral variation of topographic density can be evaluated using a simple formula in which the change of orthometric height is in a linear relation to the anomalous lateral topographic density (Heiskanen and Moritz, 1967). Adopting this relation, the effect of the anomalous topographic density to Helmert's orthometric heights was investigated, for instance, by Allister and Featherstone (2001) and Tenzer and Vaníček (2003).

In these approximate definitions of the orthometric height, the vertical gravity gradient generated by the mass density distribution below the geoid surface is approximated by the linear normal gravity gradient while disregarding the change of the normal gravity gradient with depth. Hwang and Hsiao (2003) estimated that this approximation causes the inaccuracy of orthometric heights up to several centimetres in the mountainous regions.

Tenzer and Vaníček (2003) applied the analytical downward continuation of the observed gravity in the evaluation of the mean gravity along the plumbline within the topography based on assuming the lateral topographic density distribution. They then formulated the relation between Poincaré-Prey's gravity gradient and the analytical downward continuation of gravity. A more accurate method for a determination of the mean gravity was introduced by Tenzer et al. (2005). They applied the decomposition of the mean gravity into the mean normal gravity, the mean no-topography gravity disturbance (generated by the mass density distribution below the geoid surface) and the mean values of the gravitational attractions of topographic and atmospheric masses. The mean normal gravity is evaluated according to Somigliana-Pizzetti's theory of the normal gravity field (Pizzetti, 1911; Somigliana, 1929). The mean topography-generated gravitational attraction is, in accordance with Bruns' (1878) theorem, defined in terms of the difference of gravitational potentials reckoned to the geoid and the topographic surface, multiplied by the reciprocal value of the orthometric height. The same principle was deduced for a definition of the mean atmosphere-generated gravitational attraction. The mean no-topography gravity disturbance is defined by applying Poisson's integral to the integral mean and solving the inverse to Dirichlet's boundary-value problem for the downward continuation of the no-topography gravity disturbances in prior of computing the integral mean value. In addition to the above theoretical developments, numerous empirical studies have been published on the orthometric height definition (e.g., Ledersteger, 1955; Rapp, 1961; Krakiwsky, 1965; Strange, 1982; Sünkel, 1986; Kao et al., 2000; Tenzer and Vaníček, 2003; Dennis and Featherstone, 2003).

Asserting that the topographic density and the actual vertical gravity gradient inside the Earth could not be determined precisely, Molodensky (1945, 1948) formulated the theory of normal heights. In his definition, the mean actual gravity within the topography is replaced by the mean normal gravity between the reference ellipsoid and telluroid (see also Heiskanen and Moritz, 1967, Chapter 4). The normal heights are thus defined without any hypothesis about the topographic mass density distribution.

The Molodensky normal heights and the Helmert orthometric heights are the most widely-used height systems. These two types of heights can be adopted if the levelling networks were established based on geodetic spirit levelling and gravity measurements along levelling lines. In some countries, however, the gravity values along levelling lines were calculated only approximately using the normal gravity. The vertical datum is then defined by the normal-orthometric heights.

In recent years, a considerable effort has been undertaken to unify a large number of existing vertical datum realizations around the world. The vertical datum unification typically requires the joint adjustment of interconnected levelling networks and/or the definition of the vertical datum offset with respect to the World Height System (WHS), which is defined by the geoidal geopotential value W_0. Alternatively, the vertical datum unification can be realized though the gravimetric determination of the global geoid/quasigeoid model to a high accuracy and resolution. Since the geodetic vertical systems are defined using different types of heights (and every country adopted their own height system specifications), the conversion between these types of heights is inevitable. The height conversion has been addressed extensively in geodetic literature. An approximate formula relating the normal and orthometric heights was given, for instance, in Heiskanen and Moritz (1967, Eqs. 8-103). Sjöberg (1995) slightly improved the classical definition by adding a small correction term related with the vertical derivative of the gravity anomaly. Tenzer et al. (2005) presented numerical procedures for a rigorous computation of the orthometric height and formulated an accurate relation between the (rigorous) orthometric and normal heights. An alternative method of computing the geoid-to-quasigeoid correction was given by Tenzer et al. (2006). They derived this correction based on comparing the geoidal height and the height anomaly, both defined by means of applying Bruns' (1878) theorem. A very similar expression for computing the geoid-to-quasigeoid correction was given by Sjöberg (2006). The definitions of the geoid-to-quasigeoid correction presented by Tenzer et al. (2005, 2006) and Sjöberg (2006) incorporated information on the terrain geometry, variable topographic density and mass density heterogeneities distributed below the geoid surface. Santos et al. (2006) investigated the relations between various types of the orthometric height definitions. Flury and Rummel (2009) investigated the effect of terrain geometry to the geoid-to-quasigeoid correction. They demonstrated that the consideration of the terrain geometry significantly reduces the values of the geoid-to-quasigeoid correction computed using the classical definition in which the topography is approximated by the Bouguer plate. The results of Flury and Rummel (2009) were in a good agreement with previous results over larger area in European Alps presented by Marti (2005) and Sünkel et al. (1987) (see also Hofmann-Wellenhof and Moritz, 2005) Following the work of Flury and Rummel (2009), Sjöberg (2010) derived a slightly more accurate expression for the geoid-to-quasigeoid correction, consistent with a definition of the boundary condition of physical geodesy (see also Sjöberg and Bagherbandi, 2012; Bagherbandi and Tenzer, 2013). He, however, also stated that his more refined expression could improve the accuracy not more than ~1 cm compared to the expression given by Flury and Rummel (2009). Later, Sjöberg (2012) applied an arbitrary compensation model in computing the topographic correction term. In particular, he recommended using either the Helmert or isostatic types of reductions, which provide smaller and smoother components, more suitable for interpolation and calculation, than the Bouguer reduction. It is worth mentioning herein that the conversion of the normal-orthometric to normal heights was applied, for instance, by Filmer et al. (2010) and Tenzer et al. (2011a, 2011b).

To begin with, the fundamental definitions in the theory of heights are here briefly recapitulated. With reference to these definitions, the expressions for an accurate conversion between the normal and orthometric heights (i. e., the geoid-to-quasigeoid correction) are then presented in the spatial and spectral domains. The numerical procedures of computing the geoid-to-quasigeoid correction in the spatial domain are compared. The computation of this correction in the spectral domain is realized by means of applying methods for a spherical harmonic analysis and synthesis of the gravity field and continental crustal density structures. The geoid-to-quasigeoid correction could be computed accurately only if the actual crustal density distribution within the topography is known to a sufficient accuracy. Moreover, this computation utilizes relatively complex numerical schemes which cannot routinely be applied in practice. Therefore, the rigorous definition of the orthometric height (and consequently the geoid-to-quasigeoid correction) is likely to be restricted mainly to scientific purposes, while its use in broader, more practical geodetic applications remains limited. Possible reasons are discussed in the context of (experimental) vertical datum unification in New Zealand, conducted in several processing steps. These steps comprise the levelling network adjustment, the gravimetric geoid and quasigeoid modelling, the estimation of LVD offsets, the conversion between the permanent tidal systems, the analysis of systematic errors, and the conversion between different height systems. Moreover, in order to improve the accuracy of computing the geoid-to-quasigeoid correction, digital terrain and density models are needed. For this purpose, the rock density model was compiled from existing geological maps, rock density samples, and additional geological sources. This model is then facilitated in the gravimetric forward modelling of variable topographic density.

The content is organized into eight chapters. The following three chapters provide a brief summary of the coordinate systems and transformations (Chapter 2), the Earth's gravity field (Chapter 3) and the theory of heights (Chapter 4). The explicit definition of the geoid-to-quasigeoid correction and the expressions used for computing this correction in the spatial and spectral domains are given in Chapter 5. The practical aspects of vertical datum unification in New Zealand are discussed, and numerical results presented in Chapter 6. The effect of variable topographic density on gravity field quantities is investigated in Chapter 7. The summary and major conclusions are given in Chapter 8.

2. Coordinate Systems and Transformations

The 3-D position is defined in the Cartesian coordinate system (X, Y, Z) of which the origin is identical to the mass center of the Earth, the Z-axis pass through the Conventional International Origin (CIO), and the X-axis pass through the intersection of the Greenwich meridian plane with the equatorial plane. Analogously, the 3-D position can be described by the geodetic coordinates (h, φ, λ) or the spherical coordinates (r, ϕ, λ), where φ and λ are the geodetic latitude and longitude respectively, ϕ is the spherical latitude, the spherical and geodetic longitudes λ are identical, r is the geocentric radius, and h is the geodetic (ellipsoidal) height (see Fig. 2. 1).

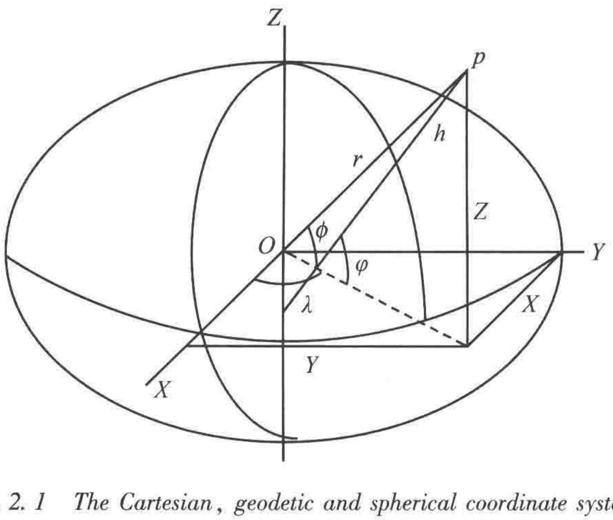

Fig. 2. 1 The Cartesian, geodetic and spherical coordinate systems