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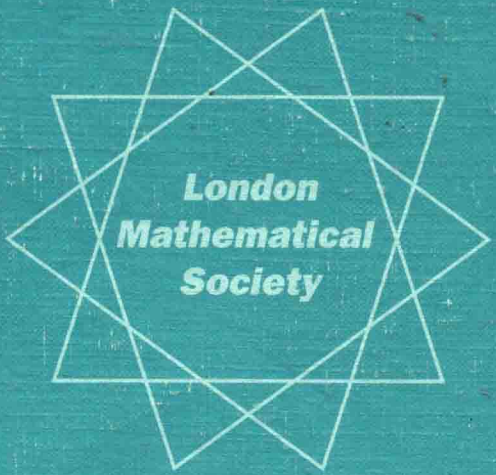
Analysis and Logic

C. Ward Henson, José Iovino,
Alexander S. Kechris, Edward Odell



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Analysis and Logic

C. Ward Henson
José Iovino
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Dedicated to Maurice Boffa (1939 – 2001)

Preface

In 1997 the Analysis and the Mathematical Logic teams of the University of Mons-Hainaut, and the Analysis team of the University of Paris 6 organized an international conference entitled "Analyse & Logique". It took place at the University of Mons-Hainaut, Mons, Belgium from 25 to 29 August 1997. The scientific committee consisted of Maurice Boffa (Mons), Gilles Godefroy (Paris 6; Columbia, Missouri), Tim Gowers (Cambridge), Boris S. Kashin (Moscow), Angus Macintyre (Oxford), Olek Pelczynski (Warsaw), Françoise Point (Mons), Alexander A. Razborov (Moscow), Stanimir Troyanski (Sofia), and Lior Tzafriri (Jerusalem). Members of the organizing committee were Robert Deville (Bordeaux), Catherine Finet (Mons), Jean-Pierre Gossez (Bruxelles), C. Ward Henson (Urbana), Chris Impens (Gent), John Jayne (London), Alain Louveau (Paris 6), Christian Michaux (Mons), André Pétry (Liège), Gilles Pisier (Paris 6; Texas A & M), Jean Schmets (Liège), and Jan Van Casteren (Antwerpen).

This conference was the third of a cycle of conferences initiated by Catherine Finet at Mons; the previous ones were held in 1987 and 1992.

The meeting was a true success; more than one hundred and twenty mathematicians from all over the world participated. The main topics discussed at the meeting were the numerous connections between Analysis and Logic.

The purpose of this volume is to report the content of the three minicourses given by C. Ward Henson (Urbana), Alexander S. Kechris (Caltech) and Edward Odell (Austin), respectively.

Plenary lectures were presented by Nigel J. Cutland (Hull), Gabriel Debs (Le Havre, Paris 6), Lou van den Dries (Urbana), Richard Haydon (Oxford), Joram Lindenstrauss (Jerusalem), Donald A. Martin (Los An-

geles), Amos Nevo (Haïfa), David Preiss (London) and Nicole Tomczak-Jaegermann (Edmonton).

There were twenty-five contributed papers presented in parallel sessions.

A special issue of the journal *Annals of Pure and Applied Logic* was devoted to the meeting. It appeared in July 2001 as Volume 111/1–2 and contains papers by some of the plenary speakers and contributors.

The conference was supported financially by the Association Stefan Banach, the Centre National de la Recherche en Belgique, the Equipe d'Analyse de l'Université de Paris 6, the Fonds National de la Recherche Scientifique, the Fonds voor Wetenschappelijk Onderzoek, the Ministère de l'Education de la Communauté Française de Belgique, the International Association for the Promotion of Cooperation with Scientists from the Independent States of the Former Soviet Union (INTAS), The United Nations Educational, Scientific and Cultural Organization (UNESCO), the University of Mons-Hainaut, and the town of Mons. We add special mention of the Belgian National Bank, which supported the project Analysis and Logic for several years through its program for fundamental research.

Last but not least, our thanks to the departmental Secretaries Lyane Bouchez and Anne-Marie Saucez. They provided calm reason in moments of panic and kept the conference on an even keel throughout. Thanks also to all of those who helped us, especially to the juniors of the department, to Jacques Lion, and to Michèle Boffa.

The Editors
Catherine Finet, *Mons*
Christian Michaux, *Mons*

December 2001

Introduction

The articles in this book had their origins in three mini-courses offered at the conference "Analyse & Logique" held August 25–29, 1997, at the University of Mons-Hainaut in Mons, Belgium. For a long time there have been rich connections between analysis and logic; these articles bear witness that this relationship is still very active, and continues to be important for both areas.

Here we briefly describe these three articles; each one has a more detailed Introduction as its first chapter.

Part One: Ultraproducts in Analysis by C. Ward Henson and José Iovino

Applications of model theory in functional analysis have been pursued since the mid 1960s, beginning with the introduction of Banach space ultraproducts by Bretagnolle, Dacunha-Castelle, and Krivine, and of nonstandard hulls by Luxemburg. These constructions have been widely and successfully used in many parts of analysis. This paper presents the basic aspects of a systematic model theoretic framework within which these tools are naturally situated.

The logic developed here has its origin in the question: "what does a normed space structure have in common with its Banach space ultrapowers?" To give a precise answer it is necessary to introduce a suitable formal language of *positive bounded formulas* together with a semantics of *approximate satisfaction.* The resulting theory is developed here for a very general class of structures based on normed spaces; our treatment has the most general context possible within functional analysis,

in which the Banach space ultraproduct and nonstandard hull constructions apply.

The model theory for normed space structures that is described here has turned out to have many unexpected features and to be a close analogue of ordinary first order model theory. A large part of the basic aspects of this theory is outlined here. This model theory has been developed since the mid 1970s and has been used from time to time in applications of nonstandard analysis (especially of the nonstandard hull construction). By now it has shown itself to be the "correct" analogue of first order logic for the normed structures that arise in functional analysis.

In this article are discussed such basic model theoretic topics as elementary equivalence and elementary extension, logical compactness, Löwenheim-Skolem theorems, unions of chains, saturated and strongly homogeneous models, axiomatizability of classes of structures using positive bounded sentences, quantifier elimination, and spaces of model-theoretic types.

Many results in the literature regarding the structure of Banach space ultraproducts resonate immediately with the model theoretic ideas presented here. Some suggestive examples of this are given in various remarks throughout the paper.

Part Two:
Actions of Polish Groups and Classification Problems
by Alexander S. Kechris

This survey concerns the theory of definable actions of Polish groups, the structure and classification of their orbit spaces, and the closely related study of definable equivalence relations. This work is motivated by basic foundational questions, like understanding the nature of complete classification of mathematical objects by invariants, up to some notion of equivalence, and creating a mathematical framework for measuring the complexity of such classification problems. This theory, which has been growing rapidly over the last few years, is developed within the context of descriptive set theory, which provides the basic underlying concepts and methods. There are natural interactions of it with other areas of mathematics, such as the theory of topological groups, topological dynamics, ergodic theory and its relationships with the theory of operator algebras, model theory, and recursion theory.

Classically, in various branches of dynamics one studies actions of special groups, such as the additive groups of integers or reals, Lie groups, or, more generally, (second countable) locally compact groups. One of the goals of the theory is to expand the scope of this subject by considering the more comprehensive class of *Polish groups* (separable completely metrizable topological groups). One of the main problems concerning a given definable action of a Polish group G on a Polish space X is to give a complete classification by invariants of members of X up to orbit equivalence. (*Orbit equivalence* being the equivalence relation induced by the orbits of the action.) This is a special case of the more general problem of completely classifying elements of a given Polish space X up to some definable equivalence relation E on that space. This means finding a set of invariants I and a map $c\colon X \to I$ such that $xEy \Leftrightarrow c(x) = c(y)$; for this to have any significance, both I, c must be "explicit" or "definable" too. Typical examples of this kind of problem are: the classification of countable models of a theory up to isomorphism, the classification of the irreducible unitary representations of a locally compact group up to unitary equivalence, the classification of measure preserving transformations up to conjugacy, etc.

Chapters 2–7 of this paper give a survey of certain aspects of the program discussed in this introduction, but with very few technical details. Chapters 8–13 give a somewhat detailed technical exposition of Hjorth's recent theory of turbulence.

Part Three: On Subspaces, Asymptotic Structure, and Distortion of Banach Spaces; Connections with Logic by Edward Odell

A number of key problems in the theory of infinite dimensional Banach spaces, those that were central to the study of the general structure of a Banach space, finally yielded their secrets in the 1990's. This survey gives an extensive discussion of these problems and their solutions, as well as a presentation of important problems that remain open. At the foundation of this work are certain deep connections between Banach space theory and logic (especially set theory).

One of these connections involves Ramsey theory. This played a direct role in such results as Rosenthal's ℓ_1-theorem. More recently, *approximate* Ramsey theorems have emerged, especially as developed by Gowers leading to the proof of his spectacular Dichotomy Theorem. Generally

speaking, Ramsey theorems assert that a function into a finite set can be restricted to some sort of substructure on which it is constant. In an approximate Ramsey theorem one seeks a substructure where the function is nearly constant in some appropriate sense; this represents a joining of analysis and Ramsey theory.

Among the topics discussed in this survey are: the unconditional basic sequence problem and its connections with distortion; Gowers' block Ramsey theorem for Banach spaces; asymptotic structure, involving a type of logical connection between the finite dimensional (or local) structure of a space X and the infinite dimensional structure of X; certain ordinal indices constructed (e.g., by Szlenk and Bourgain) to study the structure of infinite dimensional Banach spaces; and the homogeneous Banach space problem. (i.e., if X is isomorphic to all of its infinite dimensional subspaces, must X be isomorphic to Hilbert space?)

The authors of this book are grateful to Catherine Finet and Christian Michaux and their colleagues in Mons, for having organized such an interesting meeting at the interface between analysis and logic, one of a series of such meetings. The authors hope that this book will give continuing recognition of the success of these meetings, and especially of their unusual, interdisciplinary character.

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December 2001

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