resistive circuit theory

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PREFACE

Another freshman level book on circuit theory? Yes. Is it significantly different from all the others? Yes. Two departures distinguish the present text: the topics treated are restricted to resistive circuits, and the mathematical notation used is restricted to a simple well-defined system, APL.

APL is a simplification and extension of the familiar notation of vector algebra which extends its domain of application far beyond linear algebra, and enhances the role of arrays (vectors, matrices, etc.) as an important tool of organized thought. It is also a computer language, and every mathematical expression occurring in the text can be entered without change and executed on a typewriter-like APL terminal.

Three characteristics of the APL notation merit mention here:

1. It has the ability, as a notation, to suggest useful generalizations of functions and relations already defined, and thereby entend their range of validity or shed further light on their underlying structure. It is exciting to experience this aspect of APL, and one is reminded of Bertrand Russell's saying that "A good notation has a subtlety and suggestiveness which at times makes it seem almost like a live teacher".

2. The executability of APL permits a student at an APL terminal to readily experiment with a wide variety of interesting circuits, and thereby gain insight into circuit behaviour and the structure of circuit theory. Theorems can be discovered rather than encountered first via statement and proof. Speculative ideas can easily be tested.

3. Because of its simple syntax and similarity to vector algebra, APL is easy to learn; one hour spent exploring the language at an interactive terminal is sufficient to give the student or teacher a feel for the language and enable him to start using it. The importance of this ease of learning will be apparent to any teacher who has examined computer-oriented texts on circuit theory which require the student to use two systems of notation (one for the computer and one for the blackboard) and to learn inessential details of the mechanics of computers.

For the beginner - either teacher or student - who seeks a quick initial appreciation of APL, Appendix 1 contains an annotated sample terminal session. Perusal of this session or, better still, its repetition at an APL terminal, should provide the beginner with a feel for the language and an acquaintance with many of the primitive functions encountered in this book. Appendix 2 is not intended for the beginner; it is a concise summary of all the APL primitive and mixed functions and provides reference material for the reader who has begun to make serious use of APL. What form should an organized introduction to APL for students take? My own recommendation is an approach in which terminal experience constitutes the main activity, with formal lecturing kept to a minimum. A successfully tested example of this approach is offered by Kenneth Iverson's "An Introduction to APL for Scientists and Engineers", a short note* whose theme is one of guided exploration.

One significant aspect of the text which derives from the use of APL is its algorithmic and experimental flavour. At each stage in his understanding of the theory, the student is able to obtain experience of the pertinent relationships, both by executing the expressions appropriate to specific circuit examples, and by incorporating the expressions in defined functions which are then available as building blocks in further analyses. Moreover, the function definitions serve to summarize, in a precise and readable symbolic form, important relations which the student has just learned and which he will later wish to use as tools. Suitably graded examples and exercises serve to develop the student's ability to compose programs, or function definitions, as they are called in APL. I would not be surprised if such an algorithmic treatment of circuit theory led to a more ready acceptance of the subject by the more practically inclined student.

When planning this text I realized that many of the central theorems and concepts of circuit theory might best be introduced and explored within the simpler context of purely resistive circuits, leaving the dynamic behaviour of circuits incorporating inductive and capacitive elements to be treated in a subsequent (or companion) course. The inclusion of both nonreciprocal and nonlinear resistive circuits gives the student a broad and realistic experience of circuit theory. Indeed, in subject matter and approach the present text resembles the first of two courses (Resistive Circuits, and Dynamic Circuits) proposed by the COSINE Committee Commission on Engineering Education (September, 1968).

Although the material of the syllabus does not depart very much in outline from what is conventional, it differs significantly in detail. This first becomes obvious in Chapter 2, where two-terminal components are modelled by APL functions, thereby allowing the student to systematically and easily explore the nature of a component by examining

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* Iverson, K. E., "An Introduction to APL for Scientists and Engineers", IBM Philadelphia Scientific Center, Technical Report No. 320-3019, March, 1973.

the response pattern produced by a regular ordering of the excitation. He is also encouraged to build his own models, based on his measurements, and to test these models against the original functions which represented the components. Discussion is not restricted to linear resistances - exponential diodes and polynomial nonlinearities are also explored. The idea of the inverse function (yielding the voltage response to current excitation) is also introduced at this point, in preparation for its extension to the three-terminal component in Chapter 6 and its use in formal circuit analysis in Chapter 8.

Chapter 3 considers the constraints imposed by the interconnection of components. Sources and power are discussed in Chapter 4, and signals in Chapter 5. Chapter 6 concerns the measurement and modelling of 3-terminal components, introduces the use of matrices in the description of the voltage-current relation of a component, and extends the notion to n-terminal components. The idea of regarding an n-terminal circuit as an n-terminal component lays the groundwork for the later treatment of formal circuit analysis. This chapter, like Chapter 2, also gives extensive consideration to nonlinear components, in particular the transistor described by an Ebers-Moll model. Chapter 7 employs arrays in the description of circuit topology and in the concise expression of Kirchhoff's laws in terms of the incidence matrix.

Up to this point, both linear and nonlinear components have been considered in each chapter. Chapters 8, 9 and 10, however, concentrate on circuits composed solely of linear homogeneous components, show how the circuit equations are formulated and solved, and how the solution is interpreted in terms of circuit properties. Here again, the use of arrays is fully exploited, as is the common basis of component and circuit description.

Chapter 11 considers the analysis of a nonlinear resistive circuit, and describes the Newton-Raphson iterative method of analysis. By drawing upon concepts introduced earlier in the text it has been found possible to keep discussion of this topic at an acceptably simple level. A particularly useful consequence of the use of APL is the fact that the Newton-Raphson algorithm, embodied in a brief, readable function, can be introduced very simply in the context of a 2-terminal nonlinear component and then, with minimal change, be generalized to handle the analysis of an arbitrary nonlinear circuit.

Chapter 12 brings together the nonlinear circuit analysis and the analysis of linear homogeneous circuits in the treatment of the small-signal behaviour of nonlinear circuits. This chapter serves an additional purpose in calling upon the student's knowledge of the variety of topics encountered in earlier chapters.

The APL notation was invented by Kenneth Iverson, now of IBM Corporation, while teaching at Harvard, and an early description appears in his book <u>A Programming Language</u> (hence the name APL). Later work with Adin Falkoff at IBM led to refinement and extension of the notation and, in 1966, to its implementation as a language for conversational computing, since when its popularity and use has rapidly increased. It was Kenneth Iverson who, as an IBM Fellow, invited me to work with him at the IBM Philadelphia Scientific Center in order to explore the potential of APL for the teaching of circuit theory. I found this to be a stimulating experience. By his infectious enthusiasm, his quick grasp of circuit theory and, most of all, his appreciation of what teaching is all about, Iverson contributed immensely to the development of this text; it is a pleasure to record my gratitude to him.

Many other people have contributed to this text. John Goacher of Imperial College helped in the formulation of Chapter 11. Other permanent and guest workers at the Philadelphia Scientific Center were a constant source of encouragement, and I particularly wish to acknowledge the contribution of Professor Donald McIntyre of Pomona College in this respect. I would also like to thank Elizabeth Llanso for her excellent handling of the manuscript under somewhat harassing conditions, and Mr. E. E. McDonnell for a critical reading of the manuscript. Finally, I wish to express my gratitude to Mr. Clenyg Squire of IBM United Kingdom Ltd. for his cooperation in this endeavour.

In its restriction to resistive circuits, its use of APL notation, and its implicit assumption of the value of mathematical experimentation using APL terminals, the present text must be regarded as experimental. Indeed, my own experimental use of the text is at present under way, and I look forward to receiving constructive criticism from both students and colleagues.

Robert Spence

CONTENTS

CHAPTER 1

CIRCUIT DESIGN

Anyone who is encouraged to study circuit theory deserves a succinct preliminary discussion of the value of the subject and the context within which it is relevant. Only then will the associated ideas and methods be seen in their proper perspective. The present chapter attempts this and will often be referred to later to supply the motivation or context for a particular discussion.

In its normal engineering context, circuit theory is viewed as both an analytical and conceptual tool of considerable value in the design of electric circuits. It is also of wider interest as a tool for the study of some non-electrical circuits. It is, however, in the context of electrical circuit design that the subject of circuit theory will be presented in this book, and the needs of the circuit designer will provide the underlying motivation.

The designer of any object - whether it be an electrical circuit or an automobile - is supplied with a specification of the properties to be exhibited by that object, and must construct it by interconnecting a number of building blocks. The object is the <u>product</u> of the designer, and the building blocks are his <u>ingredients</u>.

Specifically, a <u>circuit designer</u> is a person who has access to a range of component types (e.g., resistors, transistors, wires) and who connects these components together in such a way as to make a <u>circuit</u> - such as an amplifier - which exhibits useful behaviour. In turn, a <u>system designer</u> will take these and other circuits and connect them together to make a larger circuit usually referred to as a <u>system</u>. To take a familiar example, components such as resistors and transistors may be suitably interconnected to form amplifiers and switches, which may then be connected to other circuits (e.g., telephone sets) to form a telephone system. Thus, in one respect, the telephone system is merely a collection of components. What will be seen to be important, however, is the idea of regarding a collection of interconnected components as a <u>new component</u> whose interior is of little interest but whose external characteristics are important.

This idea is reinforced by considering the functions of the circuit designer and system designer in the reverse order to that just discussed. In practice, the system designer specifies the function that he wishes a circuit to perform. The general form of this specification is shown in Fig. 1.1; the 4-terminal box is required to exhibit a specified voltage V between the pair of terminals X and Y when a current of value I flows between terminals A and B. Such is the system designer's view of the circuit he requires. The circuit designer's view of the same circuit is illustrated in Fig. 1.2; he is concerned with choosing components and their interconnection in such a way that when the current I flows the voltage V is generated. Normally there will be many different ways of choosing the components and interconnecting them to satisfy the system designer's specification.

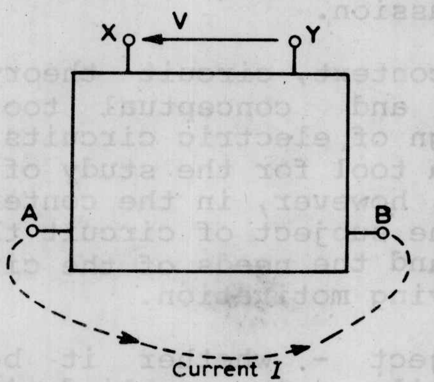

Fig. 1·1

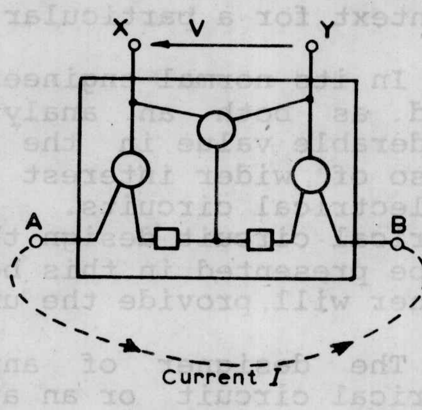

Fig. 1·2

The illustration of Fig. 1.3 shows the relative positions and functions of the circuit and system designer, and of the designer of the components used as ingredients by the circuit designer. At each interface the product of one designer constitutes some or all of the ingredients available to the other. Since the components manufactured by the component designer are identical with those contained within the circuits used as building blocks by the system designer, it could be remarked that each designer is concerned with much the same material. This is essentially true; what distinguishes the three categories of designer shown in Fig. 1.3 is their view of the product they are designing. The system designer, for example, is interested principally in the external properties (e.g., power gain, bandwidth) of an amplifier, whereas the circuit designer must concern himself with the voltages and currents associated with its internal components. He, in turn, is unconcerned with those underlying physical, metallurgical and chemical principles which lie in the domain of interest of the component designer.

The distinction between each of the three levels shown in Fig. 1.3 is principally defined by the quantities of interest to each designer. At the level of component design, quantities such as majority and minority carrier concentrations, diffusion rates and impurity profiles have been found to be useful descriptors in the processes and products involved. At the other extreme, the interest of the system designer in circuit properties such as power gain and bandwidth has already been noted. Between these two levels, the circuit designer finds it most helpful to be concerned with the voltage and current associated with each component, both in isolation and when interconnected with others to form a circuit. It is in this context that the discipline known as circuit theory is relevant, for it encompasses the principles and techniques which allow us to describe and predict the voltage-current behaviour of an electrical circuit.

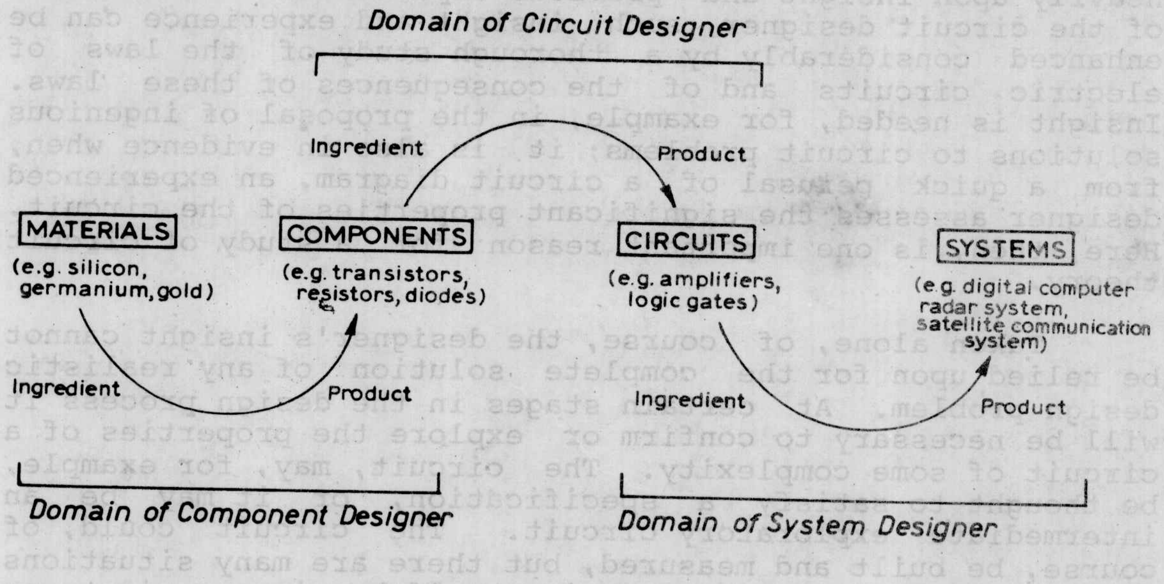

Fig. 1.3

The divisions between component, circuit and system designers are not, of course, clear-cut, and some overlap of interest is essential to communication between them. The advent of integrated circuit fabrication has, in fact, led to some merging of the functions of the component and circuit designer.

The illustration of Fig. 1.3 may also have suggested that the circuit designer is the unfortunate who is attacked from both sides, having to meet a specification laid down by the system designer with components offered by the component designer. But this would be misleading; on the one hand some knowledge of the proposed system allows him to suggest modifications to the specification and to point out inconsistencies; on the other, an acquaintance with transistor physics, for example, may enable him to provide useful feedback to the component designer concerning component fabrication. Finally, we should be aware that there is no unique starting point in the structure shown in Fig. 1.3, since invention can occur in any domain. A new component may be discovered (e.g., the transistor), a new system proposed (e.g., satellite communication) or a new circuit devised (e.g., the Eccles-Jordan bistable), and the repercussions be felt in all domains.

The transition from a collection of isolated components to a useful circuit is a creative act on the part of the circuit designer and, like all creative acts, draws heavily upon insight and previous experience. In the case of the circuit designer, such insight and experience can be enhanced considerably by a thorough study of the laws of electric circuits and of the consequences of these laws. Insight is needed, for example, in the proposal of ingenious solutions to circuit problems; it is also in evidence when, from a quick perusal of a circuit diagram, an experienced designer assesses the significant properties of the circuit. Here, then, is one important reason for a study of circuit theory.

Taken alone, of course, the designer's insight cannot be relied upon for the complete solution of any realistic design problem. At certain stages in the design process it will be necessary to confirm or explore the properties of a circuit of some complexity. The circuit, may, for example, be thought to satisfy a specification, or it may be an intermediate exploratory circuit. The circuit could, of course, be built and measured, but there are many situations in which such a course of action would be inconvenient or expensive. An integrated circuit, for example, is extremely costly if only a single copy is made. In this event the designer may choose to <u>simulate</u> circuit behaviour by means of a computer, by providing the computer with what he believes to be a satisfactory mathematical description, or <u>model</u>, of the circuit. By means of an algorithm the computer will then calculate the performance associated with the circuit model, the implication for the <u>actual</u> circuit being interpreted by the designer.

Obviously, the algorithm by means of which the computer generates the required result must be an unambiguous statement of the steps involved in the analysis of a circuit. It follows, therefore, that the associated computer program can only be written by one who understands circuit theory. Even if the circuit designer himself does not create the algorithm or program the computer, an exposure to the algorithmic approach to circuit description and analysis is invaluable in the understanding of circuit theory and hence in the development of insight.

Nevertheless, the feeling exists that the circuit designer is rarely obliged to concern himself with the steps involved in the computer analysis of circuits, and should blindly use available programs. In other words, he should supply circuit data and interpret the results he receives rather than concern himself with the precise manner in which the results were generated. But this view of the process of circuit design is unrealistic. Certainly during the interpretation of computer output the designer must confine his attention to the results of an analysis. But the competent designer must become increasingly conversant with circuit theory to understand the capabilities and limitations of existing programs, to test them effectively and use them intelligently. He may, for example, be permitted to modify or extend an existing program. What is more, with continuing improvement in man-computer communication, we are likely to see an increase in the extent of the designer's interaction with the results of a circuit analysis. He may, for example, be permitted to carry out additional and extensive circuit calculations on a conversational computer terminal. For all these reasons a good understanding of circuit theory is helpful to those involved in circuit design.

CHAPTER 2

MEASUREMENT AND MODELLING OF 2-TERMINAL COMPONENTS

The circuit designer creates a building block for the system designer by connecting together a number of components such as transistors and resistors. In order to predict the description of the resulting circuit, he must be able to describe the individual components.

By a description of a component we mean the functional relationship between the voltage(s) and current(s) associated with that component. In the present chapter we shall be concerned exclusively with components possessing only two terminals, and which are represented symbolically as in Fig. 2.1. The small circles indicate the points to which other components or measuring instruments can be connected. More fundamentally, these are the points with which an electric potential can be associated. Normally, we are concerned with, and use a voltmeter to measure, the potential difference - or voltage - between two points, so that only one voltage will be associated with a 2-terminal component.

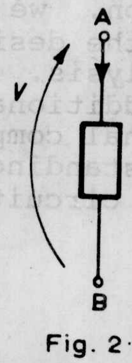

Fig. 2·1
A 2-terminal component

In order to refer to the value of a component voltage it is customary to place beside the component symbol a reference arrow, as shown in Fig. 2.1. If the potential of the terminal associated with the arrowhead is positive with respect to the other terminal then the component voltage, designated V in Fig. 2.1, is by convention said to be positive.

The presence of only two terminals also implies that only one current can be measured, since the current entering terminal A will be equal to that leaving terminal B; that this is an extremely good approximation in most circuits can be verified by measurement. Again, an arrow (Fig. 2.1) is employed to indicate the reference direction for current.

If current actually flows in the direction of the arrow its value is positive; it is negative if it flows in the opposite direction. For example, current flow in the direction B to A in Fig. 2.1 is described by a negative value of *I*.

The current and voltage reference directions can be chosen arbitrarily, since they obviously cannot influence the actual current and voltage. Nevertheless, in the interest of economy and clarity, the choice of one of the reference directions (say that for voltage) is by convention taken to imply the other reference direction (for current) in the manner shown in Fig. 2.1 (i.e., the reference arrow for current enters the terminal with which the head of the voltage reference arrow is associated). It is then only necessary to indicate either the current or the voltage reference direction.

The units of voltage and current are the volt and ampere, respectively; the latter is often abbreviated to amp. If, in the text, no unit is associated with the numerical value of a voltage or current, the units of voltage or current, respectively, are implied. Naturally, explicit mention is made of any multiples or fractions of these units (e.g., kilovolts, milliamperes).

The physical nature of a component imposes a relation between its current *I* and its voltage *V*. It is possible to associate with a component a table, such as that shown as Table 2.1, containing pairs of values of *I* and *V*. For any value of *V* there is a corresponding value of *I*, and vice versa; given one, the other is determined. More generally, we can express the current *I* as a function of the voltage *V*

I←*FV V*

or the voltage *V* as a function of the current *I*

V←*FI I*

Current *I* (amps)	Voltage *V* (volts)
.	.
.	.
.	.
1	0.5
2	2.7
3	5.1
.	.
.	.
.	.

Table 2.1

Thus, it is useful to regard the component as a function which takes either the voltage *V* or the current *I* as its argument and generates, as its result, either the current *I* or the voltage *V*, respectively. Although *FI* and *FV* are different functions, they are related; each is the inverse of the other.

The nature of the function describing a component can be found by actual measurements of its current and voltage. Such measurements are, of course, carried out in the laboratory. However, it will save a lot of time, and enable our discussions to be more effective if we study, in this book, a computer model of the component. Thus, instead of connecting a battery of known voltage to the component and observing the resulting current *I* on an ammeter (Fig. 2.2a), we employ the value of *V* as the argument of a function *FV* and display the result (Fig. 2.2b).

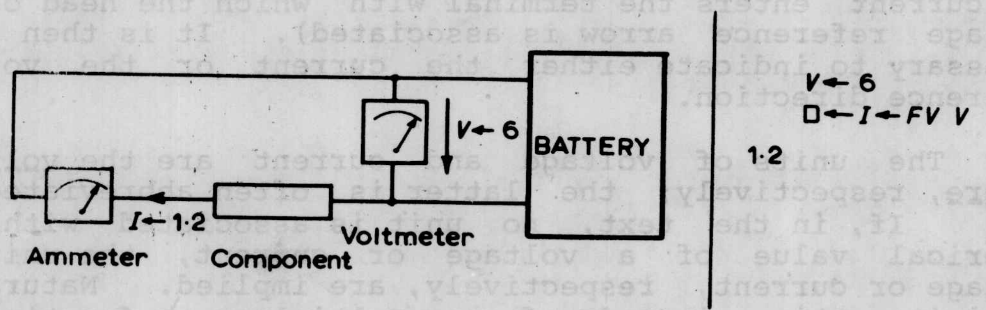

Fig. 2·2 Laboratory measurement of a component, and its computer simulation.

In this measurement the voltage is predetermined by the battery and the current is further determined by the component; we therefore refer to the voltage as the <u>excitation</u> of the component and the current as its <u>response</u>. In the computer simulation of the function we call *V* the <u>argument</u> and *I* its <u>result</u>. Although some of the measurements described in this book will undoubtedly be carried out in the laboratory, we shall henceforth be concerned solely with the computer simulation of these measurements, so that the terms excitation and argument will come to be used interchangeably, as will response and result.

Let us assume that the meters involved in the measurement of Fig. 2.2a can be read to two significant figures:

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      )DIGITS 2
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Upon connecting a battery of 1 volt to the component, we measure a current of two amperes:

```
      V←1
      I←FV V
      I
2
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