Research on Evolution Equation Compendium

Volume 2

Gaston M. N'Guérékata
Editor

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EVOLUTION EQUATIONS RESEARCH

RESEARCH ON EVOLUTION EQUATIONS COMPENDIUM

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PREFACE

This book presents and discusses new developments in the study of evolution equations. Topics discussed include global attractors for semilinear parabolic equations with delays; exact controllability for the vibrating plate equation in a non smooth domain; weighted pseudo almost automorphic solutions for some partial functional differential equations in fading memory spaces; periodic solutions to the nonlinear parabolic equation; and infinite-time admissibility of observation operators for volterra systems.

Chapter 1 - The author proves the existence and upper semicontinuity of global attractors with respect to parameters for semilinear parabolic equations with general delays. The obtained results recover and extend some known ones about a non-local PDE with the state-selective delay in.

Chapter 2 - In this work, author establish without any geometric condition, the exact controllability in cracked domain of the vibrating plate equation. We combine a square integrable boundary control whose support doesn't meet the crack tips and internal controls with support in small neighbourhoods of crack tips.

Chapter 3 - In this work, author give sufficient conditions for the existence and uniqueness of weighted pseudo almost automorphic solutions for some partial functional differential equations with infinite delay in fading memory spaces. To illustrate our main result, author study the existence of a weighted pseudo almost automorphic solution for some diffusion equation with delay.

Chapter 4 - In this paper, author prove that the equation

$$\partial_t u = \nu \Delta u + F(t, x, u(t, x), \nabla u(t, x), D_x^2 u),$$

where $\nu \neq 0$ is fixed constant, $D_x^2 u = \sum_{i=1}^{n} u_{x_i}^2$, has unique positive solution $u(t, x)$ which is ω-periodic with respect to the time variable t and $u(0, x) \in \dot{B}_{p,q}^{\gamma}(\mathbb{R}^n)$, $\gamma > 0$, $p > 1$, $q \geq 1$ are fixed constants, $x \in \mathbb{R}^n$. The period $\omega > 0$ is arbitrary chosen and fixed.

Chapter 5 - This paper investigates the infinite-time admissibility of observation operators for Volterra systems. New sufficient conditions are obtained by the embedding method.

Chapter 6 - This paper is concerned with the Cauchy problem for one-dimensional compressible Navier-Stokes equations with density-dependent viscosity coefficients. Two cases are considered here: 1) the initial density $\rho_0 \in L^1(\mathbb{R}_+)$; 2) there exists a constant $\overline{\rho} > 0$ such that $\rho_0 - \overline{\rho} \in L^1(\mathbb{R}_+)$. It is proved that the weak solutions exist globally in time and furthermore the asymptotic behaviors of the weak solutions are studied for both cases. The initial vacuum is permitted in this paper.

Chapter 7 - In this paper, author study two semigroups of bounded linear operators to show some special properties which operator semigroups may have.

Chapter 8 - The author prove Abelian-Tauberian theorems for solutions of the one-dimensional heat equation.

Chapter 9 - In this paper, author prove the existence and uniqueness of almost automorphic mild solutions of semilinear fractional differential equations on a complex Banach space $\mathbb{X}$:

$$D_t^{\alpha} u(t) = A(u(t) + D_t^{\alpha-1} F\left(t, u(t), \int_0^t \varphi(s, u(s))ds\right), \ t \in \mathbb{R}$$

where A is a linear operator of sectorial type, $\varphi : \mathbb{R} \times \mathbb{X} \to \mathbb{X}$ and $F : \mathbb{R} \times \mathbb{X} \times \mathbb{X} \to \mathbb{X}$ are two given continuous functions . The results are obtained by means of fixed point methods.

Chapter 10 - In this paper, author consider the second initial boundary value problem for hyperbolic system in infinite cylinders with non-smooth base. Some new results on the unique solvability and the smoothness with respect to variable of generalized of this problem are given.

Chapter 11 - Based on a general model for the Yosida approximation, the solvability of a general class of first-order nonlinear evolution equations is examined. The notion of the relative maximal monotonicity plays a greater role in developing a general framework for the classical Yosida approximation and its characterizations in literature. Furthermore, it seems the obtained results can be generalized to inclusion problems of the form

$$u'(t) + Mu(t) - \omega u(t) \ni b(t),$$
$$u(0) = u_0$$

for almost all $t \in (0, T)$, where T is fixed, $0 < T < \infty$ and $M : X \to 2^X$ is a set-valued mapping.

Chapter 12 - The purpose of this paper is to establish the regularity of generalized solutions of the first initial boundary value problem for hyperbolic systems in infinite cylinders with the non-smooth base. Some results on the solvability of this problem are given.

Chapter 13 - The aim of this work is to find spectral criteria for the existence of almost periodic mild solutions to the some non-autonomous Cauchy problems with inhomogeneous boundary conditions. These problems are a formulation of many evolution equations such as, retarded equations, population equations and boundary control problems. Assuming that the homogeneous part ($f = g = 0$) is 1- periodic and the forcing functions f, g are almost periodic, author give sufficient spectral conditions for the existence of almost periodic solutions to this problem. Under weaker spectral conditions, author discuss Massera problem for the existence of almost periodic mild solutions of the problem. An example of a reaction diffusion equation with periodic coefficients and inhomogeneous boundary conditions is investigated.

Chapter 14 - In this paper, author investigates the existence of S^p-almost periodic solutions to some classes of abstract functional stochastic integro-differential evolution equations in a real separable Hilbert space $\mathbb{H}$ by the means of the well-known Schauder fixed point principle.

Chapter 15 - The author correct an error made in paper (G. M. Mophou, G.M. N'Guérékata and V. Valmorin, Pseudo Almost Automorphic Solutions of a Neutral Functional Fractional Differential Equations" [IJEvE. 4(2) (2010) 249-259]), in in the proof of Theorem 3.5. We have revised the value of the constant M_θ^α and consequently some parts of the proof statement of Theorem 3.5.

Chapter 16 - The author report and correct an error in the paper: Existence of mild solutions of some neutral fractional functional evolution equations with infinite delay, Appl. Math. Computation, 216 (2010), 61-69.

Chapter 17 - In this paper, author prove by means of adapted Carleman inequalities that the null controllability problem for a linear heat equation with Fourier boundary condition and constraint on the control holds. Then author apply the results to detect the pollution in a problem of pollution governed by a semilinear parabolic equation with Fourier boundary condition. The information on the pollution is obtained by means discriminating sentinels introduced by J.L. Lions in.

Chapter 18 - In this paper, author give a thorough study of a mathematical model using differential equations in ryanodine-sensitive calcium channel dynamics in cardiomyocytes. This model consisting of three ordinary differential equations has a unique steady state to which every trajectory is convergent if all parameters are positive. All properties of the dynamical system generated by the system of equations in $\mathbb{R}^3$ are biologically meaningful.

Chapter 19 - Cancellous bone can be regarded as a lattice of asymptotically small rods and plates. In this paper, a thin plate of an orthotropic elastic material with a thickness $\varepsilon << 1$ is considered. This thickness leads to asymptotic expansions of the displacement, stress and strain tensors, and of the temporal change in the volumetric fraction of solid bone in the bone matrix. A rate equation describing the change in the volumetric fraction with respect to time governs the remodeling process of bone deposition and reabsorption. It has linear and quadratic terms of strain tensors and thus, its own asymptotic expansion. This expansion's leading term is used in some simple numerical simulations.

Chapter 20 - Consider an evolution family $\mathcal{U} = (U(t,s))_{t\geq s\geq 0}$ on a half-line $\mathbb{R}_+$ and an integral equation $u(t) = U(t,s)u(s) + \int_s^t U(t,\xi)f(\xi)d\xi$. Using the characterization of the exponential dichotomy of the evolution family by the solvability of this integral equation in admissible function spaces author study the robustness of the exponential dichotomy of evolution families on a half-line under perturbations belonging to such admissible function spaces.

Chapter 21 - In this paper, author are concerned with the relationship between the behavior of solutions of continuous dynamical systems that are restricted to a discrete time scale and that of the original solutions. We generalize a classical result of Fink [5, Theorem 9.7] by extending the theory of almost periodic sequence defined on discrete time $\{t_n\}_{n\in\mathbb{Z}}$ which may not have subgroup structure. We also derive a necessary and surfficient condition for the boundedness of solution based on the discrete observations. Some applications to the time-invariant systems and the evolution equations are given.

Chapter 22 - Psoriasis is a common T-Cell mediated inflammatory chronic skin disease.

The reddish, scaly plaque formed in Psoriasis is characterized by hyperproliferation of epidermal Keratinocytes. The immune system play an important role in the pathogenesis of Psoriasis. Clinical researches show both genetic and environmental triggers are responsible for the development of Psoriasis. Also Keratinocytes together with excessive nitric oxide are one causal to the Psoriatic lesions. Here author consider a mathematical model for the disease Psoriasis, consists of a set of differential equations involving the T-Lymphocytes cells, Dendritic Cells/Macrophages and the epidermal Keratinocytes. Cell biological research on Psoriasis so far established that suppression of epidermal T-cell density inhibits Psoriatic pathogenesis. We implement such T-cell suppression in our mathematical model. Also author observe the dynamical behavior of the system if the suppression can be made on Dendritic cells. Our analytical and numerical consequences represent that, on account of control effect, drug attempt impedes the crossing point, wedged sandwiched between T-Cells and Dendritic Cells. As a matter of fact, author would resemble to recognize the impact of Control between T-Cells and Dendritic Cells on this representation of the dynamical process. Concomitantly, author wish to put a ceiling on provoking skin disorder, Psoriasis, through applying optimally control (drug) remedial approach, by restraining the interaction between T-Cells and Dendritic Cells. Our comparative analysis shows that a suppressed epidermal T-cell and DCs density drastically reduces Keratinocytes densities, which signifies an eventual inhibition of Psoriatic pathogenesis.

Chapter 23 - The author study in this paper the anisotropic nonlinear boundary value problem
$-\sum_{i=1}^{N}\frac{\partial}{\partial x_i}a_i\left(x,\frac{\partial u}{\partial x_i}\right) = \mu$ in Ω, $u = 0$ on $\partial\Omega$, where Ω is a smooth bounded open domain in $\mathbb{R}^N$, $N \geq 3$ and μ a bounded Radon measure. We prove the existence of a weak energy solution for this anisotropic nonlinear elliptic problem with different variable exponents, so that, the functional setting involves anisotropic variable exponent Sobolev spaces and Marcinkiewicz spaces.

Chapter 24 - In this chapter, author discuss the existence and uniqueness of square mean pseudo almost periodic solution of a stochastic functional differential equations. Using the theory of semigroup of linear operators author establish existence and uniqueness of a pseudo almost periodic mild solution.

Chapter 25 - In this paper, author investigates the existence and uniqueness of mild solution of a class of nonlocal Cauchy problem for nonlinear fractional integro-differential equations in Banach spaces. New results are given.

Chapter 26 - In this paper, author presents a characterization of admissibility of some pair spaces for some linear differential equations by means of exponential dichotomies, also author give some results in connexion with pseudo almost automorphic coefficients and the exponential trichotomy.

Chapter 27 - In this paper, author obtain the problems of boundedness properties of stochastic set solutions of stochastic set control differential equations (SSCDEs) with selectors, for example, or different kinds of controls: any admissible controls, feedbacks and contraction feedbacks with selectors.

Chapter 28 - In this paper, author studies the problem

$$\frac{\partial u}{\partial t} - \operatorname{div}(a(x,t,u,Du)) + H(x,t,u,Du) = f \quad \text{in } \Omega\times]0,T[,$$

$$u(x,0) = u_0 \quad in \ \Omega$$

$$u = 0 \ \ in \ \ \partial\Omega\times]0,T[$$

in the framework of weighted Sobolev space. The main contribution of the author's work is to prove the existence of a renormalized solution without the sign condition and the coercivity condition on $H(x,t,u,Du)$, the critical growth condition on H is with respect to Du and no growth with respect to u. The second term f belongs to $L^1(Q)$ and $u_0 \in L^1(\Omega)$.

CONTENTS

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Chapter 1

GLOBAL ATTRACTORS FOR SEMILINEAR PARABOLIC EQUATIONS WITH DELAYS

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Abstract

We prove the existence and upper semicontinuity of global attractors with respect to parameters for semilinear parabolic equations with general delays. The obtained results recover and extend some known ones about a non-local PDE with the state-selective delay in [10].

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Keywords and Phrases: PDE with delay, weak solution, global attractor, Galerkin method, upper semicontinuity

1. Introduction

The understanding of the asymptotic behavior of dynamical systems is one of the most important problems of modern mathematical physics and biology. One way to treat this problem for a system having some dissipative properties is to analyse the existence and structure of its global attractor. The existence of the global attractor has been derived for a large class of PDEs and ODEs with delays (see e.g. [2, 3, 11, 15] and references therein). However, as far as we know, not many papers have been published dealing with the existence of attractors for PDEs with delays.

PDEs with delays are often considered in the model such as maturation time for population dynamics in mathematical biology and other fields. Such equations are natural more

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difficult since they are infinite dimensional both in time and space variables. We refer to the monograph [16] for a theory of PDEs with delays. Recently, the model of PDEs with state-dependent delay has attracted the attention of many researchers [6–10].

In this paper, motivated by [10], we study the following semilinear parabolic equation with general delay:

$$\begin{aligned} \frac{\partial}{\partial t}u(t,x) + Au(t,x) + f(u(t,x)) &= F(u_t)(x) + g(x), \quad x \in \Omega, \quad t > 0, \\ u(0+,x) &= u^0(x), \quad x \in \Omega, \\ u(\theta,x) &= \varphi(\theta,x), \quad \theta \in (-r,0), \quad x \in \Omega. \end{aligned} \tag{1.1}$$

Here Ω is a bounded domain in $\mathbb{R}^N$ and other symbols satisfy the following conditions:

$(H1)$ A is a densely-defined self-adjoint positive linear operator with domain $D(A) \subset L^2(\Omega)$ and with compact resolvent (for example, $-\Delta$ with the homogeneous Dirichlet condition);

$(H2)$ $f : \mathbb{R} \to \mathbb{R}$ is a C^1 function such that

$$C_1|u|^p - C_0 \le f(u)u \le C_2|u|^p + C_0, \quad p \ge 2, \tag{1.2}$$

$$f'(u) \ge -C_3, \quad \text{for all } u \in \mathbb{R}, \tag{1.3}$$

where C_0, C_1, C_2 and C_3 are positive constants;

$(H3)$ $F : L^2(-r,0;L^2(\Omega)) \to L^2(\Omega)$ is locally Lipschitz continuous for the initial data, i.e., for any $M > 0$, there exists $L_{F,M}$ such that for all $u, v \in L^2(-r,0;L^2(\Omega))$ satisfying $(u(0),u), (v(0),v) \in B(0,M)$, one has

$$\|F(u) - F(v)\| \le L_{F,M}\Big(\|u(0) - v(0)\|^2 + \|u - v\|^2_{L^2(-r,0;L^2(\Omega))}\Big)^{\frac{1}{2}}, \tag{1.4}$$

and there exist $k_1, k_2, k_3 \ge 0$, such that for all $\xi \in L^2(-r,0;L^2(\Omega))$, $\eta \in L^2(\Omega)$, one has

$$|\langle F(\xi), \eta\rangle| \le k_1\|\eta\|^2 + k_2 \int_{-r}^{0} \|\xi(\theta)\|^2 d\theta + k_3; \tag{1.5}$$

hereafter we denote the norm and the inner product in $L^2(\Omega)$ by $\|.\|$ and $\langle , \rangle$, respectively;

$(H4)$ The initial data $u^0 \in L^2(\Omega)$, $\varphi \in L^2(-r,0;L^2(\Omega))$ and the external force $g \in L^2(\Omega)$ are given.

Let us now give some comments about the above conditions. The nonlinearity f is assumed satisfying the polynomial type growth and a standard dissipative condition. A typical example of f which satisfies condition $(H2)$ is that

$$f(u) \sim u|u|^{p-2}, \quad p \ge 2.$$

Although the nonlinear term $f(u)$ is allowed to grow faster than u^{p-1} with $p \geq 2$, but condition (1.2) keeps the solution bounded. The delay term contains the general delay, which is a generalization of the distributed state-dependent delay in [10] (see Section 5 for details).

Given $T > 0$ and $u : [-r, T] \to L^2(\Omega)$, for each $t \in [0, T]$ we denote by u_t the function defined on $[-r, 0]$ by the relation $u_t(\theta) = u(t + \theta)$, for all $\theta \in [-r, 0]$. In this paper, following the general lines of the approach in [10], we will construct the dynamical system associated with (1.1) in the space $H \equiv L^2(\Omega) \times L^2(-r, 0; L^2(\Omega))$, so the pair $(u(t), u_t) \in H$ presents the state of the system. Then we prove the existence and upper semicontinuity of the global attractor with respect to the parameters of the dynamical system. In particular, we recover the recent known results in [10] for PDEs with the distributed state-dependent delay.

Since $A : D(A) \to L^2(\Omega)$ is a densely-defined self-adjoint positive linear operator with domain $D(A) \subset L^2(\Omega)$ and with compact resolvent, A has a discrete spectrum that only contains positive eigenvalues $\{\lambda_k\}_{k=1}^{\infty}$ satisfying

$$0 < \lambda_1 \leqslant \lambda_2 \leqslant \cdots, \quad \lambda_k \to \infty, \text{ as } k \to \infty,$$

and the corresponding eigenfunctions $\{e_k\}_{k=1}^{\infty}$ compose an orthonormal basis of the Hilbert space $L^2(\Omega)$ such that

$$(e_j, e_k) = \delta_{jk} \text{ and } Ae_k = \lambda_k e_k, \quad k = 1, 2, \ldots .$$

Hence we can define the fractional power spaces and operators as

$$X^\alpha = D(A^\alpha) = \left\{ u = \sum_{k=1}^{\infty} c_k e_k \in H : \sum_{k=1}^{\infty} c_k^2 \lambda_k^{2\alpha} < \infty \right\},$$

$$A^\alpha u = \sum_{k=1}^{\infty} c_k \lambda_k^\alpha e_k, \text{ where } u = \sum_{k=1}^{\infty} c_k e_k.$$

It is known (see e.g. [2]) that if $\alpha > \beta$ then the space $D(A^\alpha)$ is compactly embedded into $D(A^\beta)$. In particular

$$D(A^{\frac{1}{2}}) \hookrightarrow L^2(\Omega) \hookrightarrow D(A^{-\frac{1}{2}}),$$

where the injections are dense and compact.

Note that by the Riesz Representation Theorem, we have

$$\|F(\xi)\| = \|F(\xi)\|_{op} = \sup_{\|\eta\|=1} |\langle F(\xi), \eta \rangle| \leq k_1 + k_2 \int_{-r}^{0} \|\xi(\theta)\|^2 d\theta + k_3, \tag{1.6}$$

which implies that F is a bounded map from $L^2(-r, 0; L^2(\Omega))$ to $L^2(\Omega)$.

The rest of this paper is organized as follows. In Section 2, we prove the existence and uniqueness of a weak solution to problem (1.1) by using the Galerkin method. The global attractor is discussed in Section 3 (existence of the attractor) and in Section 4 (dependence of the attractor on the parameter). In the last section, we reconsider the non-local PDE model for population dynamics with state-selective delay proposed in [10] as an illustrative example of above results.

2. Existence and Uniqueness of Weak Solutions

Denote

$$\Omega_T = [0,T] \times \Omega,$$
$$W = L^2(0,T;D(A^{\frac{1}{2}})) \cap L^p(\Omega_T),$$
$$W^* = L^2(0,T;D(A^{-\frac{1}{2}})) + L^q(\Omega_T),$$

where q is the conjugate of p, i.e., $\frac{1}{p}+\frac{1}{q}=1$.

Definition 2.1. A function u is called a weak solution of problem (1.1) on an interval $[0,T]$ if $u \in L^2(-r,T;L^2(\Omega)) \cap W$, $\dfrac{\partial u}{\partial t} \in W^*$, $u(0)=u^0$, $u(\theta)=\varphi(\theta)$ for $\theta \in (-r,0)$, and

$$\int_0^T \left(\left\langle \frac{\partial u}{\partial t}, v \right\rangle + \left\langle A^{\frac{1}{2}}u, A^{\frac{1}{2}}v \right\rangle + \langle f(u), v\rangle \right) dt = \int_0^T \left(\langle F(u_t), v\rangle + \langle g, v\rangle \right) dt \tag{2.1}$$

for all test functions $v \in W$.

Theorem 2.2. *Under conditions* $(H1)-(H4)$, *problem* (1.1) *has a unique weak solution* $u(t)$ *on every given interval* $[0,T]$, *which satisfies*

$$u(t) \in C([0,T];L^2(\Omega)). \tag{2.2}$$

Proof. (i) *Existence.* Let $\{e_k\}_{k=1}^{\infty}$ be the orthonormal basis of $L^2(\Omega)$ consisting of all eigenfunctions of A. The subspace of $L^2(\Omega)$ spanned by $e_1, e_1, \cdots, e_n$ will be denoted by V_n. Define the projector $P_n : L^2(\Omega) \to V_n$ as $P_n u = \sum\limits_{j=1}^{n} \langle u, e_j\rangle e_j$, and consider the approximate solutions

$$u^n(t) = \sum_{j=1}^{n} u^{nj}(t) e_j,$$

which satisfy

$$\begin{cases} u^n \in L^2(-r,T;V_n) \cap C^1([0,T];V_n), \\ \left\langle \dfrac{\partial u^n}{\partial t}, e_j \right\rangle + \langle Au^n, e_j\rangle + \langle f(u^n), e_j\rangle = \langle F(u_t^n), e_j\rangle + \langle g, e_j\rangle, \quad \forall\, j = \overline{1,n}, \\ u^n(0) = P_n u^0, \quad u^n(\theta) = P_n\varphi(\theta), \quad \theta \in (-r,0). \end{cases} \tag{2.3}$$

Observe that, for fixed n, equations (2.3) can be rewritten as the following system of ordinary functional differential equations in the unknown $v(t) = v^n(t) = (u^{n1}(t), \ldots, u^{nn}(t))^T$:

$$\dot{v}(t) = \hat{f}(v(t)) + \hat{F}(v_t), \tag{2.4}$$

where the function $\hat{F}$ satisfies properties similar to (1.4), (1.5) if one uses $|.|_{\mathbb{R}^n}$ instead of $\|.\|_{L^2(\Omega)}$. We notice that $\|u^n(t)\|^2_{L^2(\Omega)} = \sum\limits_{j=1}^{n} |u^{nj}(t)|^2 = |v(t)|^2_{\mathbb{R}^n}$.