

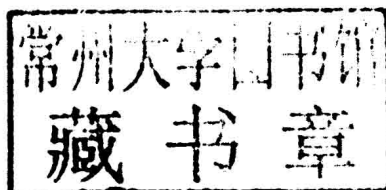
SOLUTIONS IN LIDAR PROFILING OF THE ATMOSPHERE

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WILEY

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PREFACE

Modern atmospheric science meets extremely complex and challenging problems, and thousands of researchers are enthusiastically looking for ways to find their solutions. The experimental methods and data are considered the basis for answering the scientific questions of human interest. However, the interpretation of the data remains an issue.

The most common way to interpret the experimental data in atmospheric science is by solving a number of related equations or a single equation with a number of unknowns. Unfortunately, any method of interpretation of the experimental data obtained in the real atmosphere requires assumptions. Many such assumptions have been transformed into implicit premises and now often go unmentioned. Reliable interpretation of the experimental data can have place only when all assumptions and implicit premises are met; this is where the devil hides.

The total uncertainty of the measured quantity of interest depends on how large the uncertainties in the involved parameters are. Each measurement task can only be solved within the limits established by the uncertainty of the involved elements. No way exists for determining the exact value of any atmospheric quantity of interest, only some likely value can be found. An accurate estimate of the uncertainty of an unknown quantity could only be found if the casual fluctuation of this quantity during its measurement obeyed some relatively simple laws so that existing error propagation theories can be applied. Generally, the total uncertainty includes two independent components, namely random uncertainty and systematic uncertainty. Until now, there has been no proper method for estimating systematic uncertainty, including that resulting from the often mandatory implementation of the *a priori* assumption or assumptions. Therefore, researchers do their best to, in some way, minimize systematic distortions before or during the experiment; this approach allows them to focus

on the random uncertainty. Analysis of the random phenomena still remains the main method for estimating the uncertainty in atmospheric studies. It is generally assumed that the laws governing the chance phenomena of interest are fixed in nature, and these laws are ultimately determined. In other words, it is assumed that the uncertainties obey simple “rules of the game.” The distribution of a random variable as a symmetrical or a nonsymmetrical bell-shaped graph is a typical example. The so-called normal distribution, named after the German mathematician Carl Friedrich Gauss (1777–1885), and the Poisson distribution, named after the French mathematician and physicist Simeon Denis Poisson (1781–1840), have remained the fundamental theoretical basis for the investigation of atmospheric processes for more than 200 years. Nothing new, at least with the same level of importance, has been proposed for error analysis since the nineteenth century.

Unfortunately, nature does not obey our relatively simple formulas. It obeys its own and much more complicated laws. All of our formulas are surrogates; they only approximate the atmospheric processes, proposing simplified solution schemes for these processes. It is quite rare when an accurate approximation is achieved by a simple formula, like the one for gravitational interaction or the famous Einstein formulas. A simple formula is generally valid only when the process is governed by a small number of influential parameters. In such cases, the influence of all other parameters is minor and even their significant variations do not change the essential characteristics of the process. Unfortunately, most processes in the atmosphere depend on a great number of parameters, whose variations during a measurement may have nothing in common with the relatively simple laws used in applied statistics. The actual fluctuations, and hence, the uncertainty distributions of the involved parameters are often unpredictable. No strict mathematics exist that would permit an exact evaluation of the reliability of the solutions obtained in the presence of nonstatistical uncertainties. Therefore, the assumption that relatively simple statistical laws govern chance phenomena is the compelling issue in atmospheric sciences. Meanwhile, statistical estimates may only be true under certain limited conditions, which are quite often not properly met in real atmospheres. In atmospheric physics, it is quite difficult to establish whether the phenomenon under investigation meets these conditions, and accordingly, to estimate the reliability of the applied statistics. The fluctuations of the involved unknowns may vary in an unpredictable way, often far from the assumed simple laws. The inappropriate use of statistics yields wrong conclusions, which unfortunately, often look extremely plausible and mislead both their authors and readers.

The simplest example of the doubtful use of statistics is when using temporal averaging of lidar signals during the vertical profiling of the atmosphere. The real atmosphere cannot be considered as horizontally homogeneous even in the statistical sense because the variations of the optical parameters do not obey any predictable statistical distribution. To overcome this issue, the more rigid assumption of a “frozen” atmosphere is commonly used. The time during which the atmosphere should remain “frozen” may change from some seconds to half an hour and more. This assumption is so common in lidar profiling of the atmosphere that it is rarely even mentioned in

the publications. In other words, such an assumption is now one among other implicit premises.

The principles of estimating uncertainties based on purely statistical models and conventional error propagation theory are inappropriate for investigating atmospheric processes with lidar. The conventional theoretical basis for random error estimates is very restrictive and requires rigid conditions, which are rarely satisfactorily met in real atmospheres and real lidar signals. First, the uncertainties of the involved parameters are often large, preventing the conventional transformation from differentials to finite differences used in standard error propagation. Second, the random errors of such parameters cannot always be accurately described by some simple distribution, such as Gaussian or Poisson. Third, the quantities used in lidar data processing can be correlated; the level of correlation often changes with the measurement range, and no reliable methods exist to determine the actual behavior of the uncertainty. Fourth, the measured atmospheric parameters may not be constant during the measurement period because of atmospheric turbulence, particularly for large averaging times used by deep atmospheric sounders. Apart from this, the total uncertainty can include any number of systematic errors of an unknown sign, constant, or variable that may cause large and often hidden distortions in the retrieved atmospheric profile.

The harsh reality is that instead of having truly concrete methods for uncertainty analyses, researchers are often playing a variant of the DADT game, "Don't ask about the systematic errors, — don't tell about these." A common justification for such play is the excuse that the statistical methods we have are the best that we have ...

Actually, the lidar researcher does not measure the optical parameters of the atmosphere; he measures only the sum of the backscatter signal and the background component. Therefore, today many lidar specialists avoid using the word "measurement" in their papers, or at least, in the title of their papers. Readers of scientific literature related to lidar searching of the atmosphere should notice that instead of using the term "measurement," many authors prefer using terms such as "monitoring," "profiling," "retrieving profiles," or "performing observations" of profiles of interest when describing their experimental results. These terms cloud reality, and I believe this is the proper time to dot our "i's" and cross our "t's." Truthfully, one should admit that today's lidar data processing technique implements more and more elements of a simulation rather than a measurement. In other words, lidar solutions should be considered as models, that is, simplified reflections of reality, which represent physical processes in some general way. As is known, modeling is typically used when it is impossible to create conditions in which one can accurately measure the parameter of interest. Models use assumptions and accumulated statistics while true measurements do not. Accordingly, numerical estimates in lidar observations made by using model dependencies will always be much less accurate than direct measurements, and this fact should be freely admitted. Only after such an admission can the appropriate elements of modeling technique in lidar searching be openly discussed. Triggering such a discussion is the basic goal of this book.

The term "profiling" can be defined as a reconstruction of a particular optical parameter of the atmosphere using the characteristics of the backscattered signal and some *a priori* assumptions based on statistics or sometimes just educated guesses.

The difference between measurement and profiling is that, unlike measurement, profiling gives only some general idea of the shape of the parameter of interest rather than precise details.

Comprehensive analysis shows that in any type of lidar profiling, the most significant errors occur during signal inversion, when the optical parameters of the atmosphere are extracted from the lidar signals using a number of implicit premises and *a priori* assumptions. Inverting the lidar signal, the researcher actually builds some simulation based on past lidar observations, some assumptions, implicit premises, some statistics, and finally, on the researcher's intuition and common sense. Under such conditions, the researcher can obtain only an estimate of the atmospheric profile of interest with an uncertainty that cannot be accurately quantified. In this book, instead of using the long phrase "simulation based on past lidar observations," I will also use the shorter phrase "a posteriori simulation."

Some methods used for profiling the aerosol atmosphere with lidar, as discussed in this book, have no rigid mathematical foundation; they are generally based, as the author believes, on common sense. Unfortunately, in the practice of atmospheric investigations, this is often the only way to interpret physical processes in the atmosphere in a meaningful way. Common statistics perform extremely poorly, for example, in smoky atmospheres, and this deficiency forced me to look for alternative ways for processing lidar data. Using alternative methods to invert lidar signals allows comparing results and estimating the credibility of different methods. The accuracy of the retrieved results cannot be estimated as with data that obey statistical laws. However, the use of alternative solutions gives one an estimate of how reliable the retrieved data are. This is the central premise of this book.

Fortunately, apart from the standard error-propagation procedure for statistical random errors, two alternative methods exist that allow investigating the effects of systematic and random errors without relying on common statistical laws. The first is a sensitivity study in which expected uncertainties in the involved quantities or likely signal distortions are used in numerical simulations in order to evaluate the distortion level in the output parameter of interest. In these simulations, a virtual lidar operates in a synthetic atmosphere, and its synthetic corrupted signals together with the selected *a priori* assumptions are used to retrieve the optical parameters of the atmosphere. Such a method may be used, for example, to analyze how an overestimated or underestimated backscatter-to-extinction ratio influences the accuracy of the extinction-coefficient profile extracted from elastic lidar data. To use this method, an analytical dependence may be obtained by combining and solving two inversion equations. The first equation is derived for the actual backscatter-to extinction ratio, used in the simulation, and the second is the solution obtained for the assumed incorrect ratio. Such an investigation is especially useful when making an error analysis for the case where large random or systematic errors are involved. This method provides a reasonable estimate of the total measurement uncertainty; it allows avoiding common underestimation of uncertainty when systematic distortions are ignored. The other method may be used when investigating the real lidar data, for example, the influence of the particular parameter taken *a priori*. This method may also be used to understand how an overestimated or underestimated backscatter-to-extinction ratio

influences the extinction-coefficient profile extracted from the real noisy signal in the real atmosphere under investigation.

Some recommendations in this book, which follow from such nonstandard methods of error estimation, cannot be unanimously justified. One cannot claim, for example, 68% confidence in retrieved data that includes uncertainty not treatable statistically. Considering the problem of combining random and systematic errors, Taylor (1997) wrote: "No simple theory tells us what to do about systematic errors. In fact, the only theory of systematic error is that they must be identified and reduced ... However, this goal is often not attainable ... There are various ways to proceed [the total uncertainty calculation]. None can really be rigorously justified ... Because the errors ... are surely independent ... , using the quadratic sum [of random and systematic uncertainties] is probably reasonable. The expression cannot really be rigorously justified ... Nonetheless, it does at least provide a reasonable estimate of our total uncertainty, given that our apparatus has systematic uncertainties we could not eliminate."

Defending the approaches and methods proposed in this book, I can only paraphrase Taylor by saying that there are no rigorous justifications for these methods except common sense. This principle of performing error analysis and estimation based on common sense is unavoidable and will remain the center of the author's attention in this book.

The book consists of three chapters. In Chapter 1, the basic issues of elastic-lidar-data inversion are discussed considering this task as a typical ill-posed problem. Chapter 2 discusses the specifics and the issues in separating the backscatter and transmission terms in the lidar equation. Chapter 3 considers the specifics of profiling the atmosphere with scanning lidar that operates in a multiangle mode. This book is intended for the users of atmospheric lidar, particularly newcomers who are starting their lidar investigations. The author believes that this book will allow them to see the real situation in remote sensing and current impassable restrictions in this area of atmospheric investigation.

An attentive reader will notice that the book contains a lot of repetition. The author has included such repetition deliberately. From his long experience, he knows that most readers of scientific books have neither the time nor the desire to read the book from cover to cover. Generally, they focus only on sections, in which the subject of their interest is discussed. Taking this into account, the author has tried to make the chapters and sections of the book as self-contained as possible. Therefore, the most specific and the most important points discussed in the book may be repeated in different sections, so that the reader has no need to jump from section to section to understand the points discussed in the section relevant to his or her interest.

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DEFINITIONS

α	Angstrom exponent
β	Total (molecular and particulate) scattering coefficient, $\beta = \beta_m + \beta_p$ (m^{-1} , km^{-1})
β_m	Molecular scattering coefficient (m^{-1} , km^{-1})
β_p	Particulate scattering coefficient (m^{-1} , km^{-1})
β_π	Total (molecular and particulate) backscatter coefficient, $\beta_\pi = \beta_{\pi,m} + \beta_{\pi,p}$ (m^{-1} steradian $^{-1}$)
$\beta_{\pi,m}$	Molecular backscatter coefficient (m^{-1} steradian $^{-1}$)
$\beta_{\pi,p}$	Particulate backscatter coefficient (m^{-1} steradian $^{-1}$)
β_{on}	Total (molecular and particulate) scattering coefficient at the DIAL wavelength λ_{on}
β_{off}	Total (molecular and particulate) scattering coefficient at the DIAL wavelength λ_{off}
$\beta_{\pi,\text{on}}$	Total (molecular and particulate) backscatter coefficient at the DIAL wavelength λ_{on}
$\beta_{\pi,\text{off}}$	Total (molecular and particulate) backscatter coefficient at the DIAL wavelength λ_{off}
$\beta_{\pi,R}$	Inelastic backscatter coefficient at the Raman shifted wavelength λ_R
δ_p	Distortion component of the lidar-signal multiplicative factor, ($1 + \delta_p$), caused by non-ideal transformation of the backscattered light into the output electrical signal

δ_{mult}	Ratio of the multiple scattering signal to the single scattering signal
ΔB	Remaining offset in the backscatter signal after removing the estimated constant, $\langle B \rangle$ from the total signal
$\Delta\sigma = \sigma_{\text{on}} - \sigma_{\text{off}}$	Differential absorption cross section of ozone for the <i>on</i> and <i>off</i> wavelengths
$\epsilon(h_{\text{max}})$	Criterion for the selection of the optimal maximum height, h_{max} , for the signal inversion
$\epsilon(\Delta h_i)$	Criterion for equalizing the alternative piecewise optical depths within a restricted interval, Δh_i
θ	Azimuthal angle of the scanning lidar
κ	Total (molecular and particulate) extinction coefficient, $\kappa = \kappa_m + \kappa_p$ (m^{-1} , km^{-1})
$\kappa_{p,0}$	Particulate extinction coefficient at the wavelength λ_0 of the emitted light of the Raman lidar
$\kappa_{p,R}$	Particulate extinction coefficient at the Raman shifted wavelength λ_R
$\kappa_{m,0}$	Molecular extinction coefficient at the wavelength λ_0 of the emitted light of the Raman lidar
$\kappa_{m,R}$	Molecular extinction coefficient at the Raman shifted wavelength λ_R
κ_{gr}	Total extinction coefficient at ground level
$\kappa^{(\text{dif})}$	Total extinction coefficient determined through numerical differentiation
$\kappa_p^{(\text{dif})}$	Particulate extinction coefficient determined through numerical differentiation
$\kappa_p^{(i)}(h)$	Particulate piecewise extinction coefficient within a segmented interval versus height
$\kappa_{p,j}^{(i)}$	Piecewise particulate extinction coefficient in slope direction
κ_w	Transformed extinction coefficient in the elastic lidar solution
λ	Wavelength of the emitted and backscattered light of the elastic lidar
λ_0	Wavelength of the emitted light of the Raman lidar
λ_R	Wavelength of the Raman shifted signal
λ_{on}	Wavelength within the enlarged absorption spectrum of ozone used in the DIAL profiling of the ozone concentration
λ_{off}	Wavelength outside the enlarged absorption spectrum of ozone used in the DIAL profiling of the ozone concentration
Λ	Criterion for minimizing the difference between the alternative transmission profiles within a segmented interval
Π_p	Particulate backscatter-to-extinction ratio (steradian^{-1})

Π_m	Molecular backscatter-to-extinction ratio (steradian ⁻¹)
$\Sigma\tau_p^{(0,R)}$	Total of the particulate optical depths at the wavelength, λ_0 , emitted by the laser, and at the Raman shifted wavelength, λ_R
Σw_j	Sum of the low- and high-frequency noise components in the lidar signal
$\Sigma w_{j, \text{low}}$	Sum of the low frequency noise components that remains in the signal after its smoothing
$\Sigma\kappa_p^{(0,R)}$	Sum of the particulate extinction coefficient at the wavelength, λ_0 , emitted by the laser, and at the Raman shifted wavelength, λ_R
$\frac{d\sigma_{\pi,R}}{d\Omega}$	Range-independent differential Raman backscatter cross section
ζ	Correction factor in Raman lidar equation corresponding to the variable Angstrom coefficient
ζ_0	Correction factor in Raman lidar equation corresponding to the assumed constant Angstrom coefficient
τ	Total (molecular and particulate) optical depth, $\tau = \tau_m + \tau_p$
τ_m	Molecular optical depth
τ_p	Particulate optical depth
τ_0	Total (molecular and particulate) optical depth at the emitted laser wavelength, λ_0
τ_R	Total (molecular and particulate) optical depth at the Raman shifted wavelength, λ_R
τ_{90}	Total (molecular and particulate) optical depth determined directly in zenith; $\tau_{90} = \tau_{p,90} + \tau_{m,90}$
τ_{mod}	Model optical depth used for the extrapolation of the optical depth derived with lidar down to the ground level
τ_{vert}	Total (molecular and particulate) optical depth in the vertical direction determined from the multiangle data of scanning lidar
τ_{sh}	Shaped total optical depth, which increments versus height or range are either positive or equal to zero
$\tau_p^{(i)}$	Particulate piecewise optical depth
$\tau_{p, \text{up}}$	Estimated upper limit of the shaped optical depth, $\tau_{p, \text{sh}}$
$\tau_{p, \text{low}}$	Estimated lower limit of the shaped optical depth, $\tau_{p, \text{sh}}$
v	Distortion component of the optical depth originated in additive and/or multiplicative components in the lidar signal
v_{rand}	Random noise component in the distorted optical depth profile
v_{sys}	Systematic distortion component in the distorted optical depth profile
φ	Elevation angle of vertically scanning lidar
χ	Fixed levels of the ratio function, $R_Y(h)$ (in zenith profiling) or $R_{\theta, \text{max}}(h)$ (in multiangle profiling) used for determining maximal height of the atmospheric layer with increased backscattering

a_π	Ratio of the particulate to the molecular lidar ratio, S_p/S_m
$A(h)$	Interception point of the linear fit, $Y(h)$ with y-axis at the height, h
B	Range-independent offset in the recorded lidar signal, which estimated value is $\langle B \rangle$
C	Lidar solution constant, its estimated value is $\langle C \rangle$
$EF[\delta f(x), \delta x]$	Error factor which is defined as the absolute value the ratio of the fractional error of the output function, $\delta f(x)$, to the fractional error of the input element, δx
$f_m(h)$	Temperature and pressure dependent attenuation factor for the Cabannes spectrum in the HSRL equation
f_p	Rejection ratio of the scattered light from particulates in the HSRL equation
h	Height from ground level to the scattering volume
h_{\min}	Minimum height used for the lidar signal inversion
h_{\max}	Maximum height used for the lidar signal inversion
h_b	Reference height for which an assumed boundary condition for the lidar equation solution is taken
h_s	Middle point of the interval from h to $h + s$, where s is the range resolution used for numerical differentiation
$I(r_a, r)$	Integral of the square range-corrected signal
n_{ozone}	Ozone concentration (ppb)
$N_R(T, p)$	Atmospheric number density of the Raman scattering molecules as a function of temperature (T) and pressure (p)
P_Σ	Signal recorded by lidar, which is a sum of a backscatter signal P and a constant offset, B
$P(r)$	Backscatter signal not distorted by a multiplicative and/or an additive component
$\langle P(r) \rangle$	Distorted backscatter signal used for the inversion
P_{on}	Backscatter signal of the DIAL measured at the wavelength λ_{on}
P_{off}	Backscatter signals of the DIAL measured at the wavelength λ_{off}
P_{90}	Backscatter signal in the zenith direction
$P_{\Sigma, 90}$	Total signal in the zenith direction
P_m	Backscatter signal at the output of the molecular channel of High Spectral Resolution Lidar
P_j	Backscatter signal measured under the slope angle φ
P_R	Raman signal at the shifted wavelength, λ_R
q	Overlap function of the emitted laser light beam and the cone of the receiver telescope field of view
q_{eff}	Effective overlap function determined in the multiangle mode

r	Range at which the lidar signal is considered
r_0	Distance from the lidar to the nearest point of the complete overlap area
r_b	Boundary (reference) point for the lidar signal inversion
r_{\min}	Minimum range used for the lidar signal inversion
r_{\max}	Maximum range used for the lidar signal inversion
s	Range resolution in numerical differentiation
SOR	Signal-to-offset ratio
S_m	Molecular lidar ratio (steradian)
S_p	Aerosol lidar ratio (steradian)
$\overline{S_p(h_i, h)}$	Column-integrated particulate lidar ratio over the altitude range h_i-h
$S_p^{(i)}$	Piecewise range-independent particulate lidar ratio within a restricted interval
$T_{\Sigma}^2(0, h)$	Two-way total (molecular and particulate) transmittance from ground level to the height, h ; $T_{\Sigma}^2(0, h) = T_p^2(0, h)T_m^2(0, h)$
$T_{90}^2(0, h)$	Two-way total (molecular and particulate) transmittance in the zenith direction
$T_m^2(0, h)$	Two-way molecular transmittance from ground level to the height h
$T_p^2(0, h)$	Two-way particulate transmittance from ground level to the height h
$T_0(0, r)$	One-way total (molecular and particulate) transmittance versus range in the Raman measurement at the laser wavelength, λ_0
$T_R(0, r)$	One-way total (molecular and particulate) transmittance versus range at the Raman shifted wavelength, λ_R .
$T_{\Sigma, j, \text{vert}}^2(0, h)$	Two-way total vertical transmittance determined from the signals measured along the set of slope angles, φ
$\overline{T_{\Sigma, \text{vert}}^2(0, h)}$	Average two-way total transmittance in the vertical direction determined from the multiangle data
$\overline{T_{p, \text{vert}}^2(0, h)}$	Average two-way particulate transmittance in the vertical direction determined from the multiangle data
$w(r_m, r_n)$	Weight function for the calculation of the extinction coefficient within the overlapping interval r_m-r_n using alternative two-way transmittance profiles
$w(h_i, h_{i+1})$	Weight function for the calculation of the extinction coefficient within the interval h_i-h_{i+1} using two alternative optical depth profiles
x	Independent variable in the Kano-Hamilton solution, uniquely related with the slope direction, φ [$x = (\sin \varphi)^{-1}$]