The Selected Works of Roderick S. C. Wong

Volume 1

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Preface

This collection contains a selection of Roderick Wong's research papers published during the period 1969–2013. The papers, covering topics in Asymptotic Approximations of Integrals, Singular Perturbation Theory, Difference Equations and Riemann–Hilbert Approach, underscore Wong's important contributions to mathematics, in particular, in asymptotics, over the past five decades.

Wong is a world-class mathematician in asymptotic analysis. He started to work on asymptotics almost half a century ago. It was in 1965, in Edmonton, when he began to study asymptotic approximation of integrals under Max Wyman. In the late 1970s, Wong met F. W. J. Olver who became his close friend and mentor. Olver worked on asymptotic expansions for differential equations and the construction of their associated error bounds, while Wong worked on asymptotic evaluation of integrals. Although they were working in different research areas, under the influence of Olver, Wong began to work on asymptotics from different angles. In the late 1970s, he went on to work on error bounds for integrals, and in 1990, before he returned to Hong Kong, he began to investigate difference equations. Wong started to study singular perturbation theory after he returned to Hong Kong in early 1994 and in the last 10 years, he started to work on the Riemann–Hilbert Method.

Wong's work has profound impact on the development of asymptotics. He has opened several new directions, for example, the distributional method. He has also presented a very innovative approach in tackling difference equations, a well-known difficult problem, heralding new progress in the area. Though heavily engaged in administrative duties, Wong continues to supervise students, mentor young scholars, undertake research, publish papers, write and edit books. He is the author of over 140 papers and 3 books, and is an editor of 5 books. His book, *Asymptotic Approximations of Integrals*, published in 1989 by Academic Press, and reprinted in 2001 by the Society for Industrial and Applied Mathematics (SIAM), best illustrates his contribution to the development of asymptotics. It has become a valuable and important reference among applied mathematicians, engineers and statisticians. Wong launched the journal, *Analysis and Applications*, in 2000 which plays an instrumental role in encouraging the development of new techniques in mathematical analysis and the publication of high quality original papers in the area. The journal ranks 14 out of 299 with a 5-year impact factor in the top 10%, in the 2013 SCI ranking under the subject category of Mathematics.

Wong's contributions to mathematics extend far beyond his work in asymptotics. The growth of the Department of Mathematics in CityU and the advancement of mathematics in Hong Kong owe much to his foresight, dedication and energy. In early 1994, Wong joined City University of Hong Kong as Professor of Mathematics and Head of the newly formed Department of Mathematics. He has successfully built the Department as one of the best in Hong Kong by bringing in distinguished faculty members and highly active and promising young researchers from around the world, raising its research and academic profiles. According to the Academic Ranking of World Universities in Mathematics released by Shanghai Jiaotong University in 2014, CityU ranks 28 among the top 200 universities. The most recent QS World University Rankings by Subject 2015 shows that the Department of Mathematics of CityU ranks 29^{th} in the world. In 1995, Wong established the Liu Bie Ju Centre for Mathematical Sciences to promote the development of applied mathematical sciences through research, international conferences, distinguished lectures and workshops. In a further move to recognize and promote the importance of applied mathematics, Wong set up the William Benter Prize in Applied Mathematics in 2010, which is now highly regarded internationally.

Wong's work in asymptotics demonstrates great depth and breadth, making important and lasting contributions to analysis and applied mathematics. This collection of 63 of his papers will provide a comprehensive and valuable literature of asymptotics which is conducive to the further development of research in the area. The following is a brief description of his contributions in asymptotic analysis.

Asymptotic Approximations of Integrals

Wong's work in asymptotics started with asymptotic approximations of integrals. His book on the subject, published in 1989 and reprinted in 2001, is still being widely used by researchers in and beyond the field. A major contribution that Wong and his collaborator J. P. McClure have made in this area is their distributional method; a unique feature of this method is that it naturally leads to explicit error terms associated with the asymptotic expansions. Results based on this method can be found in [7], [10] and [13]*. Other related papers include [8], [11] and [12]. It is interesting to note that recently, based on this method, E. A. Galapon and K. M. Martinez (*Proc. Roy. Soc. London A* **470**(2014) 20130529) obtained an exactification of the Poincaŕe asymptotic expansion of the Hankel transform, and found spectacular accuracy in the optically truncated asymptotic series.

Another contribution that Wong and his collaborators have made in this area is the derivation of asymptotic expansions of various orthogonal polynomials that do not satisfy a second-order differential equation. Some of these results can be found in the publications [31], [34], [40], [47] and [61]. These expansions are uniformly

*Numbered references without letters refer to the papers included in this book. For a complete list, see page 1487.

valid in intervals containing critical values. For instance, in [A34]†, Bo and Wong obtained an infinite asymptotic expansion for the Charlier polynomials $C_n^{(a)}(\beta n)$, which holds uniformly for $\beta \in [\varepsilon, M]$, where $0 < \varepsilon < M < \infty$ and ε, M are two arbitrary numbers. This result is far more global than an earlier one given by W. M. Y. Goh (1998), who divided the positive real line into seven intervals and in each interval, gave a different asymptotic formula. Furthermore, these intervals do not overlap to cover the whole interval $[\varepsilon, M]$. Moreover, Wong and his collaborators [31], [40], [47] and [A38] have also derived asymptotic formulas for the zeros of these non-classical orthogonal polynomials.

Also, Wong and his associates have obtained numerical bounds for the errors associated with the uniform asymptotic expansions of some of the orthogonal polynomials [18], [A36], [A46] and [A71]. For example, in [18], Frenzen and Wong gave a numerical bound for the error term associated with the expansion of the Jacobi polynomials. As applications of these results, Wong and his collaborators settled a long-standing conjecture of Szegö (1926) concerning the monotonicity of Lebesgue constants for Legendre series [21], and a problem of R. A. Askey (1975) on the relative extrema of the Jacobi polynomials [A31] and [A33].

Singular Perturbation Theory

Singular perturbation theory is another sub-area in asymptotics where Wong has made important contributions.

Wong's work in singular perturbation theory is mainly in using rigorous analysis to justify some formal results in the literature. In doing so, he made new observations and corrected some known results, involving both ordinary and partial differential equations. For instance, in a series of papers, he and his student H.-P. Yang constructed a uniform asymptotic solution with an error estimate to a singularly perturbed two-point boundary value problem, where the coefficient function of the first-order derivative term has a simple zero. The solution involves parabolic cylindrical functions, and by analyzing the solution they deduced a necessary and sufficient condition for the Ackerberg–O'Malley resonance; see [43].

With another student X.-H. Jiang, Wong [36] established the existence of a unique bounded solution $v(x,t)$ for all time t to a nonlinear Klein–Gordon equation on an infinite interval $-\infty < x < \infty$ with oscillatory initial data, and provided an asymptotic approximation to $v(x,t)$ with an error $O(\varepsilon^2)$ for all $0 < t < O(\varepsilon^{-2})$.

The shooting method that he and his student C.-H. Ou used is particularly effective. It was used not only to successfully demonstrate the exponentially small error estimates in some two-point boundary-value problems involving nonlinear second-order differential equations, but was also used to count the number of shock waves and spikes in the interval of concern. The relevant results are given in [45], [46], [52] and [53].

†Numbered references prefixed by letter A refer to other publications not included in this book. For a complete list, see page 1487.

Difference Equations

Wong began his research on the asymptotics of second-order linear difference equations in the 1990s. His goal was to develop a theory similar to what F. W. J. Olver had done for second-order linear dfferential equations. In [26] and [27], Wong and his student H. Li studied the case of infinity being a regular or irregular singularity of the difference equation, and they found a much easier way to establish the asymptotic results than what was available in the literature at that time.

Later in the 2000s, Wong and his other student Z. Wang developed a turning-point theory for difference equations. First, they derived the Airy-type expansion for the Bessel function $J_\nu(\nu a)$ directly from a three-term recurrence relation; the expansion holds, as $n \to \infty$, uniformly for $0 \leq a < \infty$. According to the asymptotic expansions of the coefficient functions, they classified the transition points of the linear difference equations into three cases. The first case was studied in [44], where two linearly independent, uniformly valid, asymptotic solutions were obtained in terms of Airy functions and their derivatives. In [49], they considered the third case and developed the Bessel-type expansions for a transition point. The problem of a missing case, namely, the second case when a turning point occurs at the origin, was resolved much later by his two students L.-H. Cao and Y.-T. Li (*Anal. Appl. (Singap.)*, 2014).

The latest piece of work Wong did with his student X.-S. Wang [60] was to show how to derive asymptotic formulas for a given orthogonal polynomial directly from its three-term recurrence relation; that is, how to find the two coefficients connecting the given polynomial with two linearly independent asymptotic solutions of the associated three-term recurrence relation.

Riemann–Hilbert Approach

Near the end of the last century, a new asymptotic method was introduced by P. Deift and X. Zhou (1993), namely, the Riemann–Hilbert (RH) approach. The Deift–Zhou method was originally used to find the large time asymptotics of the solution to the MKdV equation. Almost at the same time, A. S. Fokas, A. R. Its and A. V. Kitaev (1992) observed that orthogonal polynomials can be given as one of the entries of the solution to a matrix-valued RH problem. As a result of this important connection, Deift, Zhou and their associates successfully applied the RH approach to study the asymptotics of orthogonal polynomials and obtained some significant results. Nowadays, this powerful method has resolved many asymptotic problems that classical methods failed to handle and has exerted far-reaching influence on the development of approximation theory and mathematical physics.

The starting point of Wong's research in this area originates from the observation that most results obtained by using the RH approach is local in nature. Thus, the asymptotics are given in terms of different (elementary or special) functions in several different regions in the complex plane. These various representations are sometimes unnecessary in the sense that the validity of the result in one region (usually in terms of special functions) can be extended to other regions. Using a modification of the RH approach, Wong and his collaborators succeeded in obtaining asymptotics of orthogonal polynomials that hold in much more global regions. The work in this area includes [50], [51] and [57] for continuous orthogonal polynomials and [58], [59] and [62] for discrete orthogonal polynomials.

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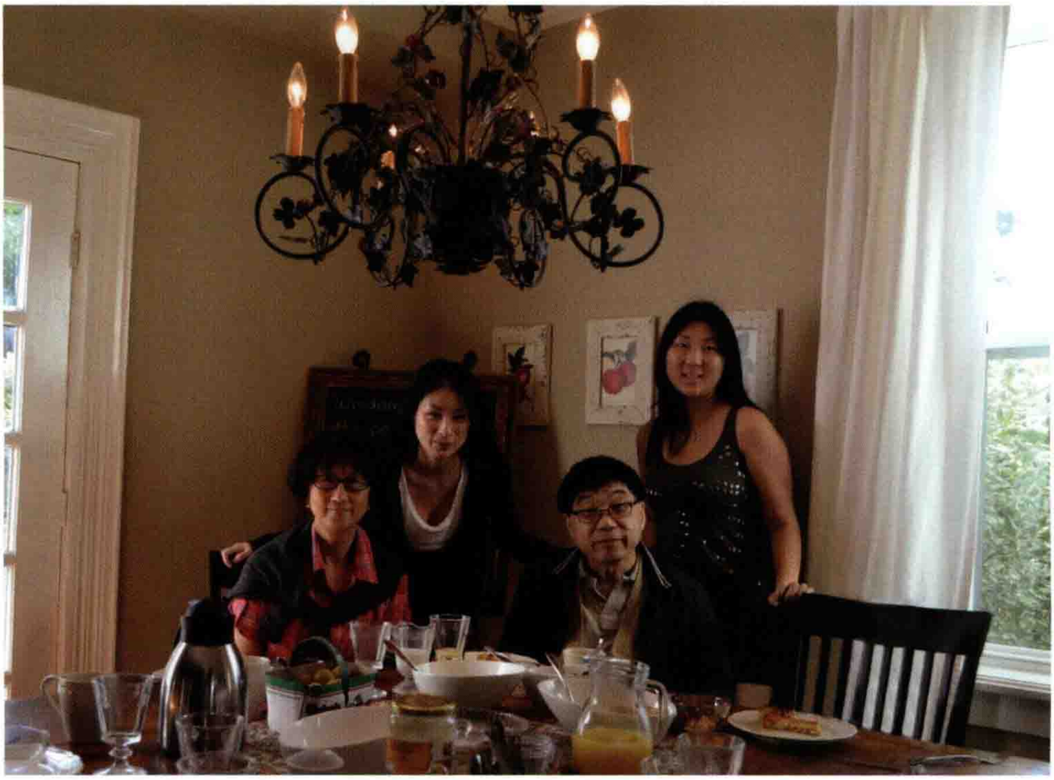

Professor Wong and his family. From left: Edwina, Letitia, Roderick and Priscilla.

Granddaughters of Professor Wong. From left: Alexandra and Audrey.

Professor Wong and the editors of *The Selected Works of Roderick S. C. Wong*. From left: Professor Hui-Hui Dai, Professor Ding-Xuan Zhou, Professor Roderick Wong, Professor Tong Yang and Dr Dan Dai.

Professor Wong and his students at a seminar in Wuhan, China. From left: Dr Xiang-Sheng Wang, Dr Dan Dai, Professor Roderick Wong, Professor Chun-Hua Ou, Dr Yu-Tian Li and Professor Zhen Wang.

Professor Wong received the *Chevalier dans l'Ordre National de la Legion d'Honneur* for his research in Mathematics, his academic standing, achievements and outstanding contributions to the furthering of links between the French and Hong Kong mathematical communities in 2004. From left: Mr Serge Mostura, the then Consul General of France in Hong Kong and Macau, and Professor Wong.