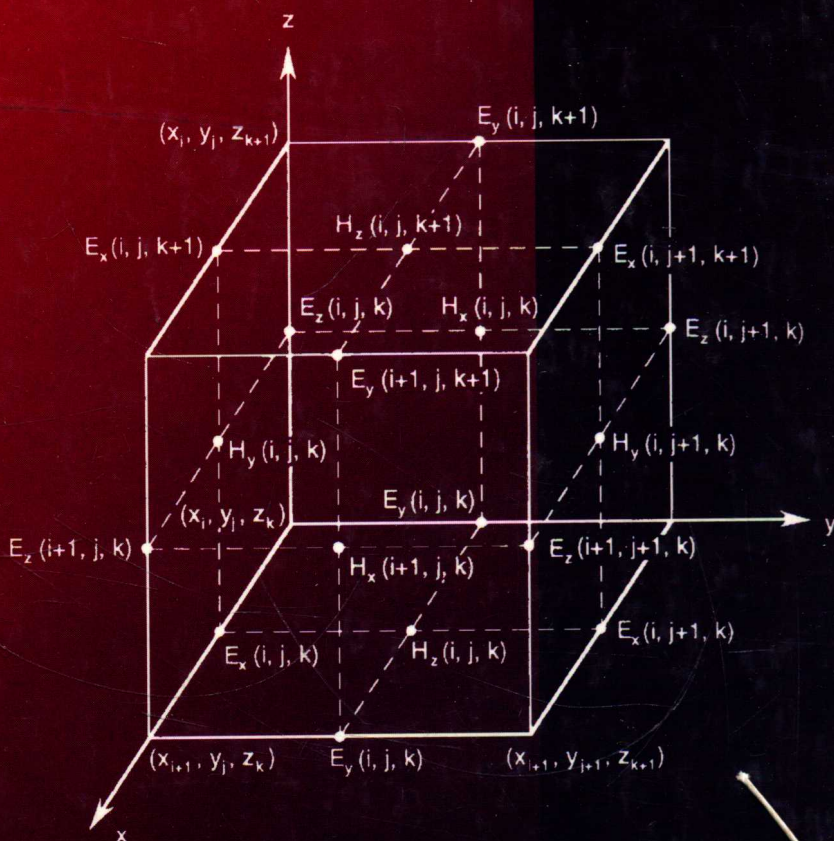


Numerical Techniques in Electromagnetics

Second Edition



Matthew N. O. Sadiku

Numerical
Techniques in

Electromagnetics

Second Edition

Matthew N. O. Sadiku, Ph.D.



CRC Press

Boca Raton London New York Washington, D.C.

Library of Congress Cataloging-in-Publication Data

Sadiku, Matthew N. O.

Numerical techniques in electromagnetics / Matthew N.O. Sadiku.—[2nd ed.].
p. cm.

Includes bibliographical references and index.

ISBN 0-8493-1395-3 (alk. paper)

1. Electromagnetism. 2. Numerical analysis. I. Title.

QC760 .S24 2000

537'.01'515—dc21

00-026823

CIP

This book contains information obtained from authentic and highly regarded sources. Reprinted material is quoted with permission, and sources are indicated. A wide variety of references are listed. Reasonable efforts have been made to publish reliable data and information, but the author and the publisher cannot assume responsibility for the validity of all materials or for the consequences of their use.

Neither this book nor any part may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, microfilming, and recording, or by any information storage or retrieval system, without prior permission in writing from the publisher.

The consent of CRC Press LLC does not extend to copying for general distribution, for promotion, for creating new works, or for resale. Specific permission must be obtained in writing from CRC Press LLC for such copying.

Direct all inquiries to CRC Press LLC, 2000 N.W. Corporate Blvd., Boca Raton, Florida 33431.

Trademark Notice: Product or corporate names may be trademarks or registered trademarks, and are used only for identification and explanation, without intent to infringe.

Visit the CRC Press Web site at www.crcpress.com

© 2001 by CRC Press LLC

No claim to original U.S. Government works

International Standard Book Number 0-8493-1395-3

Library of Congress Card Number 00-026823

Printed in the United States of America 2 3 4 5 6 7 8 9 0

Printed on acid-free paper

Numerical
Techniques in

Electromagnetics

Second Edition

Preface

The art of computation of electromagnetic (EM) problems has grown exponentially for three decades due to the availability of powerful computer resources. In spite of this, the EM community has suffered without a suitable text on computational techniques commonly used in solving EM-related problems. Although there have been monographs on one particular technique or the other, the monographs are written for the experts rather than students. Only a few texts cover the major techniques and do that in a manner suitable for classroom use. It seems experts in this area are familiar with one or few techniques and not many experts seem to be familiar with all the common techniques. This text attempts to fill the gap.

The text is intended for seniors or graduate students and may be used for a one-semester or two-semester course. The main requirements for students taking a course based on this text are introductory EM courses and a knowledge of a high-level computer language, preferably FORTRAN or C. Software packages such as Matlab and Mathcad may be helpful tools. Although familiarity with linear algebra and numerical analysis is useful, it is not required.

In writing this book, three major objectives were borne in mind. First, the book is intended to teach students how to pose, numerically analyze, and solve EM problems. Second, it is designed to give them the ability to expand their problem solving skills using a variety of available numerical methods. Third, it is meant to prepare graduate students for research in EM. The aim throughout has been simplicity of presentation so that the text can be useful for both teaching and self-study. In striving after simplicity, however, the reader is referred to the references for more information. Toward the end of each chapter, the techniques covered in the chapter are applied to real life problems. Since the application of the technique is as vast as EM and author's experience is limited, the choice of application is selective.

Chapter 1 covers some fundamental concepts in EM. Chapter 2 is intended to put numerical methods in a proper perspective. Analytical methods such as separation of variables and series expansion are covered. Chapter 3 discusses the finite difference methods and begins with the derivation of difference equation from a partial differential equation (PDE) using forward, backward, and central differences. The finite-difference time-domain (FDTD) technique involving Yee's algorithm is pre-

sented and applied to scattering problems. Numerical integration is covered using trapezoidal, Simpson's, Newton-Cotes rules, and Gaussian quadratures.

Chapter 4 on variational methods serves as a preparatory ground for the next two major topics: moment methods and finite element methods. Basic concepts such as inner product, self-adjoint operator, functionals, and Euler equation are covered. Chapter 5 on moment methods focuses on the solution of integral equations. Chapter 6 on finite element method covers the basic steps involved in using the finite element method. Solutions of Laplace's, Poisson's, and wave equations using the finite element method are covered.

Chapter 7 is devoted to transmission-line matrix or modeling (TLM). The method is applied to diffusion and scattering problems. Chapter 8 is on Monte Carlo methods, while Chapter 9 is on the method of lines.

Since the publication of the first edition, there has been an increased awareness and utilization of numerical techniques. Many graduate curricula now include courses in numerical analysis of EM problems. However, not much has changed in computational electromagnetics. A major noticeable change is in the FDTD method. The method seems to have attracted much attention and many improvements are being made to the standard algorithm. This edition adds the noticeable change in incorporating absorbing boundary conditions in FDTD, FEM, and TLM. Chapter 9 is a new chapter on the method of lines.

Acknowledgements

I am greatly indebted to Temple University for granting me a sabbatical in Fall 1998 during which I was able to do most of the revision. I specifically would like to thank my dean, Dr. Keya Sadeghipour, and my chairman, Dr. John Helferty, for their support. Special thanks are due to Raymond Garcia of Georgia Tech for writing Appendices C and D in C++. I am deeply grateful to Dr. Arthur D. Snider of the University of South Florida and Mohammad R. Zunoubi of Mississippi State University for taking the time to send me the list of errors in the first edition. I thank Dr. Reinhold Pregla for helping in clarifying concepts in Chapter 9 on the method of lines. I express my deepest gratitude to my wife, Chris, and our daughters, Ann and Joyce, for their patience, sacrifices, and prayers.

A Note to Students

Before you embark on writing your own computer program or using the ones in this text, you should try to understand all relevant theoretical backgrounds. A computer

is no more than a tool used in the analysis of a program. For this reason, you should be as clear as possible what the machine is really being asked to do before setting it off on several hours of expensive computations.

It has been well said by A.C. Doyle that "It is a capital mistake to theorize before you have all the evidence. It biases the judgment." Therefore, you should never trust the results of a numerical computation unless they are validated, at least in part. You validate the results by comparing them with those obtained by previous investigators or with similar results obtained using a different approach which may be analytical or numerical. For this reason, it is advisable that you become familiar with as many numerical techniques as possible.

The references provided at the end of each chapter are by no means exhaustive but are meant to serve as the starting point for further reading.

To my teacher

Carl A. Ventrice

and my parents

Ayisat and Solomon Sadiku

Contents

| | | |
|----------|--|-----------|
| 1 | Fundamental Concepts | 1 |
| 1.1 | Introduction | 1 |
| 1.2 | Review of Electromagnetic Theory | 2 |
| 1.2.1 | Electrostatic Fields | 3 |
| 1.2.2 | Magnetostatic Fields | 4 |
| 1.2.3 | Time-varying Fields | 5 |
| 1.2.4 | Boundary Conditions | 7 |
| 1.2.5 | Wave Equations | 7 |
| 1.2.6 | Time-varying Potentials | 9 |
| 1.2.7 | Time-harmonic Fields | 10 |
| 1.3 | Classification of EM Problems | 14 |
| 1.3.1 | Classification of Solution Regions | 14 |
| 1.3.2 | Classification of Differential Equations | 15 |
| 1.3.3 | Classification of Boundary Conditions | 18 |
| 1.4 | Some Important Theorems | 20 |
| 1.4.1 | Superposition Principle | 20 |
| 1.4.2 | Uniqueness Theorem | 21 |
| | References | 23 |
| | Problems | 23 |
| 2 | Analytical Methods | 27 |
| 2.1 | Introduction | 27 |
| 2.2 | Separation of Variables | 28 |
| 2.3 | Separation of Variables in Rectangular Coordinates | 30 |
| 2.3.1 | Laplace's Equations | 30 |
| 2.3.2 | Wave Equation | 34 |

| | | |
|-------|--|-----|
| 2.4 | Separation of Variables in Cylindrical Coordinates | 39 |
| 2.4.1 | Laplace's Equation | 40 |
| 2.4.2 | Wave Equation | 42 |
| 2.5 | Separation of Variables in Spherical Coordinates | 53 |
| 2.5.1 | Laplace's Equation | 54 |
| 2.5.2 | Wave Equation | 59 |
| 2.6 | Some Useful Orthogonal Functions | 68 |
| 2.7 | Series Expansion | 78 |
| 2.7.1 | Poisson's Equation in a Cube | 78 |
| 2.7.2 | Poisson's Equation in a Cylinder | 80 |
| 2.7.3 | Strip Transmission Line | 83 |
| 2.8 | Practical Applications | 88 |
| 2.8.1 | Scattering by Dielectric Sphere | 88 |
| 2.8.2 | Scattering Cross Sections | 92 |
| 2.9 | Attenuation Due to Raindrops | 95 |
| 2.10 | Concluding Remarks | 105 |
| | References | 106 |
| | Problems | 107 |

3 Finite Difference Methods 121

| | | |
|--------|--|-----|
| 3.1 | Introduction | 121 |
| 3.2 | Finite Difference Schemes | 122 |
| 3.3 | Finite Differencing of Parabolic PDEs | 125 |
| 3.4 | Finite Differencing of Hyperbolic PDEs | 131 |
| 3.5 | Finite Differencing of Elliptic PDEs | 134 |
| 3.5.1 | Band Matrix Method | 137 |
| 3.5.2 | Iterative Methods | 137 |
| 3.6 | Accuracy and Stability of FD Solutions | 143 |
| 3.7 | Practical Applications I — Guided Structures | 147 |
| 3.7.1 | Transmission Lines | 148 |
| 3.7.2 | Waveguides | 154 |
| 3.8 | Practical Applications II — Wave Scattering (FDTD) | 159 |
| 3.8.1 | Yee's Finite Difference Algorithm | 160 |
| 3.8.2 | Accuracy and Stability | 163 |
| 3.8.3 | Lattice Truncation Conditions | 164 |
| 3.8.4 | Initial Fields | 167 |
| 3.8.5 | Programming Aspects | 168 |
| 3.9 | Absorbing Boundary Conditions for FDTD | 177 |
| 3.10 | Finite Differencing for Nonrectangular Systems | 186 |
| 3.10.1 | Cylindrical Coordinates | 186 |
| 3.10.2 | Spherical Coordinates | 190 |
| 3.11 | Numerical Integration | 193 |
| 3.11.1 | Euler's Rule | 196 |
| 3.11.2 | Trapezoidal Rule | 197 |
| 3.11.3 | Simpson's Rule | 197 |

| | | |
|----------|--|------------|
| 3.11.4 | Newton-Cotes Rules | 198 |
| 3.11.5 | Gaussian Rules | 200 |
| 3.11.6 | Multiple Integration | 203 |
| 3.12 | Concluding Remarks | 208 |
| | References | 210 |
| | Problems | 219 |
| 4 | Variational Methods | 235 |
| 4.1 | Introduction | 235 |
| 4.2 | Operators in Linear Spaces | 236 |
| 4.3 | Calculus of Variations | 238 |
| 4.4 | Construction of Functionals from PDEs | 242 |
| 4.5 | Rayleigh-Ritz Method | 245 |
| 4.6 | Weighted Residual Method | 252 |
| 4.6.1 | Collocation Method | 253 |
| 4.6.2 | Subdomain Method | 254 |
| 4.6.3 | Galerkin Method | 254 |
| 4.6.4 | Least Squares Method | 255 |
| 4.7 | Eigenvalue Problems | 261 |
| 4.8 | Practical Applications | 268 |
| 4.9 | Concluding Remarks | 274 |
| | References | 275 |
| | Problems | 279 |
| 5 | Moment Methods | 285 |
| 5.1 | Introduction | 285 |
| 5.2 | Integral Equations | 286 |
| 5.2.1 | Classification of Integral Equations | 286 |
| 5.2.2 | Connection Between Differential and Integral Equations | 287 |
| 5.3 | Green's Functions | 290 |
| 5.3.1 | For Free Space | 292 |
| 5.3.2 | For Domain with Conducting Boundaries | 295 |
| 5.4 | Applications I — Quasi-Static Problems | 308 |
| 5.5 | Applications II — Scattering Problems | 313 |
| 5.5.1 | Scattering by Conducting Cylinder | 314 |
| 5.5.2 | Scattering by an Arbitrary Array of Parallel Wires | 317 |
| 5.6 | Applications III — Radiation Problems | 325 |
| 5.6.1 | Hallen's Integral Equation | 326 |
| 5.6.2 | Pocklington's Integral Equation | 327 |
| 5.6.3 | Expansion and Weighting Functions | 327 |
| 5.7 | Applications IV — EM Absorption in the Human Body | 338 |
| 5.7.1 | Derivation of Integral Equations | 339 |
| 5.7.2 | Transformation to Matrix Equation (Discretization) | 342 |
| 5.7.3 | Evaluation of Matrix Elements | 343 |
| 5.7.4 | Solution of the Matrix Equation | 345 |

| | | |
|----------|---|------------|
| 5.8 | Concluding Remarks | 347 |
| | References | 357 |
| | Problems | 363 |
| 6 | Finite Element Method | 377 |
| 6.1 | Introduction | 377 |
| 6.2 | Solution of Laplace's Equation | 378 |
| 6.2.1 | Finite Element Discretization | 378 |
| 6.2.2 | Element Governing Equations | 380 |
| 6.2.3 | Assembling of All Elements | 383 |
| 6.2.4 | Solving the Resulting Equations | 386 |
| 6.3 | Solution of Poisson's Equation | 397 |
| 6.3.1 | Deriving Element-governing Equations | 397 |
| 6.3.2 | Solving the Resulting Equations | 399 |
| 6.4 | Solution of the Wave Equation | 400 |
| 6.5 | Automatic Mesh Generation I — Rectangular Domains | 407 |
| 6.6 | Automatic Mesh Generation II — Arbitrary Domains | 410 |
| 6.6.1 | Definition of Blocks | 411 |
| 6.6.2 | Subdivision of Each Block | 412 |
| 6.6.3 | Connection of Individual Blocks | 413 |
| 6.7 | Bandwidth Reduction | 420 |
| 6.8 | Higher Order Elements | 424 |
| 6.8.1 | Pascal Triangle | 425 |
| 6.8.2 | Local Coordinates | 426 |
| 6.8.3 | Shape Functions | 427 |
| 6.8.4 | Fundamental Matrices | 430 |
| 6.9 | Three-Dimensional Elements | 439 |
| 6.10 | Finite Element Methods for Exterior Problems | 444 |
| 6.10.1 | Infinite Element Method | 444 |
| 6.10.2 | Boundary Element Method | 446 |
| 6.10.3 | Absorbing Boundary Conditions | 446 |
| 6.11 | Concluding Remarks | 448 |
| | References | 449 |
| | Problems | 458 |
| 7 | Transmission-line-matrix Method | 467 |
| 7.1 | Introduction | 467 |
| 7.2 | Transmission-line Equations | 469 |
| 7.3 | Solution of Diffusion Equation | 473 |
| 7.4 | Solution of Wave Equations | 477 |
| 7.4.1 | Equivalence Between Network and Field Parameters | 477 |
| 7.4.2 | Dispersion Relation of Propagation Velocity | 481 |
| 7.4.3 | Scattering Matrix | 483 |
| 7.4.4 | Boundary Representation | 486 |
| 7.4.5 | Computation of Fields and Frequency Response | 487 |

| | | |
|----------|--|------------|
| 7.4.6 | Output Response and Accuracy of Results | 487 |
| 7.5 | Inhomogeneous and Lossy Media in TLM | 493 |
| 7.5.1 | General Two-Dimensional Shunt Node | 494 |
| 7.5.2 | Scattering Matrix | 496 |
| 7.5.3 | Representation of Lossy Boundaries | 497 |
| 7.6 | Three-Dimensional TLM Mesh | 499 |
| 7.6.1 | Series Nodes | 499 |
| 7.6.2 | Three-Dimensional Node | 504 |
| 7.6.3 | Boundary Conditions | 507 |
| 7.7 | Error Sources and Correction | 517 |
| 7.7.1 | Truncation Error | 518 |
| 7.7.2 | Coarseness Error | 518 |
| 7.7.3 | Velocity Error | 519 |
| 7.7.4 | Misalignment Error | 519 |
| 7.8 | Absorbing Boundary Conditions | 519 |
| 7.9 | Concluding Remarks | 521 |
| | References | 523 |
| | Problems | 529 |
| 8 | Monte Carlo Methods | 537 |
| 8.1 | Introduction | 537 |
| 8.2 | Generation of Random Numbers and Variables | 538 |
| 8.3 | Evaluation of Error | 542 |
| 8.4 | Numerical Integration | 546 |
| 8.4.1 | Crude Monte Carlo Integration | 546 |
| 8.4.2 | Monte Carlo Integration with Antithetic Variates | 548 |
| 8.4.3 | Improper Integrals | 549 |
| 8.5 | Solution of Potential Problems | 550 |
| 8.5.1 | Fixed Random Walk | 552 |
| 8.5.2 | Floating Random Walk | 557 |
| 8.5.3 | Exodus Method | 559 |
| 8.6 | Regional Monte Carlo Methods | 574 |
| 8.7 | Concluding Remarks | 581 |
| | References | 582 |
| | Problems | 588 |
| 9 | Method of Lines | 597 |
| 9.1 | Introduction | 597 |
| 9.2 | Solution of Laplace's Equation | 598 |
| 9.2.1 | Rectangular Coordinates | 598 |
| 9.2.2 | Cylindrical Coordinates | 605 |
| 9.3 | Solution of Wave Equation | 609 |
| 9.3.1 | Planar Microstrip Structures | 612 |
| 9.3.2 | Cylindrical Microstrip Structures | 619 |
| 9.4 | Time-Domain Solution | 627 |

| | | |
|----------|---|------------|
| 9.5 | Concluding Remarks | 629 |
| | References | 629 |
| | Problems | 635 |
| A | Vector Relations | 639 |
| A.1 | Vector Identities | 639 |
| A.2 | Vector Theorems | 639 |
| A.3 | Orthogonal Coordinates | 640 |
| B | Solving Electromagnetic Problems Using C++ | 643 |
| B.1 | Introduction | 643 |
| B.2 | A Brief Description of C++ | 643 |
| B.3 | Object-Orientation | 661 |
| B.4 | C++ Object-Oriented Language Features | 665 |
| B.5 | A Final Note | 674 |
| | References | 675 |
| C | Numerical Techniques in C++ | 677 |
| D | Solution of Simultaneous Equations | 701 |
| D.1 | Elimination Methods | 701 |
| | D.1.1 Gauss's Method | 702 |
| | D.1.2 Cholesky's Method | 703 |
| D.2 | Iterative Methods | 706 |
| | D.2.1 Jacobi's Method | 706 |
| | D.2.2 Gauss-Seidel Method | 708 |
| | D.2.3 Relaxation Method | 708 |
| | D.2.4 Gradient Methods | 710 |
| D.3 | Matrix Inversion | 713 |
| D.4 | Eigenvalue Problems | 714 |
| | D.4.1 Iteration (or Power) Method | 716 |
| | D.4.2 Jacobi's Method | 717 |
| E | Answers to Odd-Numbered Problems | 725 |
| | Index | 741 |

Chapter 1

Fundamental Concepts

"Science knows no country because knowledge belongs to humanity and is the torch which illuminates the world. Science is the highest personification of the nation because that nation will remain the first which carries the furthest the works of thoughts and intelligence."

Louis Pasteur

1.1 Introduction

Scientists and engineers use several techniques in solving continuum or field problems. Loosely speaking, these techniques can be classified as experimental, analytical, or numerical. Experiments are expensive, time consuming, sometimes hazardous, and usually do not allow much flexibility in parameter variation. However, every numerical method, as we shall see, involves an analytic simplification to the point where it is easy to apply the numerical method. Notwithstanding this fact, the following methods are among the most commonly used in electromagnetics (EM).

A. Analytical methods (exact solutions)

- (1) separation of variables
- (2) series expansion
- (3) conformal mapping
- (4) integral solutions, e.g., Laplace and Fourier transforms
- (5) perturbation methods

B. Numerical methods (approximate solutions)

- (1) finite difference method
- (2) method of weighted residuals
- (3) moment method
- (4) finite element method

- (5) transmission-line modeling
- (6) Monte Carlo method
- (7) method of lines

Application of these methods is not limited to EM-related problems; they find applications in other continuum problems such as in fluid, heat transfer, and acoustics [1].

As we shall see, some of the numerical methods are related and they all generally give approximate solutions of sufficient accuracy for engineering purposes. Since our objective is to study these methods in detail in the subsequent chapters, it may be premature to say more than this at this point.

The need for numerical solution of electromagnetic problems is best expressed in the words of Paris and Hurd: “Most problems that can be solved formally (analytically) have been solved.”¹ Until the 1940s, most EM problems were solved using the classical methods of separation of variables and integral equation solutions. Besides the fact that a high degree of ingenuity, experience, and effort were required to apply those methods, only a narrow range of practical problems could be investigated due to the complex geometries defining the problems.

Numerical solution of EM problems started in the mid-1960s with the availability of modern high-speed digital computers. Since then, considerable effort has been expended on solving practical, complex EM-related problems for which closed form analytical solutions are either intractable or do not exist. The numerical approach has the advantage of allowing the actual work to be carried out by operators without a knowledge of higher mathematics or physics, with a resulting economy of labor on the part of the highly trained personnel.

Before we set out to study the various techniques used in analyzing EM problems, it is expedient to remind ourselves of the physical laws governing EM phenomena in general. This will be done in Section 1.2. In Section 1.3, we shall be acquainted with different ways EM problems are categorized. The principle of superposition and uniqueness theorem will be covered in Section 1.4.

1.2 Review of Electromagnetic Theory

The whole subject of EM unfolds as a logical deduction from eight postulated equations, namely, Maxwell’s four field equations and four medium-dependent equations [2]–[4]. Before we briefly review these equations, it may be helpful to state two important theorems commonly used in EM. These are the divergence (or Gauss’s)

¹*Basic Electromagnetic Theory*, D.T. Paris and F.K. Hurd, McGraw-Hill, New York, 1969, p. 166.

theorem,

$$\oint_S \mathbf{F} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{F} dv \quad (1.1)$$

and Stokes's theorem

$$\oint_L \mathbf{F} \cdot d\mathbf{l} = \int_S \nabla \times \mathbf{F} \cdot d\mathbf{S} \quad (1.2)$$

Perhaps the best way to review EM theory is by using the fundamental concept of electric charge. EM theory can be regarded as the study of fields produced by electric charges at rest and in motion. Electrostatic fields are usually produced by static electric charges, whereas magnetostatic fields are due to motion of electric charges with uniform velocity (direct current). Dynamic or time-varying fields are usually due to accelerated charges or time-varying currents.

1.2.1 Electrostatic Fields

The two fundamental laws governing these electrostatic fields are Gauss's law,

$$\oint \mathbf{D} \cdot d\mathbf{S} = \int \rho_v dv \quad (1.3)$$

which is a direct consequence of Coulomb's force law, and the law describing electrostatic fields as conservative,

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0 \quad (1.4)$$

In Eqs. (1.3) and (1.4), \mathbf{D} is the electric flux density (in coulombs/meter²), ρ_v is the volume charge density (in coulombs/meter³), and \mathbf{E} is the electric field intensity (in volts/meter). The integral form of the laws in Eqs. (1.3) and (1.4) can be expressed in the differential form by applying Eq. (1.1) to Eq. (1.3) and Eq. (1.2) to Eq. (1.4). We obtain

$$\nabla \cdot \mathbf{D} = \rho_v \quad (1.5)$$

and

$$\nabla \times \mathbf{E} = 0 \quad (1.6)$$

The vector fields \mathbf{D} and \mathbf{E} are related as

$$\mathbf{D} = \epsilon \mathbf{E} \quad (1.7)$$

where ϵ is the dielectric permittivity (in farads/meter) of the medium. In terms of the electric potential V (in volts), \mathbf{E} is expressed as

$$\mathbf{E} = -\nabla V \quad (1.8)$$