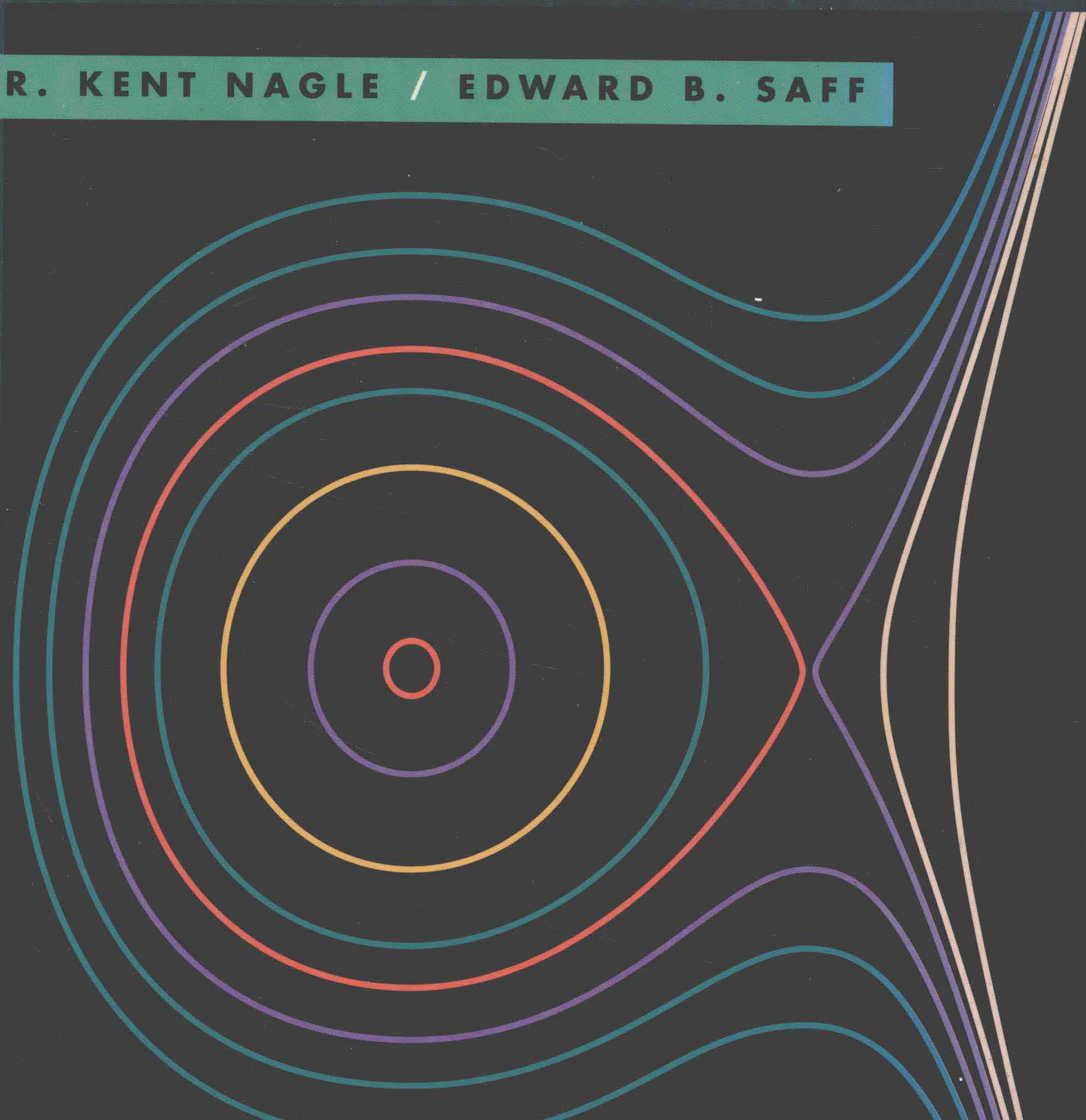
Fundamentals of

Differential Equations

Third Edition

R. KENT NAGLE / EDWARD B. SAFF



THIRD EDITION

FUNDAMENTALS OF DIFFERENTIAL EQUATIONS

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To our families,

who endured our late nights,
soothed our anxieties,
and shared our enthusiasm.

Sandy	**Loretta**
Kevin, Jeffrey	Lisa, Tracy, Alison

Preface

OUR GOAL

While an introductory course in differential equations is a mainstay of undergraduate curricula in the sciences and engineering, the flavor of such a course varies considerably because of departmental requirements, instructors' tastes, and students' mathematical maturities and backgrounds. Our goal has been to write a flexible *one-semester* text that spans a variety of topics in the basic theory as well as applications of differential equations. At the same time we have striven to make the text "user friendly" through various design features.

PREREQUISITES

While some universities make *linear algebra* a prerequisite for differential equations, many schools (especially engineering) only require calculus. With this in mind, we have designed the text so that only Chapter 6 (Higher Order Linear Differential Equations) and Chapter 10 (Matrix Methods for Linear Systems) require more than high school linear algebra. Moreover, Chapter 10 contains a review section on matrices and vectors as well as specific references for the deeper results used from the theory of linear algebra. We have also written Chapter 9 (Systems of Differential Equations and Their Applications) so as to give an introduction to systems of differential equations—including methods of solving, applications, numerical procedures, phase plane analysis, and Poincaré maps—that does not require a background in linear algebra.

IMPROVEMENTS IN THIS THIRD EDITION

- **New Sections** Three new sections have been added to the text; namely,

  §4.9 Undetermined Coefficients Using Complex Arithmetic
  §7.1 Introduction: A Simple Electrical Circuit
  §9.7 Dynamical Systems, Poincaré Maps, Strange Attractors, and Chaos.

  Engineering oriented students will find in the new optional Section 4.9 a streamlined approach to solving undetermined coefficients problems—one that appeals to the notions of complex impedance and phase angles. The new introduction, Section 7.1, to the Laplace transforms chapter emphasizes the advantages of the transform method in handling discontinuous forcing functions. The added Section 9.7 describes the Poincaré method for nonautonomous systems and provides a glimpse of several topics of current research activity. There we explain how chaotic systems arise in physical phenomena and emphasize the role of numerical computations in analyzing dynamical systems.

- **Technical Writing Exercises** Communications skills are, of course, an essential aspect of professional activities. Yet few texts provide opportunities for the reader to develop these skills. Thus we have added at the end of most chapters a set of clearly marked technical writing exercises which invite students to make documented responses to questions dealing with the concepts in the chapter. In so doing, students are encouraged to make comparisons between various methods and to present examples that support their analysis.

- **New Group Projects** Since they were well received in the first two editions, we have added several new Group Projects at the ends of the chapters. They are

  Chapter 1: *Magnetic "Dipole"*
  Chapter 2: *Designing a Solar Collector*
  Chapter 4: *Convolution Method, Nonlinear Equations Solvable by First Order Techniques*
  Chapter 6: *Transverse Vibrations of a Beam*
  Chapter 8: *Airy's Equation, Buckling of a Tower*
  Chapter 9: *Strange Behavior of Competing Species-Part I*
  Chapter 10: *Strange Behavior of Competing Species-Part II*
  Chapter 11: *Green's Function, Numerical Methods for $\Delta u = f$ on a Rectangle.*

- **Use of Computer Software** The availability of computer packages such as MATHEMATICA®, DERIVE®, and MAPLE® provide an opportunity for the student to conduct numerical experiments and tackle realistic applications that give additional insights into the subject. Consequently, we have added several exercises and projects throughout the text that are designed for the student to employ available software in phase plane analysis, eigenvalue computations, and the numerical solutions of various equations.

- **New Problems** A variety of new problems have been added to the exercises. Most of the additions are related to applications or the use of commercially available computer software.

- **Other Changes** The instructor familiar with the second edition will notice that the discussion in Sections 7.7 and 7.8 concerning the impulse response function has been significantly expanded in order to emphasize its relationship to the general solution of an initial value problem using convolution. In addition, the discussion of Cauchy-Euler equations has been dispersed throughout Chapter 4 and not confined to a separate section. Furthermore, nonlinear second order equations solvable by first order techniques is now discussed in a group project at the end of Chapter 4 and no longer is a separate section.

FEATURES FOR THE INSTRUCTOR:

- **Choice of Applications** Because of syllabus constraints, some courses will have little or no time for sections (such as those in Chapters 3, 5, and 9) that exclusively deal with applications. Therefore, we have made the sections in these chapters almost completely independent of each other. To afford the instructor even greater flexibility, we have built in a variety of applications in the exercises for the theoretical sections. In addition, we have included several projects that deal with such applications as aquaculture, designing a solar collector, and cleaning up the Great Lakes.
- **Linear Theory** We have developed the theory of linear differential equations in a gradual manner. In Chapter 4 (Linear Second Order Equations) we present the basic theory for linear second order equations and discuss various techniques for solving these equations. Higher order equations are briefly mentioned in this chapter. A more detailed discussion of linear higher order differential equations is given in Chapter 6 (Higher Order Linear Differential Equations). For a beginning course emphasizing methods of solution, the presentation in Chapter 4 is sufficient and Chapter 6 can be skipped.
- **Proofs** While more pragmatic students may balk at proofs, most instructors regard these justifications as an essential ingredient in a textbook on differential equations. As with any text at this level, certain details in the proofs must be omitted. When this occurs, we flag the instance and refer readers to either a problem in the exercises or to another text. For convenience, the end of a proof is marked by the symbol ◄◄◄.
- **Exercises** An abundance of exercises is graduated in difficulty from straightforward, routine problems to more challenging ones. Deeper theoretical questions, along with applications, usually occur toward the end of exercise sets. Throughout the text we have included problems and projects that require the use of a microcomputer. These exercises are denoted by the symbol
- **Group Projects** At the end of each chapter are group projects relating to the material covered in the chapter. A project might involve a more challenging application, delve deeper into the theory, or introduce more advanced topics in differential equations. Although these projects can be tackled by an individual student, classroom testing has shown that working in groups lends a valuable added dimension to the learning experience. Indeed, it simulates the interactions that take place in the professional arena.
- **Optional Sections** Several sections of the text are labeled (Optional). These sections can be omitted without affecting the logical development of the material. As mentioned earlier, the sections in Chapters 3, 5, and 9 are almost completely independent of each other.
- **Laplace Transforms** We provide a detailed chapter on Laplace transforms since this is a recurring topic for engineers. Our treatment emphasizes discontinuous forcing terms and includes a section on the Dirac delta function.
- **Power Series** Power series solutions is a topic that occasionally causes student anxiety. Possibly, this is due to inadequate preparation in calculus where the more

subtle subject of convergent series is (not infrequently) covered at a rapid pace. Our solution has been to provide a thorough treatment of power series solutions that also includes a review of their properties as well as a discussion of real analytic functions. Unlike many texts, we have provided an extensive section on the *method of Frobenius* (Section 8.6) and two sections on the various methods for finding a second linearly independent solution.

While we have given considerable space to power series solutions, we have also taken great care to allow for the instructor who only wishes to give a basic introduction to the topic. *An introduction to solving differential equations using power series and the method of Frobenius can be accomplished by covering the materials in Section* 8.3, *Section 8.6, and part of Section 8.7.*

- **Partial Differential Equations** An introduction to this topic is provided in Chapter 11, which covers the method of separation of variables, Fourier series, the heat equation, the wave equation, and Laplace's equation. Examples in two and three dimensions are included.

- **Syllabi** As a rough guide in designing a syllabus related to the text, we provide three sample syllabi that can be used for a 15-week course that meets three hours per week: the first emphasizes methods and applications, the second theory and methods, and the third methods and partial differential equations. Chapters 1, 2, and 4 provide the core for any course. The rest of the chapters are, for the most part, independent of each other.

	Methods and Applications	**Theory and Methods**	**Methods and Partial Differential Equations**
Week	Sections	Sections	Sections
1	1.1, 1.2, 1.3	1.1, 1.2, 2.2	1.1, 1.2, 2.2
2	2.2, 2.3	2.3, 2.4	2.3, 2.4
3	2.4, 2.6, 3.1	2.6, 4.2	4.2, 4.3
4	3.2, 3.4	4.3, 4.4, 4.5	4.4, 4.5
5	4.2, 4.3, 4.5	4.6, 4.7	4.6, 4.7
6	4.6, 4.7	4.8, 4.10	4.8, 4.10
7	4.8, 4.10	6.2	5.1, 5.2, 5.3
8	5.1, 5.2, 5.3	6.3, 6.4	7.2, 7.3
9	7.2, 7.3	6.5, 8.3	7.4, 7.5
10	7.4, 7.5	8.4, 8.5	7.6, 8.2
11	7.6, 8.2	8.6	8.3, 8.6
12	8.3, 8.6	8.7, 10.2	8.7, 9.2
13	8.7, 9.2	10.3, 10.4	9.4, 11.2
14	9.3, 9.4	10.5	11.3, 11.4
15	9.6	10.6	11.5, 11.6, 11.7

FEATURES FOR THE STUDENT:

- **Procedure Boxes** Step-by-step procedure boxes shaded in color are provided as a convenient summary and a readily accessible reference. However, the student should be cautioned not to memorize or rely solely on these outlines. Indeed, they are no substitute for a solid understanding of the subject.
- **Worked Examples** A substantial number of worked-out examples illustrate the various difficulties one might encounter in carrying out the procedures. Introductory examples are treated in greater detail. For easy identification, examples are set off by colored disks ●.
- **Numerical Algorithms** Several numerical methods for approximating solutions to differential equations are presented along with program outlines that are easily implemented on a microcomputer. These methods are introduced early in the text so that teachers and/or students can use them for numerical experimentation and for tackling complicated applications.
- **Computer Graphics** Most of the figures in the text were generated on a microcomputer with a laser printer. Computer graphics not only ensure greater accuracy in the illustrations, they demonstrate the use of numerical experimentation in studying the behavior of solutions.
- **Historical Footnotes** Throughout the text historical footnotes are set off by colored daggers (†). These footnotes typically provide the name of the person who developed the technique, the date, and the context of the original research.
- **Motivating Problem** Most chapters begin with a discussion of a problem from physics or engineering that motivates the topic presented and illustrates the methodology.
- **Modern Applications** Aside from standard applications such as Newtonian mechanics, electric circuits, and population models, we have included material on mathematical modeling, aquaculture, pollution, heating and cooling of buildings, combat models, frequency response modeling, and difference equations.
- **Chapter Summary and Review Problems** All of the main chapters contain a set of review problems along with a synopsis of the major concepts presented.

SUPPLEMENTS

Two supplements are available from Addison-Wesley. They are

- *Student's Solutions Manual to Fundamentals of Differential Equations* by John A. Banks, Jessica M. Craig, R. Kent Nagle and Edward B. Saff
- *Instructor's Guide and Answer Book to Fundamentals of Differential Equations* by R. Kent Nagle and Edward B. Saff

An expanded version of this text entitled *Fundamentals of Differential Equations and Boundary Value Problems* contains three additional chapters: Chapter 12, *Boundary Value Problems*; Chapter 13, *Stability of Autonomous Systems*; and Chapter 14, *Existence and Uniqueness Theory*.

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Tampa, Florida

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*Denotes optional sections that can be deleted without compromising the logical flow.