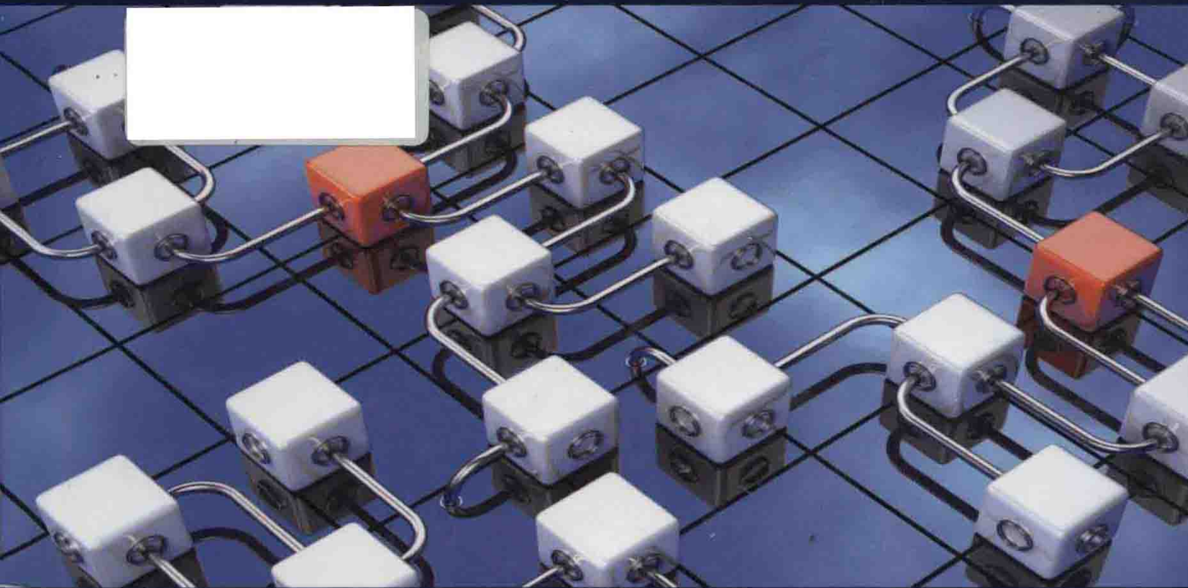
AUTOMATION – CONTROL
AND INDUSTRIAL ENGINEERING SERIES



Hybrid Systems with Constraints

Edited by Jamal Daafouz
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ISTE

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First published 2013 in Great Britain and the United States by ISTE Ltd and John Wiley & Sons, Inc.

ISTE Ltd
27-37 St George's Road
London SW19 4EU
UK

www.iste.co.uk

John Wiley & Sons, Inc.
111 River Street
Hoboken, NJ 07030
USA

www.wiley.com

Library of Congress Control Number: 2013934427

British Library Cataloguing-in-Publication Data
A CIP record for this book is available from the British Library
ISBN: 978-1-84821-527-6

Printed and bound in Great Britain by CPI Group (UK) Ltd., Croydon, Surrey CR0 4YY

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Preface

Nonlinear control systems have undergone tremendous advances in the last two decades at the levels of theory and applications. Among these, a class of particular interest is the one resulting from the interaction of a control system with a system governed by the dynamics of a different nature. This class of systems lies in the hybrid and nonlinear control systems field. In the last decade, the study of such hybrid systems, whose behavior can be mathematically described using a mixture of logic-based switching and difference/differential linear or nonlinear equations, has attracted important research efforts. The fact that many physical systems are controlled or supervised by controllers with such mixed dynamics constitutes a great motivation for such studies. We can cite many applications (such as automotive, networked control systems, energy management and biology) in which analysis and design methods for systems evolving both continuous and discontinuous components are then needed. Furthermore, among many important problems formulated in the context of hybrid systems, switched control systems have been attracting much attention in recent years. Nevertheless, many important mathematical problems remain open. These include analysis and control of hybrid systems with a periodic behavior, control of systems with actuator constraints and hybrid control design with prescribed performance. These open problems are mainly motivated by their practical impact. Hybrid systems with periodic behavior cover an important class of embedded systems. Available approaches are mainly dedicated to specific applications of these devices and today there is a serious lack of rigorous tools for analyzing and synthesizing control algorithms for such systems. To improve their performance, the objective is to go beyond the classical simplified modeling that does not capture the hetrogeneous nature of these systems.

This book deals with control theory and, in particular, discusses the problems of analysis and control design in the context of hybrid dynamical systems. This book is mainly focused on hybrid systems with constraints. Taking into account the constraints in a dynamical system, description has always been a critical issue in control theory. The book provides new tools for stability analysis and control design for hybrid systems with operating constraints and performance specifications. Hence, it is important to underline that there is no book that focuses on constraints for the analysis and control of hybrid systems. This book proposes new approaches for open problems with practical impact. We focus on the presence of constraints in hybrid systems considered as a critical issue in control theory. This includes discontinuities arising from non-smooth impacts, saturations and nested saturations on signals, positivity and interconnection structure, algebraic equations, etc. To provide a coherent panel, the book is structured into eight chapters organized in two main parts related to the kind of systems handled: switched systems (which include Chapters 1–4) and hybrid systems (which include Chapters 5–8). Chapter 6 provides, in particular, a nice overview of recent theoretical results and challenging problems.

We think that this book constitutes an add-in overview of results and techniques with respect to the recent literature. We hope that it will be a useful reference for researchers, practitioners, and graduate students in systems and control theory. We hope that readers will appreciate the open problems discussed in this book and methods that take into account various types of constraints such as positivity constraints (Chapter 1), sector nonlinearity (Chapter 2), algebraic constraints (Chapter 3), persistent excitation constraints (Chapter 4), coordination constraints (Chapter 5), actuator constraints (Chapter 7) and discontinuities issued from impacts (Chapter 8).

Table of Contents

Chapter 1

Positive Systems: Discretization with Positivity and Constraints

In this chapter, we discuss the problem of preservation of two properties pertaining continuous-time systems under discretization, namely the properties of positivity and sparsity. In the first part of the chapter, the action of diagonal Padé transformations is studied together with the preservation of copositive quadratic and copositive linear Lyapunov functions. A variation of the scaling and squaring method is then introduced and shown to be able to preserve such Lyapunov functions and positivity for small sampling times. In the second part, besides positivity, the problem of preservation of the structure (sparseness) of the continuous-time system under discretization is analyzed. The action of the standard forward Euler discretization method is discussed and a new approximation method – *mixed Euler* – ZOH (mE-ZOH) is introduced that preserves copositive Lyapunov functions, the sparseness structure and the positivity property for all sampling times.

1.1. Introduction and statement of the problem

This chapter is devoted to the study of the effects of discretization in the preservation of two properties pertaining linear systems, namely (1) positivity and (2) structure. The first property characterizes systems whose inputs, state

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and outputs take non-negative values in forward time. As part of the more general class of *monotonic* systems [ANG 03], such systems characterize the dynamic behavior of processes frequently encountered in engineering and in social, economic and biological sciences. A few monographs are now available where both the mathematical properties and the application interest of such systems are underlined [BER 94, FAR 00].

The important problem of obtaining reliable discrete-time approximations to a given continuous-time system arises in many circumstances: in simulation issues, in control system design, in certain optimization problems and in model order reduction [ANT 05, FAL 08]. While a complete understanding of this problem exists for linear time-invariant (LTI) systems [WES 01], and some results are available for switched linear systems [ROS 09, SAJ 11], the analogous problems for positive systems are more challenging since discretization methods must preserve not only the stability properties of the original continuous-time system, but also physical properties, such as state positivity. To the best of our knowledge, this is a relatively new problem in the literature, with only a few recent works on this topic [BAU 10]. In particular, we stress the importance of this issue in the framework of switched positive systems, a research field still in its infancy, but with growing importance in telecommunications, biological networks and cloud computing (see [SHO 07, SHO 06, BAR 89, HAR 02]). Generally speaking, we are interested in the evolution of the system:

$$\dot{x}_c(t) = A_{\sigma_c(t)} x_c(t),\ \sigma_c(t) \in \{1, \ldots, m\},\ x_c(0) = x_0, \qquad [1.1]$$

where $A_\sigma \in \mathbb{R}^n$ are Hurwitz stable Metzler matrices, $x_c(t) \in \mathbb{R}^{n\times 1}$ and $m \geq 1$. We are interested in obtaining from this continuous-time positive system, a discrete-time representation:

$$x_d(k+1) = F_{\sigma_d(k)}(h) x_d(k),\ \sigma_d(k) \in \{1, \ldots, m\},\ x_d(0) = x_0, \quad [1.2]$$

where $h > 0$ is the sampling interval. The first objective of this chapter is to study diagonal Padé approximations to the matrix exponential. Such a study is well motivated, as diagonal Padé approximations are methods used by control engineers. Following [ZAP 12], we deal with two fundamental questions. First, under what conditions are certain types of stability of the original positive switched system inherited by the discrete-time approximation? Second, we also ask if and when positivity itself is inherited by the discrete-time system. We give sufficient conditions under which the Padé approximation is positivity preserving, and identify a new approximation method that is guaranteed to preserve both stability and positivity.

The second objective of this chapter arises from the need of discretizing large-scale systems. In this context, we are often interested in discretization methods that preserve the structure of a dynamic system. We aim to find efficient discretization methods which preserve, for the elements of $F_{\sigma_c}(h)$, the same zero/non-zero pattern of A_{σ_c}. The attention here is focused on positive switched systems only, along the lines traced in [COL 12]. First, we analyze the properties of the forward Euler transformation, which intrinsically preserve the zero pattern of the off-diagonal entries of the dynamic matrix. However, it is well known that the forward Euler transformation can easily lead to a loss of stability even for short sampling times. We then propose a novel *mE-ZOH* discretization method that preserves the structure independently of the sampling time, with improved performance in terms of stability preservation.

The chapter is organized as follows: in section 1.2, we study Padé transformations and their properties, while in section 1.3 we propose the new *mE-ZOH* transformation and we analyze some of its properties. Section 1.4 concludes the chapter.

NOTATION. In this chapter, the following notations are used: capital letters denote matrices and small letters denote vectors. For matrices or vectors, ($'$) indicates transpose and (*) the complex conjugate transpose. For matrices X or vectors x, the notation X or $x > 0$ (≥ 0) indicates that X, or x, has all positive (non-negative) entries and it will be called a positive (non-negative) matrix or vector. The notation $X \succ 0$ ($X \prec 0$) or $X \succeq 0$ ($X \preceq 0$) indicates that the matrix X is positive (negative) definite or positive (negative) semi-definite. The sets of real and natural numbers are denoted by $\mathbb{R}$ and $\mathbb{N}$, respectively, while $\mathbb{R}_+$ denotes the set of non-negative real numbers. A square matrix A_c is said to be Hurwitz stable if all its eigenvalues lie in the open left-half of the complex plane. A square matrix A_d is said to be Schur stable if all its eigenvalues lie inside the unit disc. A matrix A is said to be Metzler (or essentially non-negative) if all its off-diagonal elements are non-negative; moreover, we say that the diagonal entries are non-positive, with at least one negative diagonal entry. A matrix B is an M-matrix if $B = -A$, where A is both Metzler and Hurwitz; if an M-matrix is invertible, then its inverse is non-negative [BER 94]. The matrix I will be the identity matrix of appropriate dimensions. Finally, we denote with $\mathcal{M}_c$ the set of Hurwitz stable Metzler matrices, and with $\mathcal{M}_d$ the set of Schur stable non-negative matrices.

1.2. Discretization of switched positive systems via Padé transformations

This section is a summary of the recent work described in [ZAP 12] and some other related papers. The interested reader is referred to [ZAP 12] for proofs and examples. Concerning the problem of obtaining a discrete-time approximation [1.2] to system [1.1], the Padé approximation can be used, where h is the sampling time. The $[L/M]$ order Padé approximation to the exponential function e^s is the rational function C_{LM} defined by:

$$C_{LM}(s) = Q_L(s)Q_M^{-1}(-s),$$

where

$$Q_L(s) = \sum_{k=0}^{L} l_k s^k,\ Q_M(s) = \sum_{k=0}^{M} m_k s^k,$$

$$l_k = \frac{L!(L+M-k)!}{(L+M)!k!(L-k)!} \text{ and } m_k = \frac{M!(L+M-k)!}{(L+M)!k!(M-k)!}.$$

Thus, given a matrix A, the diagonal Padé approximant to the matrix exponential e^{Ah} with sampling time h is given by taking $L = M = p$

$$C_p(Ah) = Q_p(Ah)Q_p^{-1}(-Ah),$$

where $Q_p(Ah) = \sum_{k=0}^{p} c_k(Ah)^k$ and $c_k = \frac{p!(2p-k)!}{(2p)!k!(p-k)!}$. It is known that diagonal Padé approximations map the open left-half of the complex plane to the interior of the unit disc, and hence are A-stable [BUT 02].

1.2.1. *Preservation of copositive Lyapunov functions*

Recently, it was shown in [SAJ 11] that quadratic Lyapunov functions are preserved for sets of matrices that arise in the study of systems of the form of equation [1.1]. We now ask whether copositive Lyapunov functions are preserved when discretizing an LTI positive system using Padé-like approximations. Since trajectories of positive systems are constrained to lie in the positive orthant, the stability of these systems is completely captured by Lyapunov functions whose derivative is decreasing for all such positive trajectories. Such functions are referred to as copositive Lyapunov functions. With this background in mind, we observe the following elementary result.