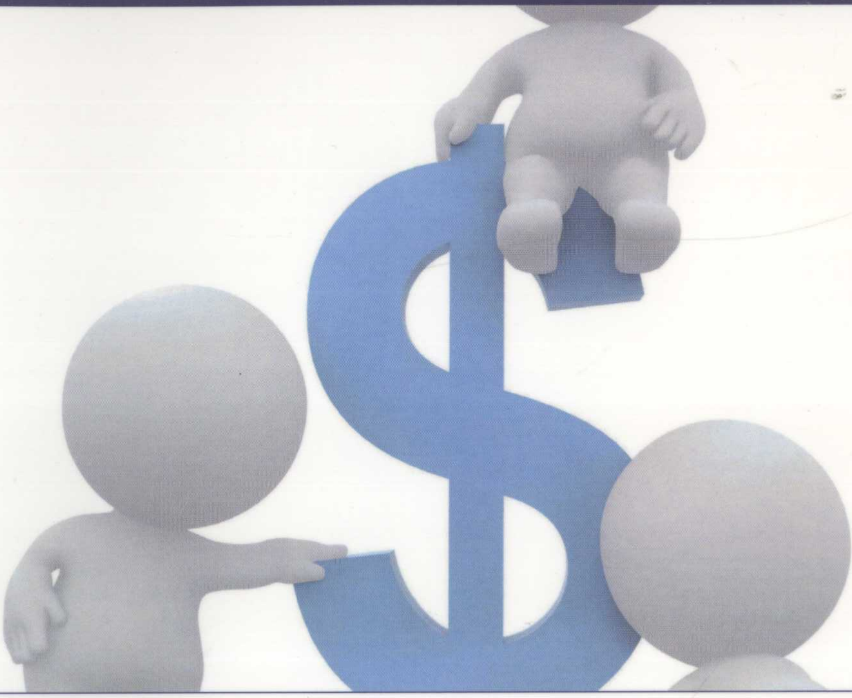

Yang Tan

Optimal Deteriorating Inventory Control and Price Theory

Methodologies and Applications

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Preface

The 21st Century Engineering Grand Challenges, identified by National Academy of Engineering, requires innovative approaches to effectively use and manage finite resources. This will positively impact the sustainability, health, security and living of different species. Of particular importance is the reduction and management of various wastes generated in the world. This book addresses this challenge by optimally controlling the deteriorating inventories and revenue management sources. Our efforts could be applied and yield positive results in a variety of sectors that stock deteriorating products. Among them are: fruits, vegetables, drugs, meat, foodstuffs, blood, perfumes, alcohol, gasoline, radioactive substances, photographic films, and electronic components. Tina Mather delineated this urgency stating that: "In a nation where millions go hungry, some of the food supermarkets throw out could feed people in need." The cost impact of waste due to deterioration is massive. According to a study by the California Integrated Waste Management Board, food products make up 63% of a supermarket's disposed waste stream. That is approximately 3,000 pounds thrown away per employee every year. Andrew Webb (2006) stated that the $1.7 billion apple industry in the U.S. loses as much as $300 million every year due to deterioration.

The current situation calls for an optimal inventory control strategy for deteriorating products. There is a severe lack of solid and comprehensive research in this area. State of the art research mostly deals with non-deteriorating products. The implicit assumption in conventional inventory models is that the stored products maintain the same utility forever, i.e., they can be stored for an infinite period of time without losing their value or characteristics. However, generally speaking, almost all products experience some sort of deterioration over time. Some products have very small deterioration rates, and henceforth the effect of such deterioration can be neglected. Some products may be subject to significant rates of deterioration. Fruits, vegetables, drugs, alcohol and radioactive materials are examples that can experience significant deterioration during storage. Therefore the effect of deterioration must be explicitly taken into account in developing inventory models for such products. Also, it can be argued that the control of deteriorating products is much more important than that of non-deteriorating products due to their loss of value over time, and how these products impact the sustainability, health, security and living of the human race. Whereas for non-deteriorating products the only cost of unsold inventory is the cost for holding inventory, for deteriorating products the unsold inventory not only incurs inventory holding cost but also the cost of spoilage. In light of this, this book focuses on developing and implementing tools to effectively manage deteriorating inventory to minimize the cost of waste due to deterioration. Our approach incorporates various real-world conditions such as periodic review, time-varying deterioration rate, waiting-time-dependent partial backlogging, time-dependent customer demand, and stochastic customer demand. Our work is a stepping stone that will create the comprehensive foundation for further research in the areas of controlling deteriorating inventory and price decision-making, adding a new branch to existing theory.

The results of this book have the potential to positively influence industrial engineering and management science curricula related to production and inventory control, large scale optimization, optimization modeling, pricing science and engineering, and supply chain management. Furthermore the work could enhance student learning by providing practical examples and by development of case-studies on the design of control systems for deteriorating inventory.

Yang Tan, Ph.D.

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CHAPTER 1

Introduction

This chapter first discusses the fundamental qustions in today's inventory management world. Then we introduce two basic types of inventory models: the Economic Order Quantity (EOQ) mode and the Newsboy model. The first is the simplest model of deterministic inventory control issues which arise when there are repeated cycles. The stocks are built up and drawn down in a deterministic mannaer. The second is the simplest model of stochastic inventory control issues, which discuss the unpredictable variability. Finally, the inventory control for deteriorating products is presented briefly.

1.1 BACKGROUND

Nowadays, for most successful, well-organized businesses, inventory control systems play a critical role in ensuring that adequate inventories are on hand to satisfy their customer demand. In general, the inventories can be classified into the following four categories (Nahmias, 2001):

(1) Raw materials. The raw materials are the resources required in the production or processing activity of a firm.

(2) Components. The components correspond to items that have not yet reached completion in the production process. Sometimes components are referred to as subassemblies.

(3) Work-in-process. This is the inventory either waiting in the system for processing or being processed. Work-in-process inventories include component inventories and may include some raw materials inventories as well.

(4) Finished goods. The finished goods are also known as end items, which are the final products of the production process.

As inventories are expensive and need careful control, a fundamental question arises: Why do organizations hold inventories? Generally speaking, the main reason why inventories are held is to provide a buffer between uncertain supply and demand. For example, if a customer places an order and there are no items available immediately, it is very likely that the customer will go somewhere else and may never return. So the uncertainty of external demand is the most important reason to hold inventories. There are also some other reasons. For instance, if the fixed setup/ordering costs are high, it would be economical to produce/order a relatively large quantity and store them for future use. Also, sometimes if the price of a product increases over a short period of time, then it will be more economical to buy a large amount of the product at current price and put into storage than to pay a higher price in future.

Since it is necessary to hold some inventories on hand, then it is essential to manage inventories economically. Inventory control consists of all the activities and procedures used to ensure the right amount of products is held in stock (Waters, 1992). A frequent objective of inventory control is to provide a moderate amount of inventories at minimum cost, so inventory control often relies on a tradeoff between conflicting costs. The relevant costs that are considered in most inventory systems are as follows (Nahmias, 2001).

(1) Fixed Order Cost. It is incurred independent of the size of the order as long as the order quantity is not zero.

(2) Unit Purchasing Cost. It is incurred on a per-unit basis.

(3) Inventory Holding Cost. It is also known as the inventory carrying cost, which is the sum of all costs that are proportional to the amount of inventory physically on hand. Some of the components of the holding cost include (i) Cost of providing the physical space to store the items. (ii) Taxes and insurance. (iii) Opportunity cost of alternative investment. The inventory holding cost ($/unit/year) is usually measured by the product of unit purchasing cost and annual interest rate. The interest rate is an aggregated term comprised of some components like cost of capital, taxes and insurance, and cost of storage.

(4) Backlogging Cost. It includes whatever bookkeeping and/or delay costs that might be involved and "loss-of-goodwill" cost. (Orders that cannot be filled immediately are held on the books until the next shipment arrives.)

(5) Penalty Cost for Lost Sales. It includes the lost profit that would have been made from the sale and "loss-of-goodwill" cost.

As one can see, the "loss-of-goodwill" cost is included in either backlogging or penalty cost, and is a measure of customer unsatisfaction. Estimation of the "loss-of-goodwill" cost can be very difficult in practice.

By using cost minimization as an optimization criterion, the following two questions which reflect the fundamental problem of inventory control can be answered.

(1) When should an order be placed?

(2) How many should be ordered?

Regarding the time of ordering, there are two distinct inventory systems with different timing of replenishment: a periodic review system and continuous review system. A periodic review system allows inventory levels to be checked at discrete times periodically, and the order size is subject to change according to the variation in demand in each period. This system is often used in supermarkets, where stocks are reviewed at the end of each day and any sold units are replaced. A continuous review system allows the level of inventory to be monitored continuously and an order is placed whenever the inventory decreases to a specified level. The time between two consecutive orders is subject to change according to the variation in demand over time. Minimizing the total costs of the inventory system can yield the optimal quantity of ordering.

1.2 INVENTORY

The control and maintenance of inventory is a problem common to all organizations in any sector of the economy. The problems of inventory do not confine themselves to profit-making institutions. Inventory problem has been encountered by each country, but it was not till the 20th century that analytical skills were developed to study it. The first impetus for analysis came from manufacturing industry. The same types of problems are encountered by social and nonprofit institutions. Inventory is common to manufacturers, wholesalers, universities, local governments and retailers. Indeed, inventory is also relevant to the family unit related to food, clothing, medicines and so forth. On an aggregate national basis, the total investment in inventory represents a sizable portion of the gross national product. In theory, inventory is an area of organizational operation that is well developed. In practice, however, it is backward. This gap will be narrowed as educational institutions integrate materials management into their course structures.

Historically, inventory control has often meant too much inventory and too little control or too little inventory and too much control. There can be severe penalties for excesses in either direction. Inventory problems have proliferated as technological progress and have increased the organizations' ability to produce products in greater quantities and with multiple design variations.

The relative significance of inventory control to an organization can be gauged by the total investment in inventory and the magnitude of the material costs for all products. The overall investment in inventory can be ascertained by reviewing the balance sheet of an organization. If the investment in inventory is a large percentage of current assets or total assets, major emphasis should be placed on inventory control. Likewise, if material costs are a large percentage of total product costs, inventory management is critical.

Inventory tie up money. For many organizations, the investment in inventories represents a sizable summation. A review of American industry balance sheets reveals that many businesses have 10-30% of their total assets tied up in inventory. Poor control of inventory can create a negative cash flow, tie up large amounts of capital, limit the expansion of an organization due to lack of capital, and reduce the return on investment by broadening the investment base. The pressure for capital and the effective utilization of resources has made decision makers more aware of its significance. Cash invested in inventories could be used elsewhere for profit making, debt servicing, or dividend distribution. Inventory control must determine how many to order and when to order each product purchased by the organization.

Inventory is material held for future use in an idle or unproductive state awaiting its intended purposes. It may consist of supplies, raw materials, in-process goods, and finished products. Supplies are inventory items consumed in the normal functioning of an organization that are not a part of the final goods. Typical supplies are pencils, chapters, and facility maintenance items. Raw materials are the inputs into the production process that will be modified or transformed into finished products. Typical raw materials for a furniture manufacturer are lumber, stain, glue and so forth. In-process goods are partially completed final products that are still in the production process. They represent both the accumulation of partially completed work and the queue of material awaiting further work. Finished goods are the final products that are available for sale and distribution.

Inventory, plant capacity, and labor are interchangeable factors of production within certain limits. Different mixes of the factors of production make It possible for demand to be independent of procurement, production and distribution. The assignment of inventory to any of these categories is dependent on the entity under study. This is because one entity's finished products may be another entity's raw material. Inventories are a kind of lubrication for the supply-production-distribution system that protects it from excessive friction. Inventories isolate one part of the system from the next to allow each to work independently, absorb the shock of forecast errors, and permit the effective utilization of resources when demand undulations are experienced. Different types of organizations have different inventory problems. Inventory exists because organizations cannot function without it. It exists because supply and the demand differ in the rates at which they respectively provide and require stock. There is an optimal level of investment in any asset, whether it is cash, plant, equipment, or inventory. Having excessive assets can impair income just as much as having too little assets. With inventories, too much can result in unnecessary holding costs and too little can result in lost sales or disrupted production.

1.3 DETERMINISTIC AND STOCHASTIC MODELS

The inventory control models in the literature can be classified according to the following criteria (Ravindran, 2008).

(1) Stocking location: Single stocking location (single location models) and more than one stocking location (multi-echelon inventory models).

(2) Supply process lead times: Deterministic lead time and stochastic lead time.

(3) Demand: Deterministic demand and stochastic demand. The stochastic demand can be stationary or nonstationary. Stationary stochastic demand means all demand parameters are constant over time. If the parameters change over time, then the demand is said to be nonstationary.

(4) Capacities: Uncapacitated inventory models and capacitated inventory models.

(5) Number of items: Single product inventory models and multiple product inventory models.

(6) Sourcing options: Single sourcing and multiple sourcing.

In this book, we are considering single stocking location, zero lead time, deterministic/stochastic demand, uncapacitated, single product, and single sourcing inventory models. One commonly used criterion is demand pattern. Based on this, the inventory control models can be classified as deterministic inventory models and stochastic inventory models.

1.3.1 Deterministic Inventory Models

Deterministic inventory models assume that the demand is fixed and known. Two most famous models of this kind are presented in the following section.

1.3.1.1 The Economic Order Quantity (EOQ) Model

The EOQ model lays the foundation for all inventory models. It is the most important analysis of inventory control and describes the important trade-off between fixed order cost and holding cost. The first reference to the EOQ model is by Harris (1913) but this model was popularized by Wilson (1934).

In this model, the demand per unit time is assumed to be a known constant R. A constant fixed cost is incurred whenever an order is placed. An order of quantity Q is placed whenever the on-hand inventory becomes zero and the replenishment time is assumed to be zero. The unit purchasing cost c is constant and known. The on-hand inventory is charged with a constant holding cost h per unit per unit time. Shortages are not allowed. The objective is to determine Q so as to minimize the total average cost. The relationship between order and on-hand inventory can be depicted in Figure 1.1, where T (= Q/R) is called the cycle time. Without loss of generality, it is assumed that the initial inventory is zero.

There are two cost components: ordering cost and inventory holding cost. Since all cycle are identical, it is only necessary to derive these costs in a cycle. The ordering cost consists of a fixed order cost K and purchasing cost cQ. The average inventory in a cycle is Q/2. Therefore, the total cost per unit time, C(Q), is

$$C(Q)=\frac{K+cQ}{T}+\frac{hQ}{2}=\frac{K+cQ}{Q/R}+\frac{hQ}{2}=\frac{KR}{Q}+Rc+\frac{hQ}{2}$$

In C(Q), only ordering quantity Q is a decision variable. The first and second order derivatives of C(Q) are given by