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FLUID MECHANICS for ENGINEERS

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SECOND EDITION

LONDON
BUTTERWORTHS
1964

First Impression 1957
Reprinted and revised 1960
Second edition 1964

Set in Monotype Baskerville type
Printed in Northern Ireland at The Universities Press, Belfast

PREFACE TO THE SECOND EDITION

HAVING received numerous suggestions over the past years, the text of this edition was subsequently improved. The number of problems has been increased and answers were provided; numerous errors were eliminated and the section dealing with momentum was enlarged.

During the years following the appearance of the first edition, the book was used to a fair extent in the United Kingdom and to a larger extent abroad. This is gratifying and the author expresses hope that interest in this edition will be unfailing.

The author wishes to express his appreication to the various users of the first edition who have offered useful suggestions; he also wishes to extend his gratitude to the various reviewers of the first edition whose fair appraisal of the text substantially enhanced its success.

PREFACE

This text was planned primarily to provide in one volume adequate coverage for undergraduates studying for a degree or diploma in Mechanical or Civil Engineering. It is hoped that parts will be useful for Higher National Certificate courses.

Emphasis is laid upon the broad representation of the fundamentals, leaving certain topics not included in the text for the choice of individual teachers. A large number of examples with complete solutions are given and these have been carefully selected to illustrate the preceding theory. In some cases more than one example illustrates the theory and wherever possible simple examples precede the more complex ones.

It is suggested that students in Mechanical Engineering may omit Flow in Open Channels whilst students in Civil Engineering may omit some details of Rotodynamic Machinery and of Compressible Fluid Flow.

The text is subdivided into self-contained chapters each covering the relevant field as completely as it was thought practicable. So far as possible, theory was kept in the neighbourhood of its application. For example, manometers are dealt with in the chapter on Fluid Metering, and moment of momentum and free vortex are treated in the later chapter on Rotodynamic Machinery. For similar reasons, Wing Theory, although an independent field, may be linked with Axial Flow Machinery and may serve as a forerunner to that chapter.

It is anticipated that certain elementary concepts such as density, pressure, force etc., have already been introduced in physics and their detailed discussion is omitted from the text.

The author expresses sincere appreciation to Messrs R. A. Bryant, B. Langevad, C. Sapsford, R. Vallentine, and A. D. Owen of the University of Technology, Sydney, to Messrs J. Palmer and K. Moore of the College of Aeronautics, Cranfield, and Royal Aircraft Establishment, Bedford, respectively, for their critical examination of the manuscript and for their useful suggestions. It is acknowledged that some of the examples and problems were adopted from past examination papers of the N.S.W. University of Technology.

Any suggestions which might improve the text will be welcomed by the author.

Cranfield, Buckinghamshire
March, 1957

CONTENTS

PART 2

PART 3

1

FLUID STATICS

1.1 Introduction

WHEN fluids are at rest the associated problems fall into the category which is generally termed Fluid Statics. These problems are far simpler and fewer than those associated with Fluid Motion. Since no relative movement is experienced between fluid layers any mutual action due to viscosity is non-existent, and calculations may ignore viscosity effects completely; because of this, solutions may be obtained by simple methods without the aid of complex experiments.

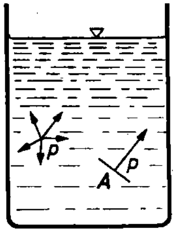

Figure 1.1

The free surface of liquids at rest lies in the gravitational equipotential planes. This is the reason why the surfaces of oceans follow the earth's curvature. For practical purposes the free surface of a liquid in a container may be considered perfectly plane provided the dimensions of the container are small (relative to the diameter of the earth). When we consider a longer channel, however, say 1,000–2,000 ft. long like the modern towing tanks used for model ship research, the curvature must be considered, as the deviation from the straight line may be of the order of, say, 1/10 of an inch.

Pressure, p, in fluids is a scalar quantity, that is a quantity without any direction, so that at any point of a fluid aggregate the pressure may not be represented by a vector. In other words, at any one point the pressure is the same in all directions. Once a surface, A, is specified the pressure acting on the surface produces a force, which is a vector of a magnitude $p \, . \, A$, and direction normal to the surface (*see Fig. 1.1*).

1.2 Fundamentals of Fluid Statics

The fundamental problem of fluid statics is the determination of the distribution of pressure in a homogeneous fluid.

Consider a tank of cross-section S filled with liquid of specific weight w as shown in *Fig. 1.2.* Going down from the free surface each layer of liquid of thickness Δh rests on the next layer, the weight of each layer being $w \,.\, \Delta h S$. It may be seen therefore that the pressure continuously increases with depth by an amount

$$\Delta p = \frac{\text{weight}}{\text{area}} = \frac{w\,\Delta h S}{S} = w \,.\, \Delta h$$

for each distance Δh traversed.

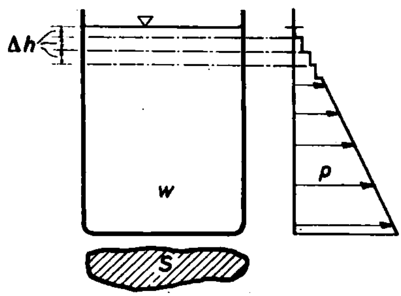

Figure 1.2—Distribution of pressure in a tank

Hence

$$w = \lim \frac{\Delta p}{\Delta h} = \frac{\mathrm{d}p}{\mathrm{d}h} \qquad \ldots.(1.1)$$

Integration yields

$$p = \int w \,\mathrm{d}h \qquad \ldots.(1.2)$$

Equation 1.2 at once brings up the question of compressibility, because in order to integrate the equation a relationship between w and h must be given. Liquids are considered incompressible so that w is independent of h. Therefore for liquids

$$p = w \,.\, h \qquad \ldots.(1.3)$$

which represents a linear relationship between depth and pressure (h being measured from the free surface).

Gaseous fluids on the other hand are compressible and follow the law of the gas equation

$$pv = RT \qquad \text{where } v = \frac{1}{w}$$

so that Eq. 1.1 becomes

$$-\frac{\mathrm{d}p}{\mathrm{d}h}=\frac{p}{RT} \quad \text{or} \quad \frac{\mathrm{d}p}{p}=-\frac{\mathrm{d}h}{RT} \qquad \ldots (1.4)$$

The negative sign is due to p decreasing with increasing h.

Integration of Eq. 1.4 leads to the vertical pressure distribution of the atmosphere. However, in order to integrate the right hand side of Eq. 1.4 a relationship between h and T must be given.

The international standard atmosphere is defined by the assumption that the relationship between the temperature T and height h, measured from sea level, is a linear one given by the lapse rate

$$a=-\frac{\mathrm{d}T}{\mathrm{d}h}=0{\cdot}00198°\mathrm{C} \text{ per ft.}$$

This holds fairly accurately up to 36,000 ft. Integration leads to

$$T=T_0-0{\cdot}00198h$$

Since the standard temperature at sea level is adopted as 15°C, $T_0=273{\cdot}2+15=288{\cdot}2°\mathrm{K}$.

Hence $$T=288{\cdot}2-0{\cdot}00198h.$$

Substituting the lapse rate into Eq. 1.4, one obtains

$$\frac{\mathrm{d}p}{p}=\frac{1}{aR}\cdot\frac{\mathrm{d}T}{T}$$

Integration leads to

$$\log p=\log T^{\frac{1}{aR}}$$

or

$$\frac{p}{p_0}=\left(\frac{T}{T_0}\right)^{\frac{1}{aR}}$$

Since $$1/aR=1/0{\cdot}00198\times 96=5{\cdot}256$$

one finally obtains

$$\frac{p}{p_0}=(1-0{\cdot}00000687h)^{5{\cdot}256} \qquad \ldots (1.5)$$

where p_0 is the pressure at sea level. Similar relation exists for the density change with height. Values of p, based on Eq. 1.5 are given in tables published by the N.A.C.A.†

† National Advisory Committee for Aeronautics, Report 1235/1955.

1.3 Pressure Forces on Submerged Plane Surfaces. Centre of Pressure

Practical application of Eq. 1.3 may be found when pressure forces on surfaces submerged in liquids are calculated. Examples of such surfaces are lock gates and valves. The problem involves the calculation of the total force and its location which is frequently called the centre of pressure.

Consider a flat plate *A–B* of surface area S which may be of arbitrary shape, covering a submerged opening in the side of a reservoir (*see Fig. 1.3*). The plane of the plate makes an angle α with the free liquid surface and at a depth h, taken from the free surface, the pressure $p = wh$. The force acting on an elementary area dS of the plate $dF = dSp = dSwh$.

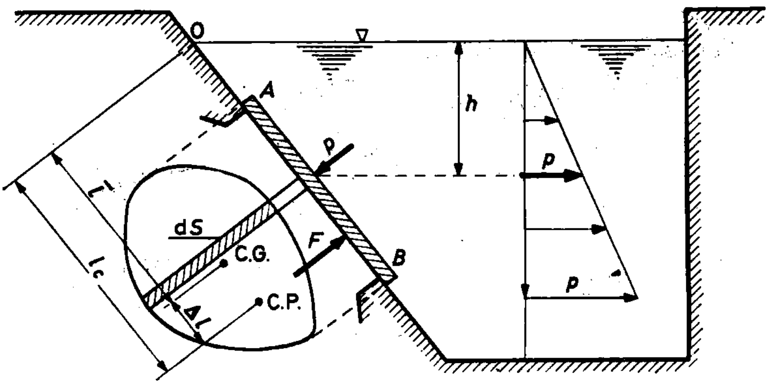

Figure 1.3—Pressures on a submerged plane surface

The total force on the plate may be obtained upon integration

$$F = w \int_0^s h \, dS = w \sin \alpha \int_0^s l \, dS$$

where the expression $\int l \, dS = \bar{l}S = I_I$ is called the first moment of area S about the horizontal axis 0.
Therefore

$$F = I_I w \sin \alpha \qquad \ldots.(1.6)$$

The location of the total force, called the centre of pressure, may be obtained by taking moments of the force elements about the axis 0 and equating the sum of these moments to the moment of the total force about the same axis. Thus

$$\int wh \, dSl = F \,.\, l_c \qquad \ldots.(1.7)$$

where l_c is the distance of the centre of pressure from 0. Since $h = l \,.\, \sin\alpha$, the left hand side of Eq. 1.7 becomes $w \sin\alpha \int^{s} l^2\, dS$. This integral is called the second moment of the area around 0. Denoting this by I_{II} we have

$$F \,.\, l_c = w \sin \alpha I_{II}$$

Substituting for F from Eq. 1.6 one obtains the centre of pressure

$$l_c = \frac{I_{II}}{I_I} = \left.\frac{\text{second moment}}{\text{first moment}}\right\} \begin{array}{l}\text{of area } S\\ \text{about 0.}\end{array} \qquad \ldots .(1.8)$$

The depth of the centre of pressure is given by

$$h_c = l_c \sin \alpha$$

It may be shown that the centre of pressure lies always below the centre of gravity. Denoting the distance between the two by Δl and considering that $I_{II} = I_{C.G} + l^2 S$, where $I_{C.G}$ is the second moment of the area about the horizontal axis through the centre of gravity, then

$$\Delta l = \frac{I_{C.G} + l^2 S}{lS} - l = \frac{I_{C.G}}{l \,.\, S} \qquad \ldots .(1.9)$$

For plane surfaces which are asymmetric about the vertical centre line, the lateral position of the C.P. or C.G. may be obtained by taking moments laterally about a convenient axis.

Example

1.1. A submerged circular opening, cut in the side of a water tank, is provided with a cover plate. The diameter of the opening is 6 ft. and its centre lies 10 ft. below the free surface of the water. Calculate the magnitude and location of the total force acting on the surface, if the angle of inclination of the side is 45°.

Solution.—The first moment of the circular area

$$I_I = \frac{10}{\sin 45} \times \frac{6^2\pi}{4} = 400 \text{ ft.}^3$$

Hence the total force

$$F = 400 \times 62{\cdot}4 \times 0{\cdot}707 = 17650 \text{ lb.}$$

The second moment of the circular area about a diameter is given by $\frac{d^4\pi}{64}$, hence $I_{IIC.G} = \frac{6^4\pi}{64} = 63{\cdot}8 \text{ ft.}^4$

The second moment about 0

$$I_{II} = 63{\cdot}8 + \frac{6^2\pi}{4} \times 14{\cdot}2^2 = 5744$$

Distance of centre of pressure from 0

$$l_c = \frac{5744}{400} = 14{\cdot}36$$

Hence

$$\Delta l_c = 14{\cdot}36 - 14{\cdot}2 = 0{\cdot}16 \text{ ft.}$$

1.4 Pressure Forces on Submerged Curved Surfaces

Consider now a curved surface shown in *Fig. 1.4.* The pressure forces at any point act perpendicularly to the surface and there

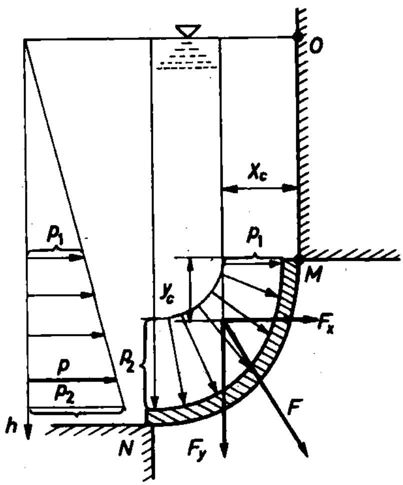

Figure 1.4

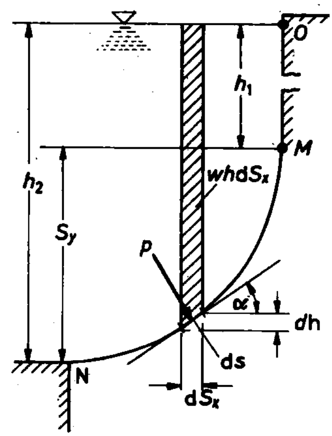

Figure 1.5

Pressures on a submerged curved surface

will be a resultant force F which may be resolved into two components, F_x being the horizontal and F_y the vertical component. These forces may be obtained from the following considerations.

At a point P of the surface an element dS is inclined to the horizontal at an angle α (*see Fig. 1.5*).

The horizontal component of the elementary pressure force p dS (acting on unit width) is p dS . sin α; and since dS sin α = dh the total horizontal force

$$F_x = \int_1^2 p \, dh = w \int_{h_1}^{h_2} h \, . \, dh = \frac{w}{2}(h_2^2 - h_1^2) = wS_y\bar{h} \quad \ldots . (1.10)$$

where

$$\bar{h} = \frac{h_1 + h_2}{2}$$

and the projection of the curved surface onto a vertical plane

$$S_y = h_2 - h_1$$

Since $S_y \bar{h}$ is the first moment of the projected area about the axis 0, the horizontal force is identical with the result of Section 1.2 and the centre of pressure may be found on similar lines. It may be seen that the horizontal force is independent of the shape of the curved surface.

The vertical component of the elementary pressure force acting on dS is p d$S \cos \alpha$ and since d$S \cos \alpha = $ dS_x the total vertical force

$$F_y = \int p \, dS_x = w \int h \, dS_x \qquad \ldots.(1.11)$$

Eq. 1.11 represents the weight W of the vertical liquid column over the curved surface extending to the free liquid surface.

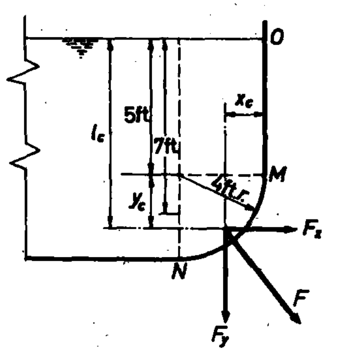

Figure 1.6

Taking moments of the vertical forces about a convenient axis gives the location of F_y. The line of action of F_y must pass through the centre of gravity of the liquid column over M–N. To find distance x_c equate the sum of the moments to $F_y x_c$.

Example

1.2. The dimensioned cross-section of a water tank is shown in *Fig. 1.6.* Calculate the force acting on the segment M–N and find its location. The tank is 25 ft. long.

Solution.—Vertical area $S_y = 4 \times 25 = 100$ ft.^{2}

Horizontal force $F_x = 62{\cdot}4 \times 100 \times 7 = 43700$ lb.

$$I_{C.G} \text{ of vertical surface } = \frac{25 \times 4^3}{12} = 133{\cdot}5 \text{ ft.}^4$$

Hence vertical location of C.P.

$$l_c = 7 + \frac{133{\cdot}5}{7 \times 100} = 7{\cdot}19 \text{ ft.} \quad \therefore \quad v_c = 2{\cdot}19 \text{ ft.}$$

Vertical force

$$F_y = \left[(4 \times 5) + \frac{4^2\pi}{4}\right] 25 \times 62{\cdot}4 = 50800 \text{ lb.}$$

Equilibrium of moments about 0 gives

$$62{\cdot}4 \times 25[(5 \times 4) \times 2 + (\tfrac{1}{4} \times 4^2\pi \times 0{\cdot}576 \times 4)] = 107{,}400 \text{ lb.}$$

thus $$x_c = \frac{107{,}400}{50800} = 2{\cdot}12 \text{ ft.}$$

The total force

$$F = \sqrt{(43700^2 + 50800^2)} = 67800 \text{ lb.}$$

1.5 Buoyancy of Submerged and Floating Bodies

Consider a prismatic body submerged in a fluid. (*see Fig. 1.7*). If the specific weight of the body is w_b its weight is $W = w_b V$, where V is its volume. The buoyant force $F_B = S(p_2 - p_1) = S(h_2 - h_1) \,.\, w$, where w is the specific weight of the liquid. Since $S(h_2 - h_1) = V$ the net force

$$W - F_B = V(w_b - w) \qquad \ldots.(1.12)$$

and is independent of the depth of submergence.

It follows from Eq. 1.12 that if $w_b < w$ the body rises to the surface and if $w_b > w$ the body sinks in the fluid.

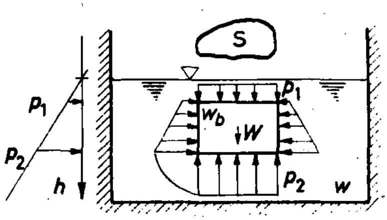

Figure 1.7—Pressures on a submerged body

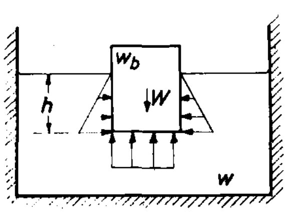

Figure 1.8—Pressures on a floating body

The weight 'lost' by the body is actually equal to the weight of the volume of fluid displaced (*Archimedes' Law*).

The same law applies to floating bodies (*see Fig. 1.8*) which are able to float only if the weight of the fluid displaced is equal to total weight of the body.

When a balloon filled with a gas lighter than air begins to ascend, the net buoyant force is proportional to the difference between the specific weights of air and gas. Since the air density decreases with