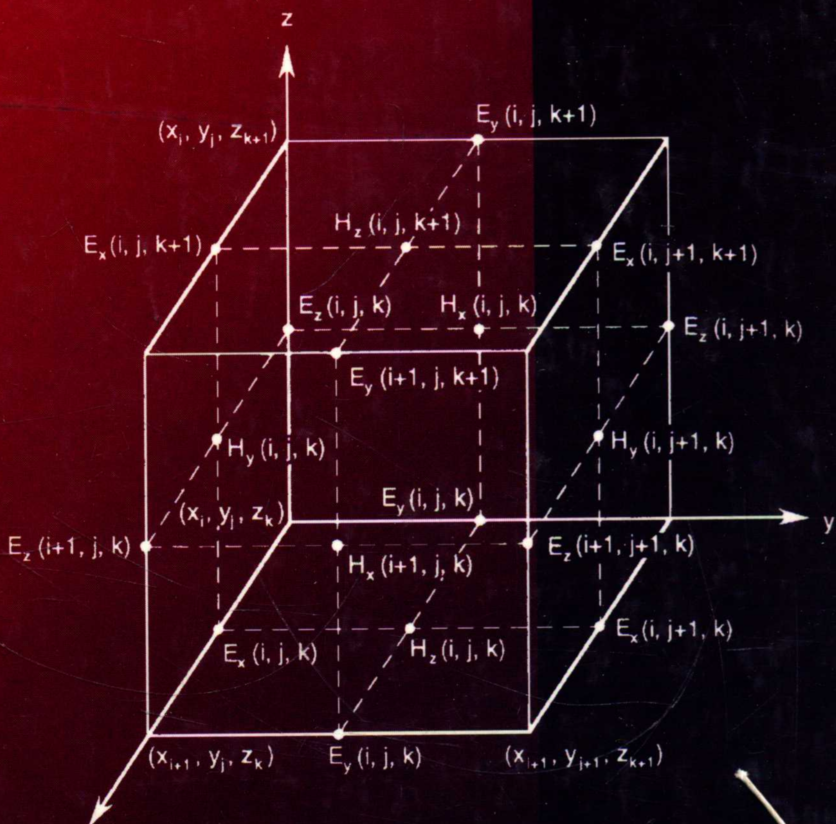
# Numerical Techniques in Electromagnetics

## Second Edition

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# *Preface*

The art of computation of electromagnetic (EM) problems has grown exponentially for three decades due to the availability of powerful computer resources. In spite of this, the EM community has suffered without a suitable text on computational techniques commonly used in solving EM-related problems. Although there have been monographs on one particular technique or the other, the monographs are written for the experts rather than students. Only a few texts cover the major techniques and do that in a manner suitable for classroom use. It seems experts in this area are familiar with one or few techniques and not many experts seem to be familiar with all the common techniques. This text attempts to fill the gap.

The text is intended for seniors or graduate students and may be used for a one-semester or two-semester course. The main requirements for students taking a course based on this text are introductory EM courses and a knowledge of a high-level computer language, preferably FORTRAN or C. Software packages such as Matlab and Mathcad may be helpful tools. Although familiarity with linear algebra and numerical analysis is useful, it is not required.

In writing this book, three major objectives were borne in mind. First, the book is intended to teach students how to pose, numerically analyze, and solve EM problems. Second, it is designed to give them the ability to expand their problem solving skills using a variety of available numerical methods. Third, it is meant to prepare graduate students for research in EM. The aim throughout has been simplicity of presentation so that the text can be useful for both teaching and self-study. In striving after simplicity, however, the reader is referred to the references for more information. Toward the end of each chapter, the techniques covered in the chapter are applied to real life problems. Since the application of the technique is as vast as EM and author's experience is limited, the choice of application is selective.

Chapter 1 covers some fundamental concepts in EM. Chapter 2 is intended to put numerical methods in a proper perspective. Analytical methods such as separation of variables and series expansion are covered. Chapter 3 discusses the finite difference methods and begins with the derivation of difference equation from a partial differential equation (PDE) using forward, backward, and central differences. The finite-difference time-domain (FDTD) technique involving Yee's algorithm is presented and applied to scattering problems. Numerical integration is covered using trapezoidal, Simpson's, Newton-Cotes rules, and Gaussian quadratures.

Chapter 4 on variational methods serves as a preparatory ground for the next two major topics: moment methods and finite element methods. Basic concepts such as inner product, self-adjoint operator, functionals, and Euler equation are covered. Chapter 5 on moment methods focuses on the solution of integral equations. Chapter 6 on finite element method covers the basic steps involved in using the finite element method. Solutions of Laplace's, Poisson's, and wave equations using the finite element method are covered.

Chapter 7 is devoted to transmission-line matrix or modeling (TLM). The method is applied to diffusion and scattering problems. Chapter 8 is on Monte Carlo methods, while Chapter 9 is on the method of lines.

Since the publication of the first edition, there has been an increased awareness and utilization of numerical techniques. Many graduate curricula now include courses in numerical analysis of EM problems. However, not much has changed in computational electromagnetics. A major noticeable change is in the FDTD method. The method seems to have attracted much attention and many improvements are being made to the standard algorithm. This edition adds the noticeable change in incorporating absorbing boundary conditions in FDTD, FEM, and TLM. Chapter 9 is a new chapter on the method of lines.

## Acknowledgements

I am greatly indebted to Temple University for granting me a sabbatical in Fall 1998 during which I was able to do most of the revision. I specifically would like to thank my dean, Dr. Keya Sadeghipour, and my chairman, Dr. John Helferty, for their support. Special thanks are due to Raymond Garcia of Georgia Tech for writing Appendices C and D in C++. I am deeply grateful to Dr. Arthur D. Snider of the University of South Florida and Mohammad R. Zunoubi of Mississippi State University for taking the time to send me the list of errors in the first edition. I thank Dr. Reinhold Pregla for helping in clarifying concepts in Chapter 9 on the method of lines. I express my deepest gratitude to my wife, Chris, and our daughters, Ann and Joyce, for their patience, sacrifices, and prayers.

## A Note to Students

Before you embark on writing your own computer program or using the ones in this text, you should try to understand all relevant theoretical backgrounds. A computer is no more than a tool used in the analysis of a program. For this reason, you should be as clear as possible what the machine is really being asked to do before setting it off on several hours of expensive computations.

It has been well said by A.C. Doyle that "It is a capital mistake to theorize before you have all the evidence. It biases the judgment." Therefore, you should never trust the results of a numerical computation unless they are validated, at least in part. You validate the results by comparing them with those obtained by previous investigators or with similar results obtained using a different approach which may be analytical or numerical. For this reason, it is advisable that you become familiar with as many numerical techniques as possible.

The references provided at the end of each chapter are by no means exhaustive but are meant to serve as the starting point for further reading.

*To my teacher*

*Carl A. Ventrice*

*and my parents*

*Ayisat and Solomon Sadiku*

# Contents

# Chapter 1

## Fundamental Concepts

*"Science knows no country because knowledge belongs to humanity and is the torch which illuminates the world. Science is the highest personification of the nation because that nation will remain the first which carries the furthest the works of thoughts and intelligence."* Louis Pasteur

### 1.1 Introduction

Scientists and engineers use several techniques in solving continuum or field problems. Loosely speaking, these techniques can be classified as experimental, analytical, or numerical. Experiments are expensive, time consuming, sometimes hazardous, and usually do not allow much flexibility in parameter variation. However, every numerical method, as we shall see, involves an analytic simplification to the point where it is easy to apply the numerical method. Notwithstanding this fact, the following methods are among the most commonly used in electromagnetics (EM).

A. Analytical methods (exact solutions)

   (1) separation of variables
   (2) series expansion
   (3) conformal mapping
   (4) integral solutions, e.g., Laplace and Fourier transforms
   (5) perturbation methods

B. Numerical methods (approximate solutions)

   (1) finite difference method
   (2) method of weighted residuals
   (3) moment method
   (4) finite element method

(5) transmission-line modeling

(6) Monte Carlo method

(7) method of lines

Application of these methods is not limited to EM-related problems; they find applications in other continuum problems such as in fluid, heat transfer, and acoustics [1].

As we shall see, some of the numerical methods are related and they all generally give approximate solutions of sufficient accuracy for engineering purposes. Since our objective is to study these methods in detail in the subsequent chapters, it may be premature to say more than this at this point.

The need for numerical solution of electromagnetic problems is best expressed in the words of Paris and Hurd: "Most problems that can be solved formally (analytically) have been solved."[1] Until the 1940s, most EM problems were solved using the classical methods of separation of variables and integral equation solutions. Besides the fact that a high degree of ingenuity, experience, and effort were required to apply those methods, only a narrow range of practical problems could be investigated due to the complex geometries defining the problems.

Numerical solution of EM problems started in the mid-1960s with the availability of modern high-speed digital computers. Since then, considerable effort has been expended on solving practical, complex EM-related problems for which closed form analytical solutions are either intractable or do not exist. The numerical approach has the advantage of allowing the actual work to be carried out by operators without a knowledge of higher mathematics or physics, with a resulting economy of labor on the part of the highly trained personnel.

Before we set out to study the various techniques used in analyzing EM problems, it is expedient to remind ourselves of the physical laws governing EM phenomena in general. This will be done in Section 1.2. In Section 1.3, we shall be acquainted with different ways EM problems are categorized. The principle of superposition and uniqueness theorem will be covered in Section 1.4.

## 1.2 Review of Electromagnetic Theory

The whole subject of EM unfolds as a logical deduction from eight postulated equations, namely, Maxwell's four field equations and four medium-dependent equations [2]–[4]. Before we briefly review these equations, it may be helpful to state two important theorems commonly used in EM. These are the divergence (or Gauss's)

[1] *Basic Electromagnetic Theory*, D.T. Paris and F.K. Hurd, McGraw-Hill, New York, 1969, p. 166.

theorem,

$$\oint_S \mathbf{F} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{F}\, dv \tag{1.1}$$

and Stokes's theorem

$$\oint_L \mathbf{F} \cdot d\mathbf{l} = \int_S \nabla \times \mathbf{F} \cdot d\mathbf{S} \tag{1.2}$$

Perhaps the best way to review EM theory is by using the fundamental concept of electric charge. EM theory can be regarded as the study of fields produced by electric charges at rest and in motion. Electrostatic fields are usually produced by static electric charges, whereas magnetostatic fields are due to motion of electric charges with uniform velocity (direct current). Dynamic or time-varying fields are usually due to accelerated charges or time-varying currents.

### 1.2.1 Electrostatic Fields

The two fundamental laws governing these electrostatic fields are Gauss's law,

$$\oint \mathbf{D} \cdot d\mathbf{S} = \int \rho_v \, dv \tag{1.3}$$

which is a direct consequence of Coulomb's force law, and the law describing electrostatic fields as conservative,

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0 \tag{1.4}$$

In Eqs. (1.3) and (1.4), $\mathbf{D}$ is the electric flux density (in coulombs/meter$^2$), $\rho_v$ is the volume charge density (in coulombs/meter$^3$), and $\mathbf{E}$ is the electric field intensity (in volts/meter). The integral form of the laws in Eqs. (1.3) and (1.4) can be expressed in the differential form by applying Eq. (1.1) to Eq. (1.3) and Eq. (1.2) to Eq. (1.4). We obtain

$$\nabla \cdot \mathbf{D} = \rho_v \tag{1.5}$$

and

$$\nabla \times \mathbf{E} = 0 \tag{1.6}$$

The vector fields $\mathbf{D}$ and $\mathbf{E}$ are related as

$$\mathbf{D} = \epsilon \mathbf{E} \tag{1.7}$$

where $\epsilon$ is the dielectric permittivity (in farads/meter) of the medium. In terms of the electric potential $V$ (in volts), $\mathbf{E}$ is expressed as

$$\mathbf{E} = -\nabla V \tag{1.8}$$